École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-621 Microeconomics (Prof. Weber)

Problem Set 5

Autumn 2021

Issued: Wednesday, September 22, 2021	**Due:** Monday, September 27, 2021

Problem 5.1 (Cournot Oligopoly) Consider the case when n firms *simultaneously* choose their production quantities $q_1, \ldots, q_n$ in a market for undifferentiated widgets. The equilibrium price p is determined by a linear inverse demand curve,

$$p(q) = [a - bQ(q)]_+ ,$$

where $a, b > 0$ are constants, $Q(q) = q_1 + \ldots + q_n$ is the aggregate production quantity of widgets, and $q = (q_1, \ldots, q_n)$ is the vector of firm outputs. Assume for simplicity that the firms' unit production costs are identical and equal to $c > 0$ and that $c < a$.

(i) Find the firms' best-response correspondences $q_i^*(q_{-i})$ and the unique NE tuple q^* as a function of n. What can you say for $n \to \infty$?

(ii) Do firms have an incentive to collude and/or to merge? Explain for the Cournot duopoly (i.e., for $n = 2$).

(iii) Determine the socially optimal choice(s) of q and compare to your findings in (i) for the noncooperative Cournot oligopoly game.

(iv) Provide intuition on how you could obtain a Cournot duopoly which does not have a unique NE. What would that imply in practice? (A well-explained picture would be enough here, but you can also try to modify the functional form of $p(q)$ and do everything explicitly.)

Problem 5.2 (Horizontal Product Differentiation) Consider two sellers of widgets who at time $t = 0$ can decide where to locate on a street of unit length, i.e., each seller $i \in \{1, 2\}$ simultaneously chooses a location $x_i \in I = [0, 1]$ and sets the product price to one. $N > 0$ consumers are uniformly distributed on I and at time $t = 1$ each consumer goes to the seller located most closely. Assume without loss of generality that the sellers' location choices are such that $x_1 \leq x_2$.

(i) Determine seller i's payoffs as a function of $x = (x_i, x_{-i})$.

(ii) Find all NE of the game.

(iii) Based on your findings in part (ii) explain Hotelling's "principle of minimum differentiation."

Problem 5.3 (Tragedy of the Commons) Consider a lake that can be accessed freely by fishermen. The cost of sending out a boat on the lake is $r > 0$. When $b \geq 0$ boats are sent out onto the lake, $f(b) = \sqrt{b}$ fish are caught in total. Assume that the market price p of fish remains unaffected by the level of catch from this lake.

(i) Characterize how many boats are sent out onto the lake in equilibrium. [Hint: assume that there is free entry.]

(ii) Characterize the socially optimal number of boats on the lake, and compare the outcome with your result in part (i).

(iii) Describe different ways in which the government could intervene to implement an efficient (i.e., welfare-maximizing) outcome.

(iv) What per-boat fishing tax would restore efficiency?

Problem 5.4 (Positional Goods and Prestige) Consider a continuum of agents indexed by a prestige parameter or 'type' $\theta \in [0,1] = \Theta$. We assume that the agents' types are uniformly distributed on the 'type space' Θ. Each agent can build at most one house, either in neighborhood 1 where it costs $c_1 = 1/8$ to build, or in a more expensive neighborhood 2 where building costs are $c_2 = 7/8$. The utility of an agent of type θ who builds in neighborhood $k \in \{1, 2\}$ is given by

$$u(\theta, k) = (1 + \theta)(1 + \bar{\theta}_k) - c_k,$$

where the 'prestige' $\bar{\theta}_k$ corresponds to the (arithmetic) *average* of all types who decide to build in neighborhood k.

(i) Show that in an equilibrium (where all consumer types make their location-choice decisions simultaneously) both neighborhoods must be occupied. [Hint: you can show this by contradiction.]

(ii) Show that in an equilibrium there exists exactly one marginal consumer type $\theta_m \in (0, 1)$ who is indifferent between the two neighborhoods, and that consumers of type $\theta < \theta_m$ choose the cheap neighborhood to build their houses.

(iii) Determine the prestige levels in equilibrium, and the utilities of all consumers as a function of θ.

(iv) Is it possible to Pareto-improve the equilibrium outcome in part (iii)? Explain.

(v) How would a social planner implement the location choices to maximize the sum of all consumers' utilities? Compare your answer with the equilibrium outcome.

(vi) What instruments could a social planner use to implement the efficient outcome in part (v) as an equilibrium outcome, i.e., an outcome that would result if all agents choose their locations simultaneously in a self-interested way?