

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

College of Management of Technology

MGT-621 MICROECONOMICS (PROF. WEBER)

Problem Set 4

Autumn 2023

Issued: Wednesday, September 20, 2023

Due: Friday, September 22, 2023

Problem 4.1 (Market Power and Monopoly) Consider a firm with market power that is producing a single good. The demand and cost functions of the good are given by $D(p) = [5 - 2p]_+$ and $C(q) = 10 + q/4$, respectively, where $q \geq 0$ denotes the quantity and $p \geq 0$ is the price of the good.

- (i) Find the inverse demand function $p(q)$.
- (ii) Find the quantity that the firm chooses to supply.
- (iii) Find the market price of the good.
- (iv) Is the market price smaller, equal or bigger than the marginal cost? Explain.
- (v) Find the competitive output.
- (vi) Determine the Lerner index at the monopolist's optimum output.
- (vii) Find the 'deadweight loss' (DWL) both graphically and analytically.

Problem 4.2 (Lerner Index) Consider a firm that produces one good with the cost function $C(q) = \alpha + \beta \log q$, where $q \geq 0$ denotes the output. Assume that the price of that good is given by p . Find the values of the output q , for which ...

- (i) ... the firm has maximum market power;
- (ii) ... the firm has no market power.

Problem 4.3 (Market Power and Price Discrimination) Consider a market for widgets which are being sold to women and men alike. Given a price $p > 0$, the total demand by women for these widgets is $q_w(p) = a - b_w p$ and the total demand by men is $q_m(p) = a - b_m p$, where $a > 0$ and $0 < b_w < b_m$. The unit production cost for widgets is $c > 0$.

- (i) If the market for widgets is competitive, find the equilibrium price and quantity sold. For all following parts we assume that there is a single monopolist provider of widgets.
- (ii) Find the profit-maximizing price p^* if the monopolist cannot price discriminate among different consumers. Compare your result to your answer in part (i) and discuss. [Hint: is it always optimal to serve both markets?]

- (iii) Given a total output Q of widgets, what would be the welfare-maximizing way to distribute this output between all consumers?
- (iv) What are the profit-maximizing prices p_w^* and p_m^* if the monopolist is able (and permitted) to charge different prices to women and men for the widgets? Compare your result to your answer in part (iii) and discuss. [Hint: see the hint in part (ii).]

Problem 4.4 (Optimal Insurance) Joe is a risk-averse individual with initial wealth $w = 1000$. He faces a probability θ of suffering a loss of size $L = 500$. You work for a monopolist insurance company and would like to offer Joe an insurance contract of the form (c_1, c_2) , where c_1 is Joe's leftover wealth in case there is no loss and c_2 is the amount Joe has left to consume, if a loss occurs. The insurance premium in the event of no loss is therefore $w - c_1$ and the net payment received from the insurance if a loss occurs, is $c_2 - (c_1 - L)$. Joe's preferences over nonnegative wealth levels $x \geq 0$ are represented by the utility function $u(x) = \sqrt{x}$.

- (i) Find the first-best insurance contracts (each of the form (c_1^{FB}, c_2^{FB})) as a function of θ when the loss probability θ is observable by all parties.
- (ii) Unfortunately you have no way of exactly knowing θ as it pertains to Joe's private information. For simplicity assume that $\tilde{\theta} \in \{\theta_L, \theta_H\}$ with $\theta_H = 3/4$ and $\theta_L = 1/4$ and a prior belief of $\text{Prob}(\tilde{\theta} = \theta_H) = \mu \in [0, 1]$. What are the the optimal second-best contract offerings of the form (c_1^*, c_2^*) that you can offer? Draw a picture in (c_1, c_2) -space. How does your solution in part (ii) vary with μ ?
- (iii) Consider your solution in part (ii). Is one type of the individual being "rationed" in terms of being able to acquire insurance? Describe. Comment also on the externality between the two types.
- (iv) Determine the information rent in part (ii) as a function of μ . Who gets it?