École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-621 Microeconomics (Prof. Weber)

Problem Set 3

Autumn 2023

Issued: Friday, September 15, 2023	**Due:** Wednesday, September 20, 2023

Problem 3.1 (Risk Attitude) Nadine has wealth $w = 0$ and a money lottery which pays an uncertain amount $\tilde{y} = [1/2, y_1; 1/2, y_2]$, where the outcomes are $y_1 = 0$ and $y_2 = -16$. Her utility function for any monetary outcome y is given by $u(y) = \sqrt{(16 + y)_+}$.[1]

(i) Determine Nadine's certainty equivalent $\text{CE}(\tilde{y})$ of the money lottery $\tilde{y}$. Sketch the solution graphically as well.

(ii) Determine Nadine's risk premium $\pi(\tilde{y})$ of the money lottery $\tilde{y}$. Sketch the solution graphically as well.

(iii) Provide a utility function $\hat{u}$ that would make Nadine more risk averse than she currently is.

(iv) Show that Nadine is not always risk averse. Provide a money lottery $\tilde{z} = [p_1, z_1; p_2, z_2]$ that is "riskier" than $\tilde{y}$ but for which Nadine has a higher expected utility than for $\tilde{y}$.[2]

(v) Provide a practical real-world example where agents have "kinked" utility functions similar to the representation of Nadine's preferences. Can you imagine a consequence of this in financial markets, especially in light of your answer in part (iv)?

Problem 3.2 (Portfolio Investment) Consider an investor with wealth $w > 0$ who can invest an amount $a \in [0, w]$ into a risky asset A of random return $\tilde{r}_A \in \{r_1, r_2\}$, where $r_1 < 0 < r_2$. The probability that return r_i realizes is p_i for $i \in \{1, 2\}$, where $p_1 + p_2 = 1$. Any amount not invested in the risky asset earns a guaranteed return of zero. We assume that the expected return of the asset is positive, i.e., that $E[\tilde{r}_A] = p_1 r_1 + p_2 r_2 > 0$, and that the investor's utility function is of the form $u(x) = 1 - \exp(-\rho x)$ for some constant absolute risk-aversion coefficient $\rho > 0$. (i) Show that if the investor invests a in the risky asset, then her ex-post wealth is a lottery of the form $L(a) = [p_1, w + a r_1; p_2, w + a r_2]$. (ii) Determine the investor's certainty equivalent $\text{CE}(a)$ of the lottery $L(a)$ and the corresponding risk premium $\pi(a)$. Interpret certainty equivalent and risk premium. How do they relate to the asset's expected return? (iii) Determine the investor's expected-utility-maximizing choice of $a \in [0, w]$. (iv) Can it ever be optimal to invest nothing (everything) in the risky asset? Explain. (v) Provide a return distribution $\tilde{r}_B = [\hat{p}_1, \hat{r}_1; \hat{p}_2, \hat{r}_2]$ of an asset B that first-order (second-order) stochastically dominates asset A. Would the investor prefer asset B over asset A? How would her optimal investment change?

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[1] The *nonnegative part* of a real number $x \in \mathbb{R}$ is given by $[x]_+ = \max\{0, x\}$.

[2] We call a random variable $\tilde{z}$ "riskier" than random variable $\tilde{y}$ if $E[\tilde{y}] = E[\tilde{z}]$ and $\tilde{y}$ second-order stochastically dominates $\tilde{z}$, i.e., $\tilde{z} \preceq_{\text{SOSD}} \tilde{y}$. In other words, $\tilde{z}$ is riskier than $\tilde{y}$ if it results from $\tilde{y}$ by a mean-preserving spread.

Problem 3.3 (Production Technology) Consider a price-taking firm with a production possibilities set

$$Y = \left\{(-z_1, -z_2, q) : (z_1, z_2, q) \in \mathbb{R}^3_+, z_1^\alpha z_2^\beta \geq q\right\},$$

where α, β are positive constants, such that $\alpha + \beta < 1$. (i) Determine the firm's production function. (ii) Given a price vector $(w_1, w_2, p) \gg 0$, determine the firm's cost $C(q)$ as a function of its output $q \geq 0$. (iii) Does the firm's production technology exhibit increasing/decreasing/constant returns to scale? Explain. (iv) Determine the firm's profit-maximizing production vector $y^* \in Y$. (v) How does y^* change if the price p for the end product increases? (vi) [Bonus] How do your answers in parts (i)–(v) change if the firm must use at least one unit of input 1, i.e., if its production possibilities set is of the form

$$\hat{Y} = \left\{(-z_1, -z_2, q) : (z_1, z_2, q) \in \mathbb{R}^3_+, z_1^\alpha z_2^\beta \geq q, z_1 \geq 1\right\}?$$

How can this additional requirement be interpreted and how could this arise in practice?

Problem 3.4 (Production and Learning) In some industries cost reductions are achieved over time, simply by learning. Consider a monopolist that produces widgets over two time periods, $t \in \{1, 2\}$. In both periods the demand for the widgets is given by the demand curve

$$D(p_t) = a - p_t$$

for some $a \in (0, 1)$. The monopolist's cost at time $t = 1$ is $C_1(q_1) = cq_1$, where $c \in (0, a)$ is some constant. The cost at time $t = 2$ is $C_2(q_1, q_2) = c(1 - \gamma q_1)q_2$ for some $\gamma \in [0, 1]$. In other words, the cost in period 2 decreases in the production in period 1, which is a "learning curve" effect. The monopolist maximizes the total discounted profit,

$$\Pi(q_1, q_2) = q_1 p_1 - C_1(q_1) + \delta\left(q_2 p_2 - C_2(q_1, q_2)\right),$$

where $\delta \in (0, 1)$ is some discount factor and p_1, p_2 are the market prices in periods 1 and 2.

(i) Determine the firm's inverse demand curve, $p_t(q_t)$, for the two periods $t \in \{1, 2\}$.

(ii) Given some first-period production level $q_1 > 0$, determine the firm's optimal output $q_2^*(q_1)$ at time $t = 2$. Is that output increasing or decreasing in q_1? Explain.

(iii) Using your answer in part (ii) maximize the firm's profit $\Pi(q_1, q_2^*(q_1))$ with respect to q_1, and find the monopolist's profit-maximizing production plan (q_1^*, q_2^*). [Assume that $0 < c < 2/(\gamma\sqrt{\delta})$.]

(iv) [Bonus] How does q_1^* change in the parameters a, γ, δ (i.e., does it increase or decrease)? What happens as $\gamma \to 0+$? [Hint: this question can be answered without having computed the optimal solution. Intuitive answers are sufficient for this part; rigorous answers earn another bonus point.]

(v) [Bonus] Compare the prices p_1^*, p_2^* under monopoly with the prices p_1^c, p_2^c that would obtain in a competitive market, where prices are set equal to marginal cost.