ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

College of Management of Technology

MGT-621 MICROECONOMICS (PROF. WEBER)

Problem Set 2

Autumn 2023

Issued: Wednesday, September 13, 2023	**Due:** Friday, September 15, 2023

Problem 2.1 (Revenue Maximization and Consumer Surplus) You are in charge of setting the entry price to the new community pool. The monthly demand, $D(p)$, as a function of entry price p (in dollars) is $D(p) = 60 - 10p$.

(i) How many people would attend the pool each month if it was free?

(ii) How much revenue would be generated each month if the entry price was three dollars?

(iii) What loss of consumer surplus is associated with the price in part (ii)? (Compare consumer surplus at a price of three dollars to the consumer surplus at a price of zero.)

(iv) Find the revenue-maximizing price p^*, the associated optimal revenue R^*, and the resulting consumer surplus CS^*.

(v) Imagine now that the monthly demand $D = D_1 + D_2$ for the pool is in fact composed of children, who have the demand function $D_1(p) = 40 - 8p$, and adults, with demand function $D_2(p) = 20 - 2p$. Can you increase revenues for the community pool by charging different prices for children and adults? Determine a revenue-maximizing set of prices, and compare the associated revenue and consumer surplus to your results in part (iv). Explain your solution by drawing a diagram (in addition to your computations). [Hint: make sure that demand stays nonnegative.]

Problem 2.2 (Hicksian Welfare Measures) Rudy likes Italian food. He spends an amount of y each week on consuming pizza (x_1) and pasta (x_2). His overall enjoyment of the food depends on the quality q of the music that his neighbor plays in the evening. (Rudy does not own a stereo system and so during dinner always opens his window and listens to whatever music his neighbor is listening to.) The quality of the music ranges from zero to one (i.e., $q \in [0, 1]$), and Rudy's preferences are represented by the utility function

$$u(q, x) = \left(x_1^{\alpha} x_2^{1-\alpha}\right)^{1-q}.$$

(i) For a given level of $q \in (0, 1)$, and a given vector $p = (p_1, p_2) \gg 0$ of prices for pizza and pasta, determine Rudy's Walrasian demand vector $x(p, y)$. Does it also depend on q? Explain.

(ii) Find Rudy's indirect utility $v(p, q, y)$.

(iii) For $\alpha = 1/4$ and $p = (1, 2)$, what is Rudy's compensating variation $C(y)$ (i.e., his willingness to pay) for an increase in his neighbor's music quality from $q_0 = 1/4$ to $q_1 = 3/4$? What is Rudy's equivalent variation $E(y)$ for a decrease in music quality from q_1 to q_0?

(iv) Explain any differences in the welfare measures $C(y)$ and $E(y)$ (or lack thereof) that you may have encountered in part (iii).

(v) Based on part (iv), imagine two incarnations of Rudy, called Ann and Bert. Ann currently listens to good music of quality q_1. Bert currently listens to bad music of quality q_0. Is there an amount, τ, such that Bert is willing to pay τ to trade his place for Ann's, and Ann is willing to trade her place for Bert's upon receiving τ? Why or why not?

(vi) After Rudy pays his willingness to pay $C(y)$ to his neighbor for changing the music from q_0 to q_1, how much money $\hat{y}$ has he left for Italian food? Given that Italian-food budget, what is his equivalent variation $E(\hat{y})$ (i.e., his willingness to accept) from his neighbor to change the music back from q_1 to q_0? (The neighbor actually likes q_0 better than q_1, for her quality perception is exactly opposite Rudy's.)

(vii) Explain any differences in the welfare measures $C(y)$ and $E(\hat{y})$ (or lack thereof) that you may have encountered in part (vi).

Problem 2.3 (Welfare Impact of a Price Change) Consider again Rudy's consumption of pizza and pasta in the last problem, and assume that he has convinced his neighbor to set $q = q_1$ permanently (by offering to mow her lawn for free in return for the favor). The price of pasta in the supermarket has recently been cut in half. We now investigate the change in Rudy's welfare as a consequence of the price change from $p = (1, 2)$ to $\hat{p} = (1, 1)$.

(i) Determine Rudy's expenditure function $e(p, U)$ by solving his expenditure minimization problem for any given price vector $p = (p_1, p_2) \gg 0$ and utility level $U > 0$. What is Rudy's Hicksian demand $h(p, U)$?

(ii) Determine Rudy's compensating variation $C(y)$ and his equivalent variation $E(y)$ for the price change from p to $\hat{p}$. Explain any differences (or lack thereof) between the two welfare measures.

(iii) Determine the change in Rudy's consumer surplus $\Delta\text{CS}(y)$ as a consequence of the shift from p to $\hat{p}$.

(iv) Can you order the measures $C(y)$, $E(y)$, and $\Delta\text{CS}(y)$ in terms of their magnitudes? Does your answer depend on Rudy's Italian-food budget $y > 0$? Explain.

(v) In your own words, can you explain any problems that might arise when trying to compute changes in consumer surplus as a consequence of price changes. Why do these problems disappear when using compensating and/or equivalent variation as welfare measures?