École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-621 Microeconomics (Prof. Weber)

Problem Set 1

Autumn 2023

Issued: Monday, September 11, 2023	**Due:** Wednesday, September 13, 2023

Problem 1.1 (Walrasian and Hicksian Demand) John and Jane each have a budget of one hundred dollars to spend on apples and bananas at the local farmers market. Apples and bananas cost one dollar each. John and Jane have different preferences, represented by the utility functions

$$u_{\text{John}}(x, y) = \sqrt{x^2 + y^2} \quad \text{and} \quad u_{\text{Jane}}(x, y) = xy,$$

respectively, where x denotes the number of apples in an individual's basket, and y the number of bananas in that basket. We will now determine utility-maximizing consumption bundles (x, y) for John and Jane as a function of market prices and budget constraints.

(i) Without explicitly solving a utility maximization problem explain why $(10, 80)$, i.e., ten apples and eighty bananas, cannot be a utility-maximizing bundle for either John or Jane.

(ii) Explain (or prove) why $\hat{u}_{\text{John}}(x, y) = x^2 + y^2$ can be used as an alternate utility function for John.

(iii) Determine the budget set $B(p, w)$, where $w = 100$ is an individual's budget and $p = (1, 1)$ is the vector of fruit prices. Draw this set in the commodity space.

(iv) Sketch John and Jane's indifference curves that pass through the point (10,80).

(v) Solve for the utility maximization problem for John and Jane. In particular, show that John has two optimal bundles and demonstrate that both John and Jane are worse off if they trade their optimal baskets. Describe, in words, the nature of John and Jane's preferences: what ratios of apples to bananas do they prefer most and which do they like least?

(vi) If we remove either one apple or one banana from Jane's optimal consumption bundle, how many of the other fruit must we offer to her to compensate her for this loss. [Hint: determine first Jane's marginal rate of substitution at the optimal consumption bundle, and then relate it to the corresponding price ratio.] Repeat this exercise for John, and explain any discrepancy or lack thereof.

(vii) We now consider the more general case when John and Jane each have a positive amount w to spend on fruit. Determine their optimal consumption bundles as a function of w, and sketch the corresponding Engel curves (a.k.a. 'wealth expansion paths').

(viii) The banana vendor is considering changing her prices (from one dollar) to p_B. If $p_B > 1$, then John has a unique optimal bundle. Determine this bundle.

(ix) Determine Jane's Walrasian demand (i.e., her optimal consumption bundle) $(x^*(p, w), y^*(p, w))$, as a function of the fruit price vector $p = (p_A, p_B) \gg 0$, where p_A is the price of an apple, p_B is the price of a banana, and her budget is $w > 0$. Are apples and bananas substitutes or complements for Jane? Explain.

(x) Now find Jane's Hicksian demand $(h_A^*(p, U), h_B^*(p, U))$ for a utility level $U > 0$ by minimizing the expenditure needed to attain at least this utility level. What is the relation between her Hicksian and her Walrasian demand when $U = u_{\text{Jane}}(x^*(p, w), y^*(p, w))$? Explain.

Problem 1.2 (More on Walrasian Demand) Given a consumption set $X = \left\{x \in \mathbb{R}_+^3 : x \geq b\right\}$ and a continuously differentiable utility function $u : X \to \mathbb{R}$, with

$$u(x) = (x_1 - b_1)^\alpha (x_2 - b_2)^\beta (x_3 - b_3)^\gamma,$$

and $b = (b_1, b_2, b_3) \gg 0$ as well as $\alpha, \beta, \gamma > 0$, consider the standard utility maximization problem

$$\max_{x \in X} u(x), \quad \text{s.t.} \quad p \cdot x \leq w,$$

where $p = (p_1, p_2, p_3) \gg 0$ corresponds to the (constant) price vector for any commodity bundle $x = (x_1, x_2, x_3) \in X$ and $w > p \cdot b$ is the consumer's wealth. Assume that the base consumption vector $b = (b_1, b_2, b_3)$ lies in the interior of the budget set, so that $0 < p \cdot b < w$. (i) Show that without loss of generality we can assume that $\alpha + \beta + \gamma = 1$. Use this relation in the following parts. (ii) Write down the first-order necessary optimality conditions for a solution of the utility maximization problem. (iii) Derive the Walrasian demand vector $x^*(p, w)$. Show that it is homogeneous of degree zero, and satisfies Walras' law.

Problem 1.3 (Excise vs. Income Taxes) A consumer has a Cobb-Douglas utility function $u(x) = x_1^\alpha x_2^{1-\alpha}$ (with some constant parameter $\alpha \in (0, 1)$) for two-good bundles $x = (x_1, x_2)$, and an income of $w > 0$.

(i) Given any price vector $p = (p_1, p_2) \gg 0$, determine the consumer's Walrasian demand vector $x^*(p, w)$.

(ii) How does the consumer's Walrasian demand change when an excise tax is imposed on good 1 (i.e., instead of p_1, good 1 now costs $p_1 + t$ for some $t > 0$)?

(iii) How does the consumer's Walrasian demand change when an income tax of T dollars is imposed (where $0 < T < w$)?

(iv) Determine the government's tax revenue from imposing either an excise tax or an income tax.

(v) Show that when the government's tax revenues from an excise tax and an income tax are equal, the consumer is strictly worse off facing the excise tax. Explain. [Hint: a graphical analysis in this and the next part is acceptable.]

(vi) [Bonus] Determine a class of utility functions, for which the finding in part (v) is valid.