MGT-621 Microeconomics

Value of a Call Option

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A risk-neutral agent owns a European call option on an asset. The value of the underlying asset at the exercise date is described by a random variable $\tilde{x}$ that takes values in $[a, b]$. The strike price of the option is equal to $y \in [a, b]$. We show that the agent prefers an *increase in risk* in the value of the underlying asset. Indeed, the (risk-neutral) agent's expected payoff is $E[\tilde{x} - y]_+$. More specifically, consider two random variables $\tilde{x}$ (with cdf F) and $\tilde{z}$ (with cdf G) on $[a, b]$ with $E\tilde{x} = E\tilde{z}$, which each could describe the value of the underlying asset. The agent's expected profit with respect to $\tilde{x}$ is

$$\begin{aligned}\Pi_x(y) &= \int_y^b (x-y)dF(x) = b - y - \int_y^b F(x)dx = b - y - \left(\int_a^b F(x)dx - \int_a^y F(x)dx\right) \\ &= b - y - \left((b - E\tilde{x}) - \int_a^y F(x)dx\right) = E\tilde{x} - y + \int_a^y F(x)dx.\end{aligned}$$

Similarly, the agent's expected profit with respect to $\tilde{z}$ is

$$\Pi_z(y) = E\tilde{z} - y + \int_a^y G(z)dz.$$

Hence, $\Pi_x(y) \geq \Pi_z(y)$ if and only if

$$E[\tilde{x} - \tilde{z}] + \int_a^y F(x)dx - \int_a^y G(z)dz \geq 0.$$

In other words, since $E\tilde{x} = E\tilde{z}$ by assumption, the agent prefers an increase in (pure) risk, i.e.,

$$\Pi_x(y) \geq \Pi_z(y) \text{ for all } y \in [a, b] \quad \Leftrightarrow \quad \tilde{x} \preceq_2 \tilde{z},$$

which completes the proof. Note that if the strike price y is fixed, then second-order stochastic dominance of $\tilde{z}$ over $\tilde{x}$ is only sufficient (but not necessary) for the agent to prefer $\tilde{x}$ over $\tilde{z}$.

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