MGT 621 – MICROECONOMICS

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8. General Equilibrium, Part I

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École Polytechnique Fédérale de Lausanne
College of Management of Technology

AGENDA

General Equilibrium: The Standard Model

Pure Exchange

Production Economies

Key Concepts to Remember

THE STANDARD MODEL

Basic Assumptions:

- N **goods**, $i \in \{1,\dots,N\}$
- C **consumers**, $c \in \{1,\dots,C\}$
  - Each consumer c has a *rational* (i.e., complete and transitive) *preference ordering* over $\Re_+^N$ representable by a *continuous utility function* $u_c : \Re_+^N \to \Re$. Consumers are **price takers**
- F **firms**, $f \in \{1,\dots,F\}$
  - Each firm f has a **production set** $Y_f \subset \Re^N$
    - *Nonempty* and *closed*
    - *No free lunch* ($Y_f \cap \Re_+^N \subset \{0\}$; can't produce something from nothing)
    - *Possibility of inaction* ($0 \in Y_f$)
    - *Free disposal* ($Y_f - \Re_+^N \subset Y_f$)
    - *Irreversibility* ($y \in Y_f \setminus \{0\} \Rightarrow -y \notin Y_f$)
- **Initial endowments**: each consumer c starts with an endowment vector $\omega^c \in \Re_+^N$ and a fractional share distribution $\theta^c = \left(\theta_1^c,\dots,\theta_F^c\right)$, where $\theta_f^c \in [0,1]$ for each firm f with

$$\sum_{c=1}^{C} \theta_f^c = 1 \qquad \text{(Private Ownership Economy)}$$

WALRASIAN EQUILIBRIUM

Definition: A **Walrasian equilibrium** (WE) is a specification of a **price vector** $p \in \Re_+^N$, a **demand vector** $x^c \in \Re_+^N$ for each consumer c , and a **supply vector** $y^f \in Y_f$ for each firm f, such that

- **profit maximization**, i.e., $y^f \in \arg\max_{y \in Y_f} p \cdot y$
- **utility maximization**, i.e., $x^c \in \arg\max_{x \in B(p,I^c)} u_c(x)$

  where *consumer c's* **budget set** is given by $B_c = B(p,I^c) = \{x \in \Re_+^N : p \cdot x \le I^c\}$

  with **total income** $$I^c = p \cdot \omega^c + \sum_{f=1}^{F} \theta_f^c (p \cdot y^f)$$

- **demand = supply**, $$\sum_{c=1}^{C} x^c = \sum_{c=1}^{C} \omega^c + \sum_{f=1}^{F} y^f \qquad \text{(i.e., allocation is feasible)}$$

hold.

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PURE EXCHANGE IS A SPECIAL CASE OF THE STANDARD MODEL

Consider an "exchange economy" *without firms* and *without production*

- **Two goods, 1 and 2**
- **M consumers of each of two types, 1 and 2**
- **Each consumer of type $c \in \{1,2\}$ begins with an allocation $\omega^c \in \Re_+^2$ (his endowment) and solves a utility maximization problem given a price vector $p \in \Re_+^2$, which defines his "offer curve",**

$$x^c(p) \in \arg \max_{x \in B(p, p \cdot \omega^c)} u_c(x)$$

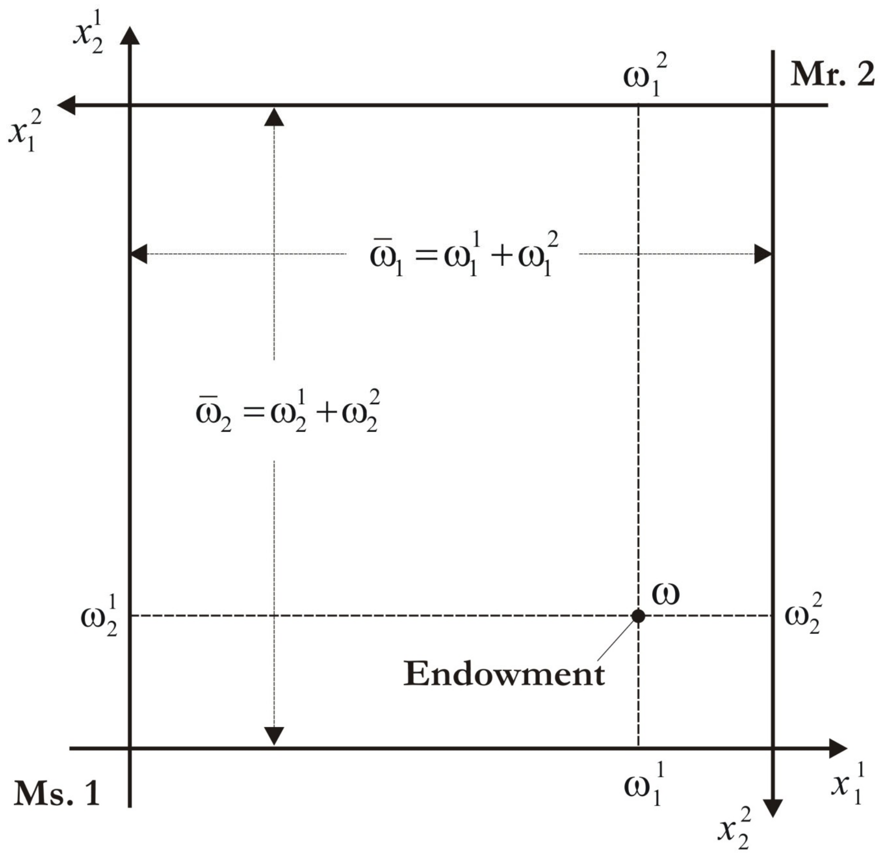
- **Any feasible allocation satisfies** $x^1 + x^2 = \omega^1 + \omega^2$

This two-consumer two-good exchange economy can be represented graphically using a "Edgeworth box" (sometimes also referred to as "Edgeworth-Bowley Diagram").

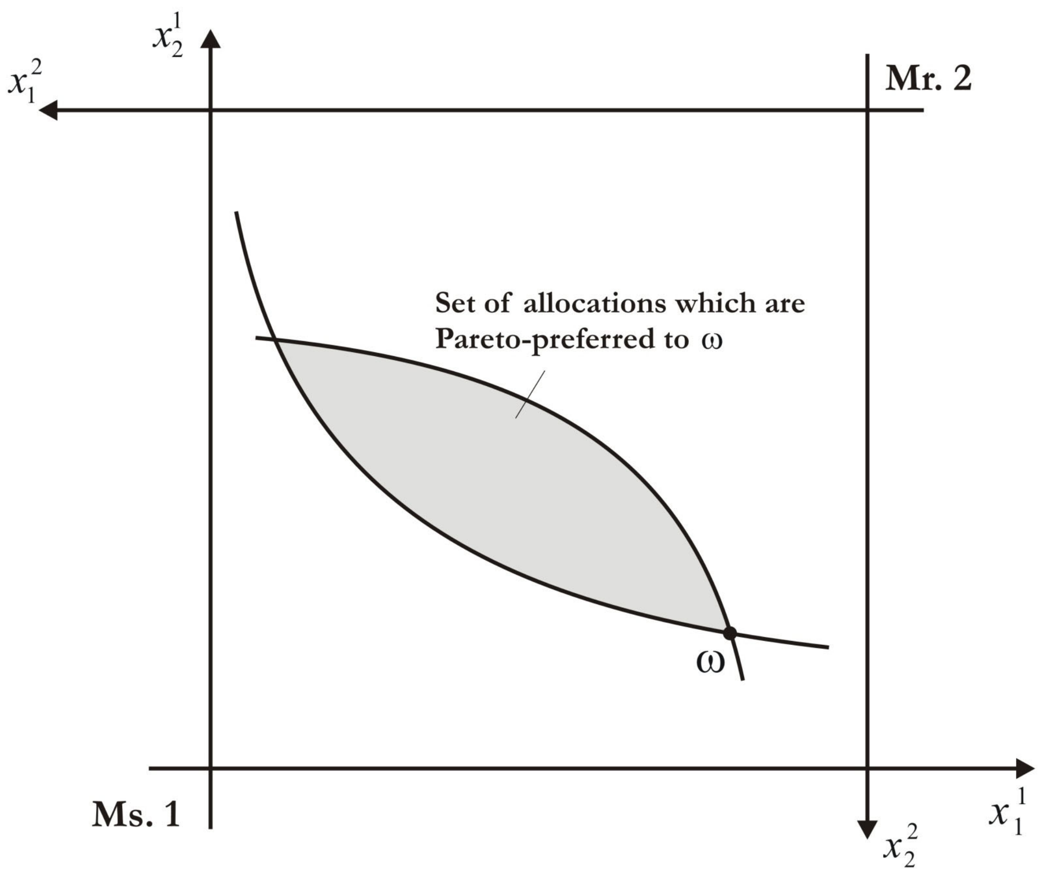
EDGEWORTH BOX

Consider one consumer of each type, Ms. 1 and Mr. 2

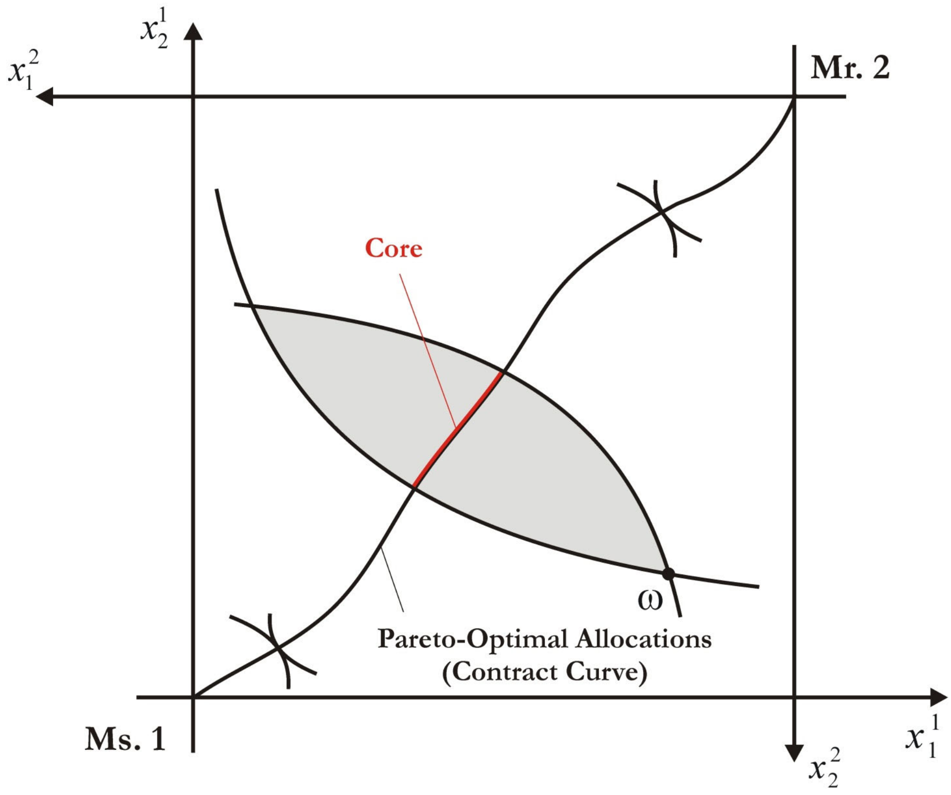
Any allocation of the *totally available quantities* $\overline{\omega}_i = \omega_i^1 + \omega_i^2$ of good i can be represented as a point in the Edgeworth box



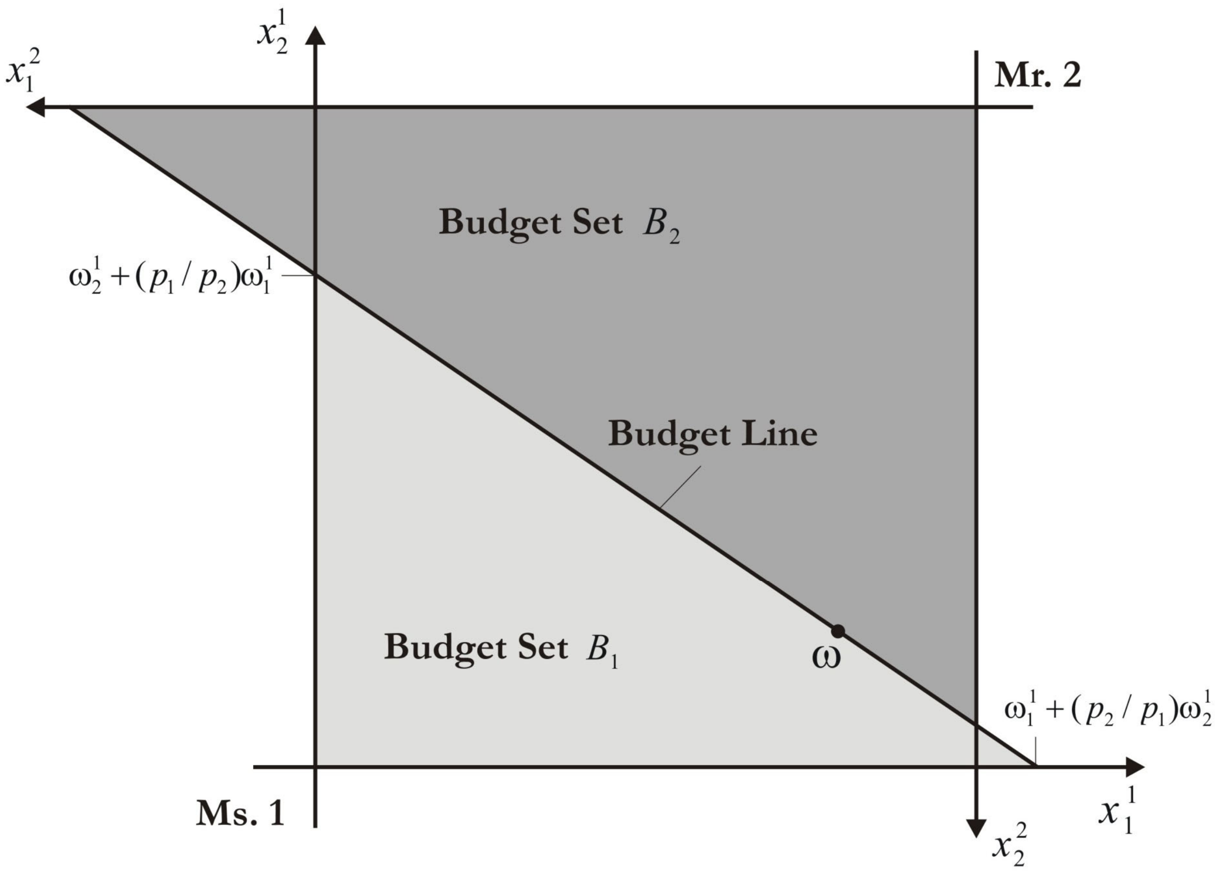
THERE MAY BE GAINS FROM TRADE



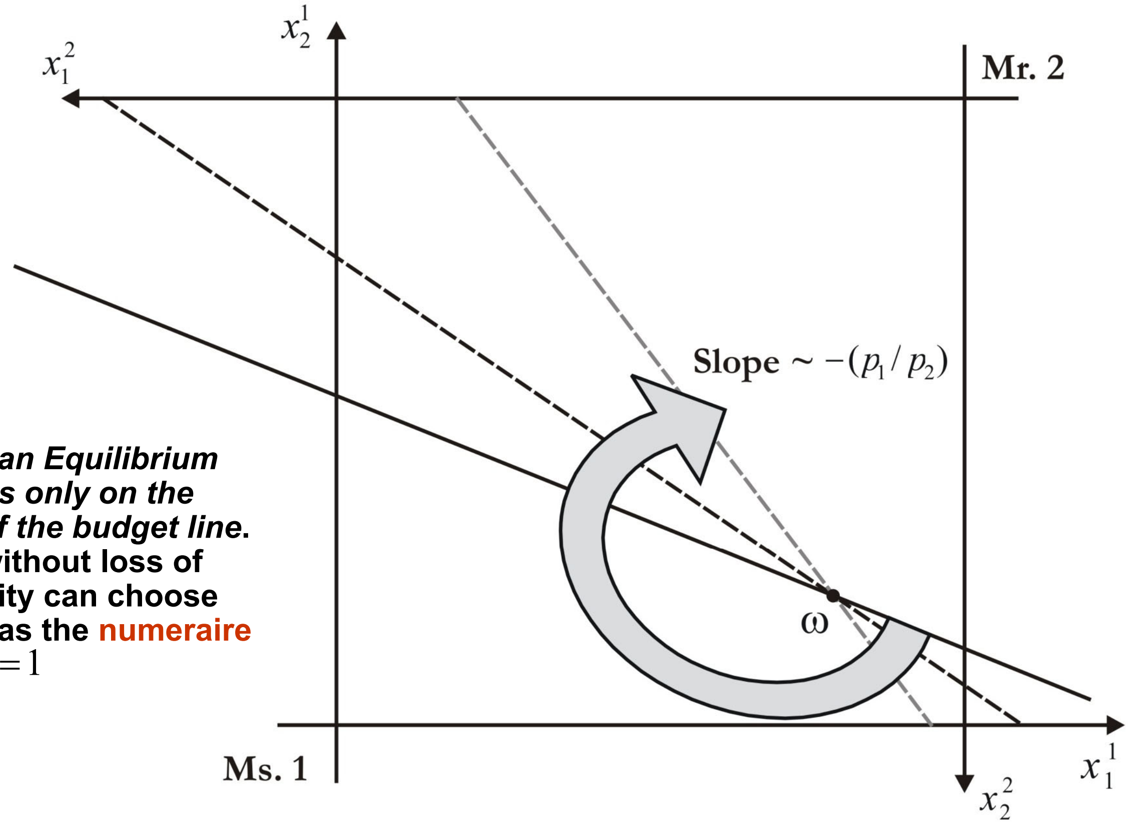
PARETO-OPTIMAL ALLOCATIONS AND CONTRACT CURVE



BUDGET LINE AND BUDGET SETS



THE SLOPE OF THE BUDGET LINE IS $(-p_1/p_2)$



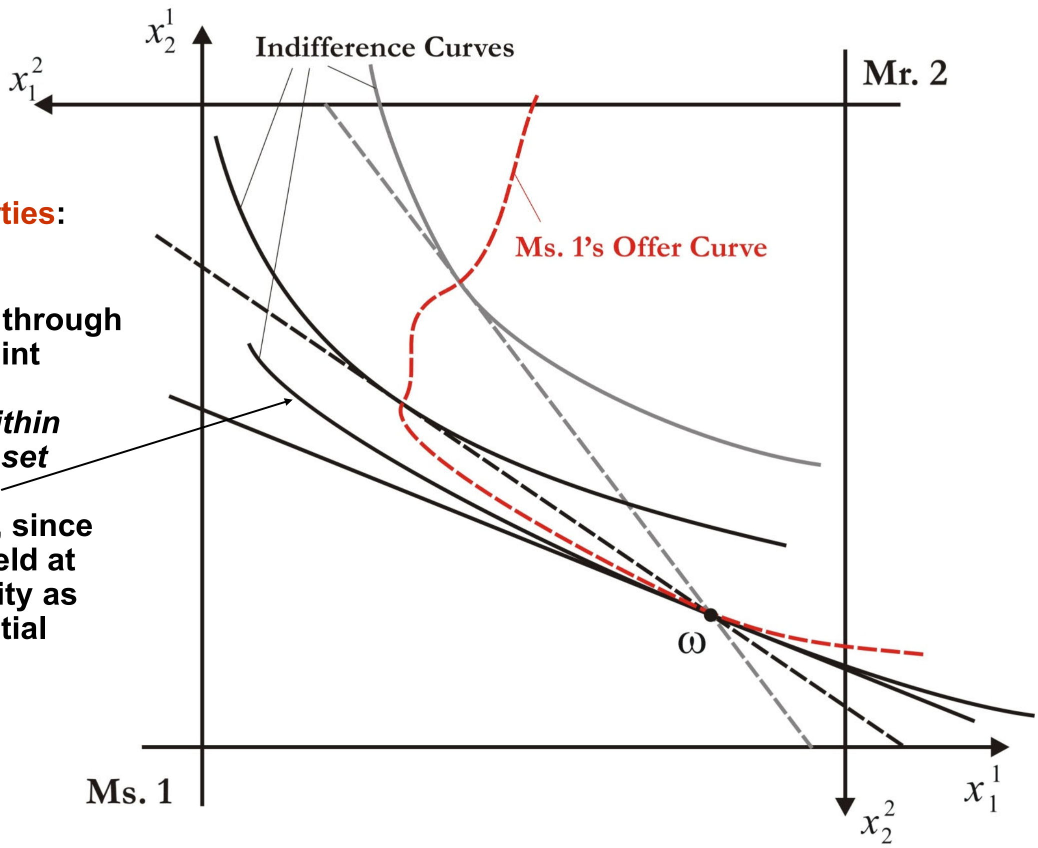
***Walrasian Equilibrium depends only on the slope of the budget line*.** **Thus, without loss of generality can choose good 1 as the numeraire with $p_1 = 1$**

THE OFFER CURVE DESCRIBES THE OPTIMAL CONSUMPTION CHOICE AS A FUNCTION OF MARKET PRICES

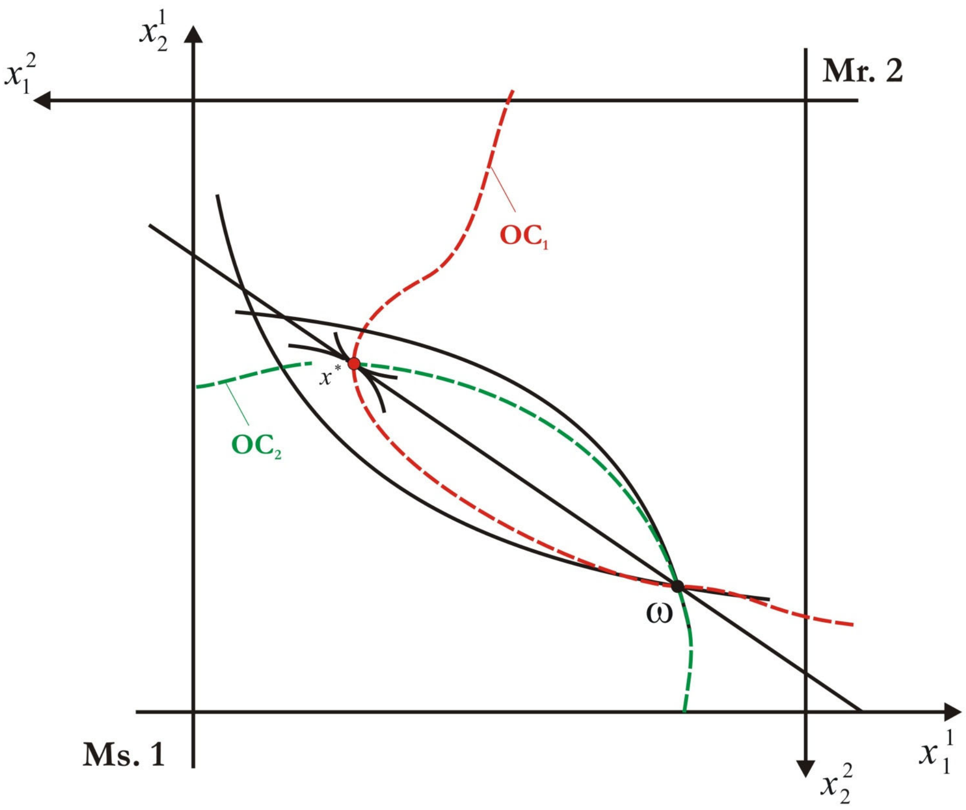
Offer Curve Properties:

OC is *tangent to indifference curve* through the endowment point

It also *has to lie within the upper contour set* delineated by that indifference curve, since any offer has to yield at least the same utility as the consumer's initial endowment



IN A WALRASIAN EQUILIBRIUM THE OFFER CURVES INTERSECT



EXAMPLE: EXCHANGE WITH COBB-DOUGLAS UTILITIES

Compute the Walrasian Equilibrium

Consider two consumers, 1 and 2, with Cobb-Douglas utility functions

$$u_c(x_1^c, x_2^c) = (x_1^c)^{\alpha}(x_2^c)^{1-\alpha}$$

where $\alpha \in (0,1)$ **is a constant and** $c \in \{1,2\}$ **. The endowment vectors are** $\omega^1 = (1,2)$ **and** $\omega^2 = (2,1)$ **respectively**

Given a price vector $p = (p_1, p_2)$**, consumer 1's utility maximization problem can be stated (after taking the logarithm) in the form**

$$x^1(p) = \arg \max_{(x_1,x_2)\in B(p,p_1+2p_2)} \{\alpha \log x_1 + (1-\alpha)\log x_2\} = \underbrace{\left(\frac{\alpha(p_1+2p_2)}{p_1}, \frac{(1-\alpha)(p_1+2p_2)}{p_2}\right)}_{\text{Consumer 1's offer curve}}$$

Consumer 1's offer curve

Similarly, we find *consumer 2's offer curve*,

$$x^2(p) = \left(\frac{\alpha(2p_1+p_2)}{p_1}, \frac{(1-\alpha)(2p_1+p_2)}{p_2}\right)$$

EXCHANGE WITH COBB-DOUGLAS UTILITIES (cont'd)

Using the Demand = Supply condition for the Walrasian equilibrium we can clear the market for good 1,

$$x_1^1(p) + x_1^2(p) = \omega_1^1 + \omega_1^2$$

or equivalently,

$$\frac{\alpha(p_1 + 2p_2)}{p_1} + \frac{\alpha(2p_1 + p_2)}{p_1} = 3$$

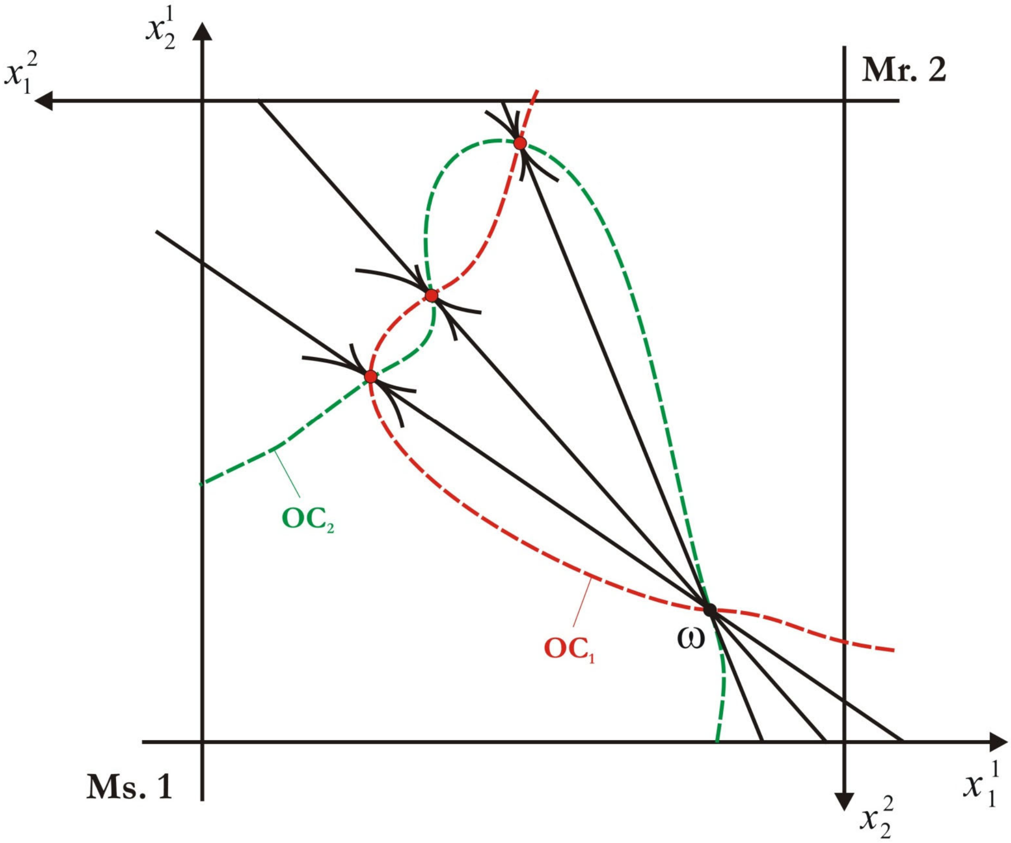
which yields $\dfrac{p_1}{p_2} = \dfrac{\alpha}{1-\alpha}$ **and** $\hat{x}_1^1 = \hat{x}_2^1 = 2-\alpha, \quad \hat{x}_1^2 = \hat{x}_2^2 = 1+\alpha$

In other words, *the price ratio in equilibrium is equal to the marginal rate of substitution between the two goods at the equilibrium allocation.*(1)

Note also that market clearing for good 1 implies market clearing for good 2 (why?)

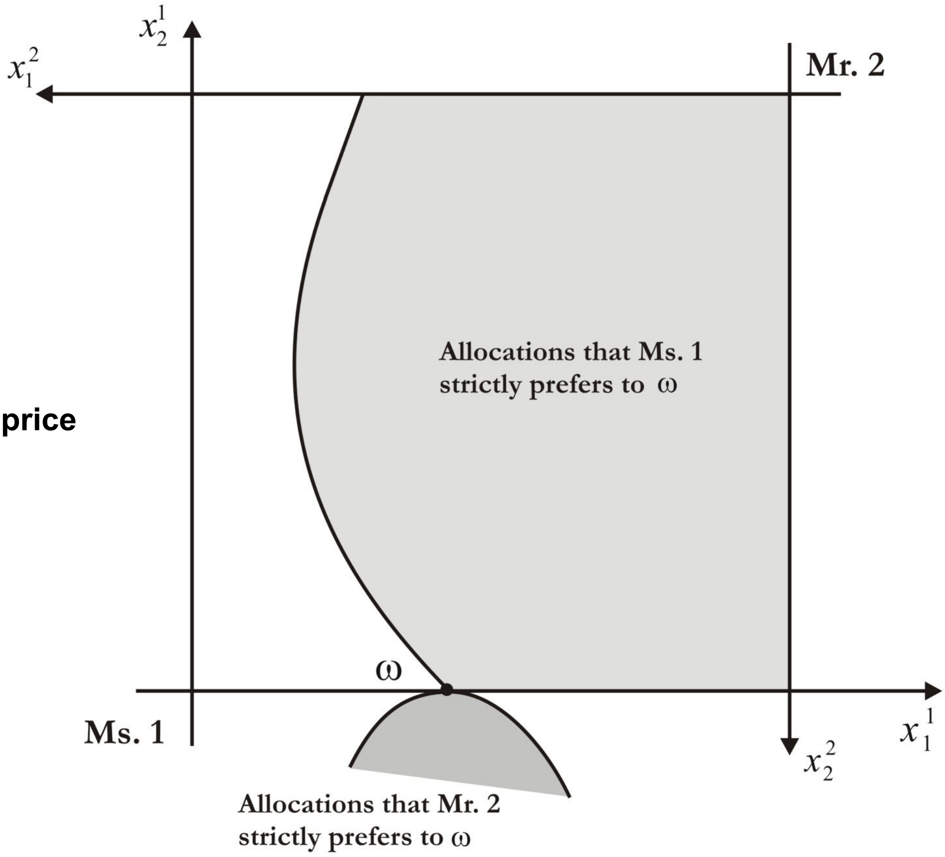
(1) The marginal rate of substitution for consumer 1 between goods 1 and 2 is $MRS_{1,2}(\hat{x}^1) = \dfrac{\left.\frac{\partial u_1}{\partial x_1^1}\right|_{(\hat{x}_1^1,\hat{x}_2^1)}}{\left.\frac{\partial u_1}{\partial x_2^1}\right|_{(\hat{x}_1^1,\hat{x}_2^1)}} = \dfrac{\alpha}{1-\alpha}\dfrac{\hat{x}_1^1}{\hat{x}_2^1} = \dfrac{\alpha}{1-\alpha}\dfrac{2-\alpha}{2-\alpha} = \dfrac{\alpha}{1-\alpha}$.

THERE MAY BE DIFFERENT WALRASIAN EQUILIBRIA
Nonuniqueness



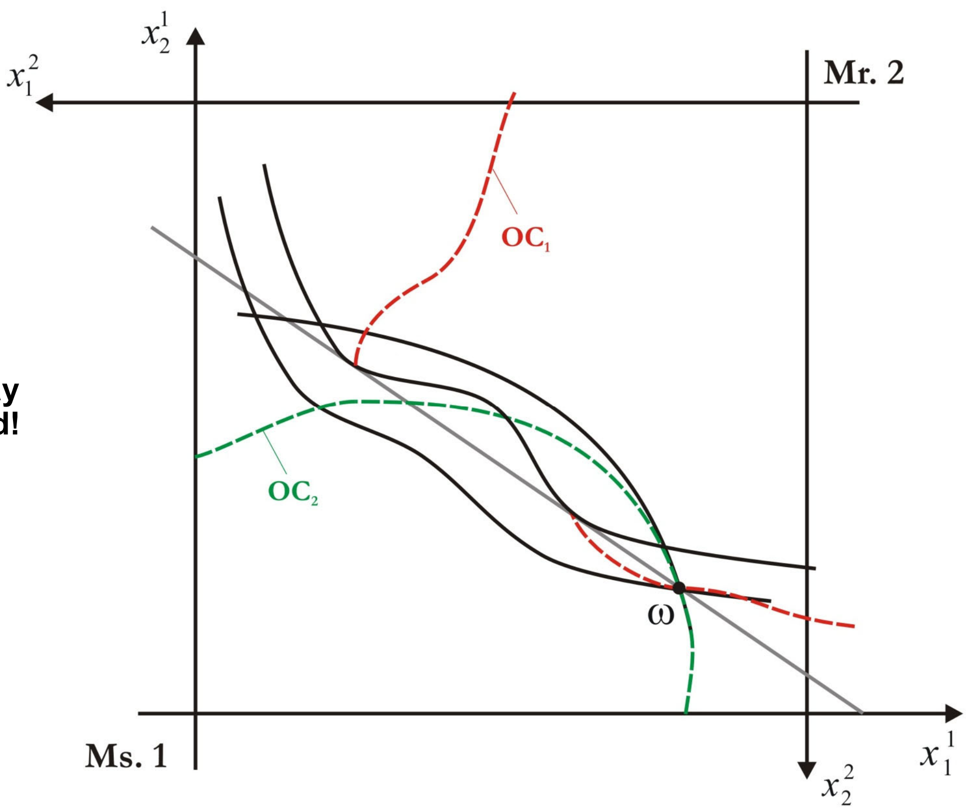
A WALRASIAN EQUILIBRIUM MAY NOT EXIST
Example: Preferences Not Strictly Monotone

No supporting price vector exists!



A WALRASIAN EQUILIBRIUM MAY NOT EXIST (cont'd)
Example: Nonconvex Preferences

Offer curves may be disconnected!



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ROBINSON CRUSOE ECONOMY

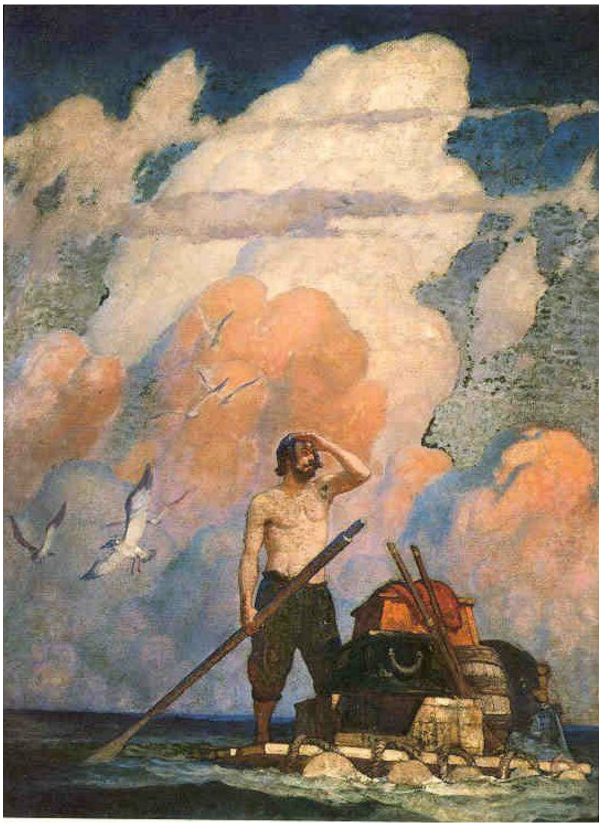
Robinson is alone on an island. He can *either* rest (i.e., consume leisure x_1) *or* use his own labor to pick yummy coconuts x_2

You can think of the firm "Robinson Crusoe Enterprises" producing coconuts using Robinson's labor $z = \bar{L} - x_1$ as the only production input, i.e., the firm maximizes profits

$$\pi(w,p) = \max_{z \geq 0}\{pf(z) - wz\}$$

where $f(\cdot)$ is the firm's *production function*,[(1)] w is the *wage* Robinson pays himself, p is the price of coconuts, and $\bar{L} > 0$ is a constant

Question: If Robinson maximizes his utility $u(x_1, x_2)$, *what is the Walrasian equilibrium of his private economy*?



(1) For simplicity assume that the production function is strictly concave, increasing, and continuously differentiable; we also assume that Robinson's utility is smooth. In addition it is sometimes useful to add the assumptions that $f(0) = 0$ (to all for the possibility of inaction), and the so-called Inada conditions $f'(0) = \infty, f'(\bar{L}) = 0$ (to guarantee strict interiority of the optimizer).

ROBINSON CRUSOE ECONOMY (cont'd)

Answer: Naturally, Robinson owns all of "Robinson Crusoe Enterprises," whence we obtain his utility maximization problem,

$$x(w,p) = \arg \max_{(x_1,x_2)\in B((p,w),I)} u(x_1,x_2)$$

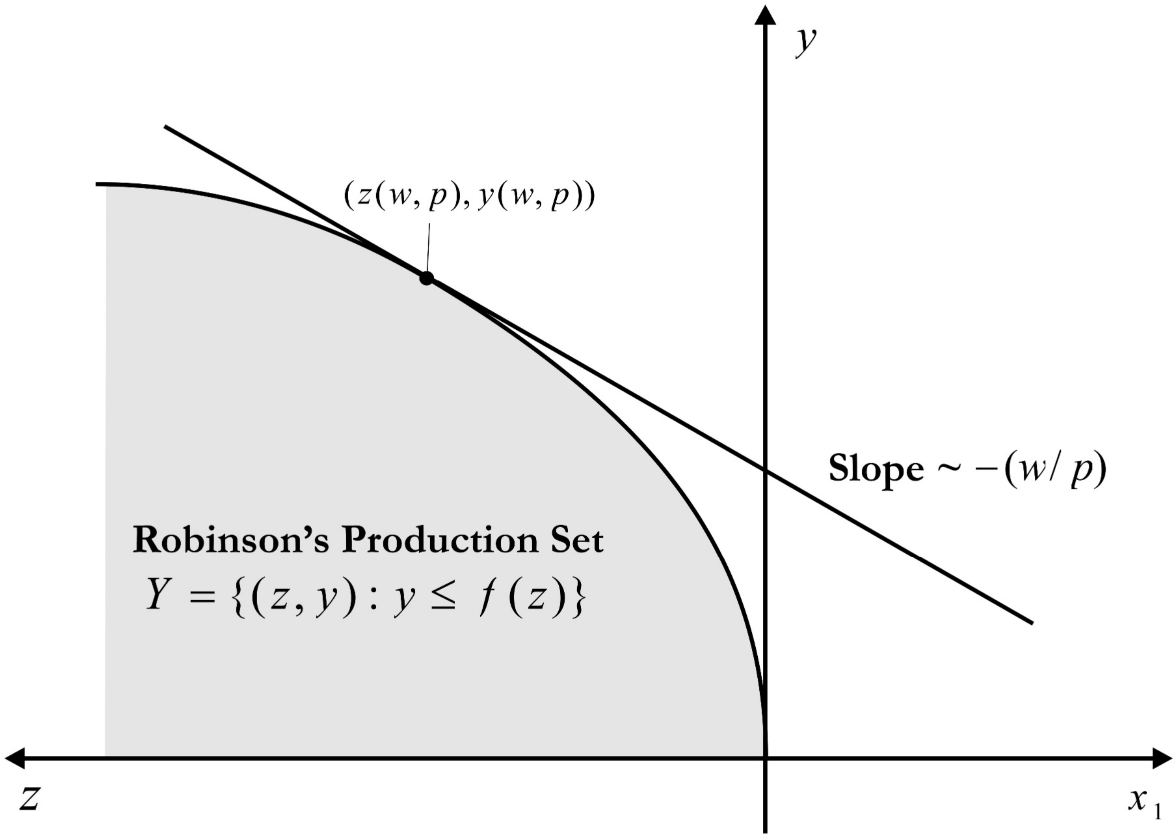
where $$B((w,p),I) = \{(x_1,x_2)\in \Re^2_+ : px_2 \le w(\overline{L}-x_1)+\pi(w,p)\}$$

From the definition of a Walrasian equilibrium, we obtain the following conditions:

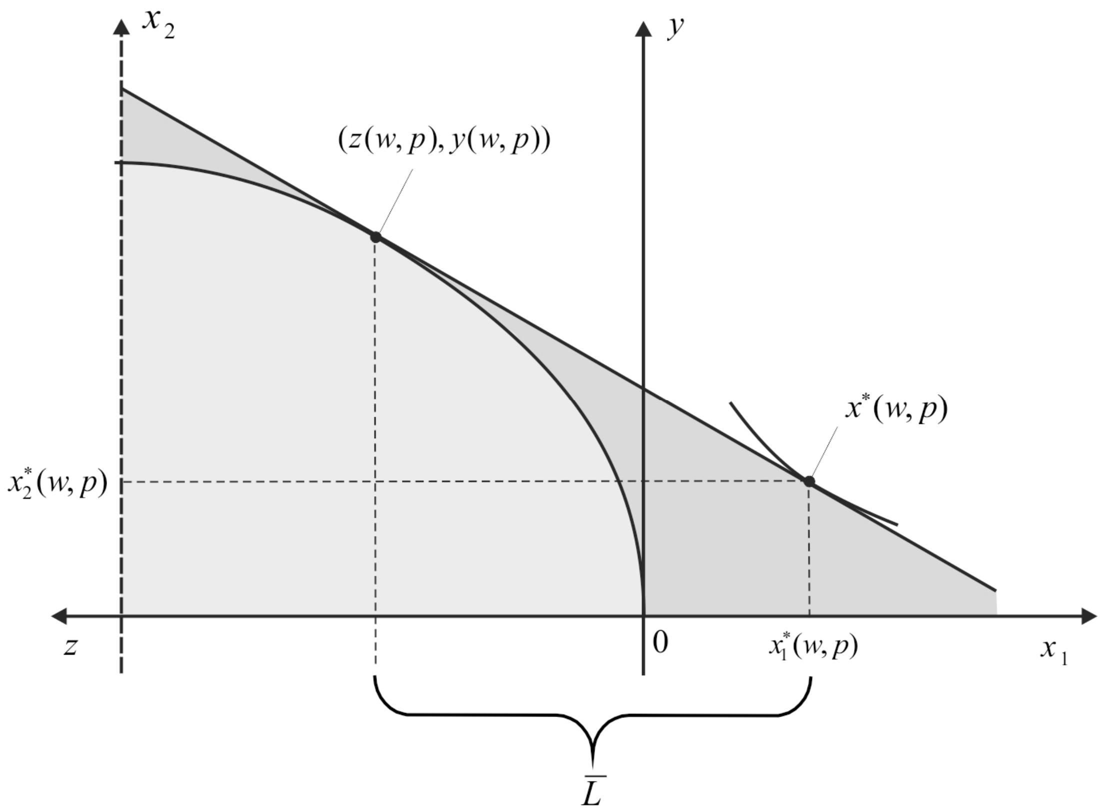
- **Profit maximization:** $f'(z^*(w,p)) = \dfrac{w}{p}$
- **Utility Maximization:** $\left[\dfrac{\partial u}{\partial x_1} - \left(\dfrac{w}{p}\right)\dfrac{\partial u}{\partial x_2}\right](x_1^*, \dfrac{w}{p}(\overline{L}-x_1^*) + y^*(w,p) - \dfrac{w}{p} z^*(w,p)) = 0$
  where $y^*(w,p) = f(z^*(w,p))$ **is the *equilibrium production quantity* of coconuts**
- **Demand = Supply:** $x_1^* = \overline{L} - z^*$, $x_2^* = y^*$

 Note: Without loss of generality we can *take coconuts as the numeraire good* (Robinson pays himself in coconuts), i.e., $p=1$.

ROBINSON CRUSOE ECONOMY (cont'd)
Production Problem



ROBINSON CRUSOE ECONOMY (cont'd)
Walrasian Equilibrium



2X2 PRODUCTION ECONOMY

Consider now a simple two-input two-output production economy:

- **2 Outputs, $j \in \{1,2\}$: there are 2N firms *producing one output each* (1)**
- **2 Inputs, *capital* K^j and *labor* L^j**
- **The production function of each firm for *output* j is given by**

$$y^j = f^j(K^j, L^j)$$

where $f^j(0,0) = 0$ (*possibility of inaction*) and f^j is strictly concave

- **N consumers *of each of two types*, $c \in \{1,2\}$, with increasing and strictly quasi-concave utility functions $u_c(x_1^c, x_2^c)$, where x_j^c is the amount a consumer of type c consumes of product j**
- **We assume for simplicity that consumers do not want to consume either capital or labor**
- **Consumers start with zero endowments in the production goods $j \in \{1,2\}$ and with $\overline{K}_c$ units of capital and $\overline{L}_c$ units of labor. In addition, each consumer of type c owns the fraction θ_j^c of the outstanding shares in the firms producing output j**

(1) More precisely, we assume that there are N firms producing output 1 and N firms producing output 2.

2X2 PRODUCTION ECONOMY (cont'd)

Definition: A symmetric allocation is one in which *all consumers of type c* receive the *same consumption vector* $x^c = (x_1^c, x_2^c)$ and all firms of type j produce the same output level y^j using the same input vector (K^j, L^j).

A *symmetric allocation* $((x^1, x^2), (y^1, K^1, L^1), (y^2, K^2, L^2))$ is feasible, if demand = supply, i.e.,

$$y^j = x_j^1 + x_j^2 \quad (1)$$

$$\overline{K}_c = K_c^1 + K_c^2 \quad (2)$$

$$K^j = K_1^j + K_2^j \quad (3)$$

$$\overline{L}_c = L_c^1 + L_c^2 \quad (4)$$

$$L^j = L_1^j + L_2^j \quad (5)$$

$$y^j = f^j(K^j, L^j) \quad (6)$$

Let F be the set of feasible allocations.

2X2 PRODUCTION ECONOMY (cont'd)

Definition: A symmetric Walrasian equilibrium is a specification of a price p_j for each output j, a *price* r *of capital*, a *wage* w *for labor*, a *consumption vector* $\hat{x}^c$ for each consumer type c, and a *production vector* $(\hat{y}^j, \hat{K}^j, \hat{L}^j)$ for each type of firm, such that the following three conditions are satisfied.

- **Utility maximization** $\hat{x}^c = \arg\max_{x \in B_c} \{u_c(x)\}$

$$B_c = \{(x_1^c, x_2^c) \in \Re_+^2 : p_1 x_1^c + p_2 x_2^c \le r\overline{K}_c + w\overline{L}_c + \theta_1^c \pi_1 + \theta_2^c \pi_2\}$$

- **Profit maximization** $(\hat{y}^j, \hat{K}^j, \hat{L}^j) = \arg\max_{y^j = f^j(K,L)} \{p_j y^j - rK^j - wL^j\}$

- **Demand = supply, i.e.,** $((\hat{x}^1, \hat{x}^2), (\hat{y}^1, \hat{K}^1, \hat{L}^1), (\hat{y}^2, \hat{K}^2, \hat{L}^2))$ **is a *symmetric feasible allocation* for each $j \in \{1,2\}$, where**

$$\pi_j = p\hat{y}^j - r\hat{K}^j - w\hat{L}^j$$

is firm j's equilibrium profit

2X2 PRODUCTION ECONOMY (cont'd)

First-order necessary optimality conditions **hold in a symmetric WE:**

- **Utility maximization**

$$\frac{\partial u_1(\hat{x}^1)/\partial x_1^1}{\partial u_1(\hat{x}^1)/\partial x_2^1} = \frac{\partial u_2(\hat{x}^2)/\partial x_1^2}{\partial u_2(\hat{x}^2)/\partial x_2^2} = \frac{p_1}{p_2} \quad (7)$$

- **Profit maximization**

$$p_1 \frac{\partial f^1(\hat{K}^1,\hat{L}^1)}{\partial K^1} = p_2 \frac{\partial f^2(\hat{K}^2,\hat{L}^2)}{\partial K^2} = r \quad (8)$$

$$p_1 \frac{\partial f^1(\hat{K}^1,\hat{L}^1)}{\partial L^1} = p_2 \frac{\partial f^2(\hat{K}^2,\hat{L}^2)}{\partial L^2} = w \quad (9)$$

From (7)—(9) we obtain

$$\frac{\partial u_c(\hat{x}^c)/\partial x_1^c}{\partial u_c(\hat{x}^c)/\partial x_2^c} = \frac{p_1}{p_2} = \frac{\partial f^2(\hat{K}^2,\hat{L}^2)/\partial K^2}{\partial f^1(\hat{K}^1,\hat{L}^1)/\partial K^1} = \frac{\partial f^2(\hat{K}^2,\hat{L}^2)/\partial L^2}{\partial f^1(\hat{K}^1,\hat{L}^1)/\partial L^1}$$

All marginal rates of substitution are determined by the equilibrium price ratio of the traded market commodities.

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KEY CONCEPTS TO REMEMBER

- **Edgeworth Box**
- **Pareto Optimality**
- **Budget Line**
- **Endowment**
- **Numeraire Good**
- **Offer Curve**
- **Walrasian Equilibrium (Competitive Equilibrium) (w/ or w/o transfers)**
- **Pareto Set**
- **Contract Curve/Core**
- **Pure Exchange / Production Economy**
- **Private Ownership Economy**
- **Price-Taking Behavior**
- **Walras' Law**