

# MGT 621 – MICROECONOMICS

Thomas A. Weber

## 6. Oligopoly

Autumn 2023

École Polytechnique Fédérale de Lausanne  
College of Management of Technology

Copyright © 2023 T.A. Weber  
All Rights Reserved

## OLIGOPOLY THEORY Introduction

So far in this course we have not emphasized strategic interactions between firms.

- We have seen that externalities can lead to significant distortions of the market outcome, even if all firms are price takers
- When a monopolist has market power, it can use second-degree price discrimination to segment a heterogeneous consumer base. For that analysis we did consider strategic interactions, but obtained a pure optimization problem, since the monopolist is able to move first by committing to a pricing scheme, anticipating the consumers' actions

When multiple firms select their actions simultaneously, and those actions directly influence each others' payoffs (i.e., there are externalities), then we need **game theory** to produce reasonable predictions about the outcome of the interaction.

**Game theory is a fundamental tool in the analysis of strategic interactions between multiple firms with market power.**

# AGENDA

## What is Game Theory?

### Building Blocks and Key Assumptions

### Market Structure & Strategy Analysis

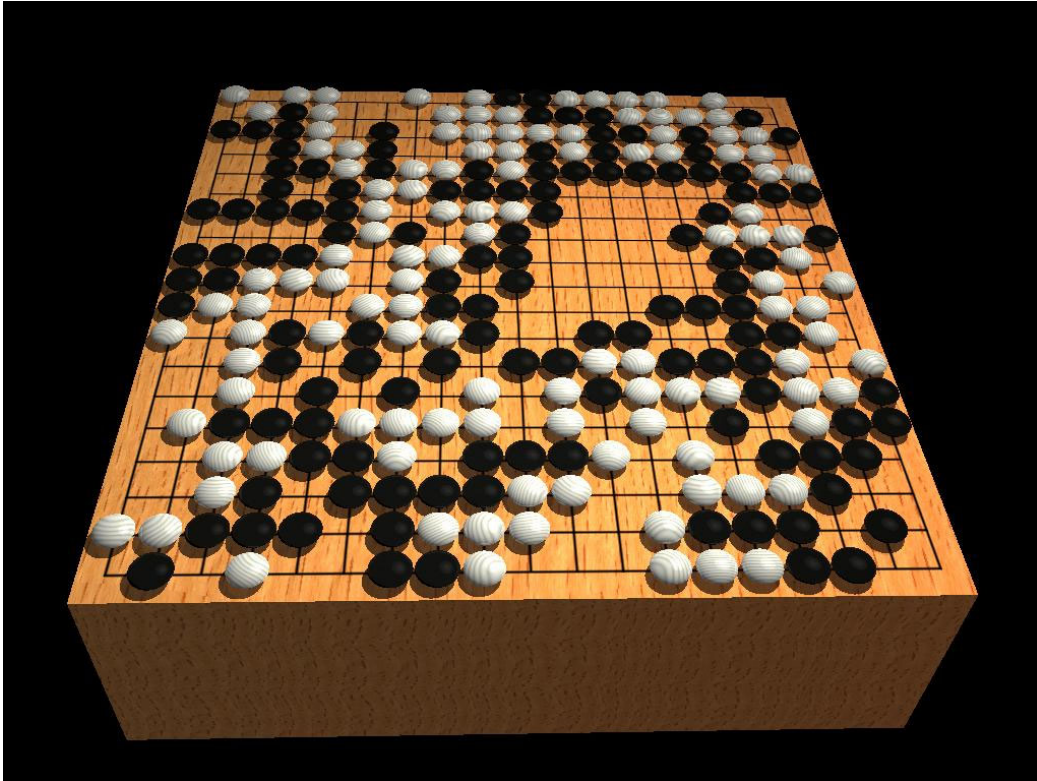
- Cournot Quantity Competition
- Bertrand Price Competition

### Key Concepts to Remember

## GAME THEORY



# GAME THEORY



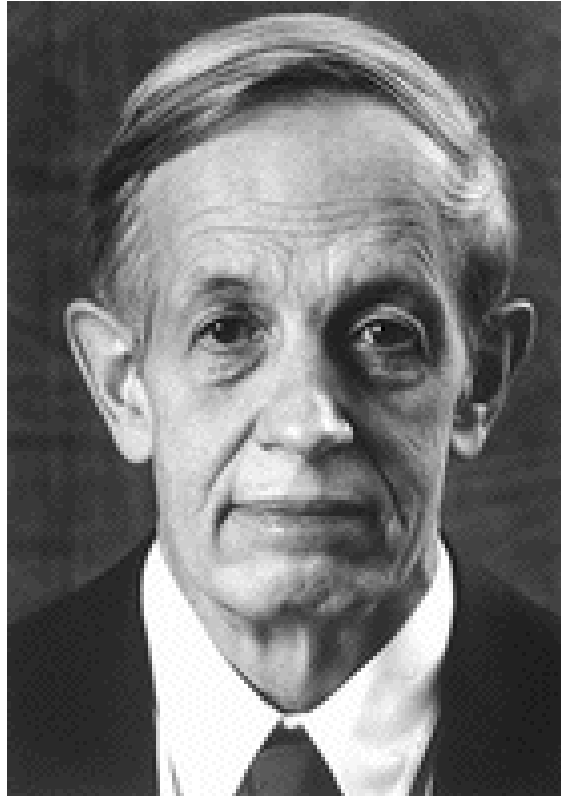
## JOHN VON NEUMANN (1903 – 1957)



**Oskar Morgenstern**  
(1902 – 1976)



## JOHN FORBES NASH (1928 – 2015)



MGT-621-Spring-2023-TAW

- 7 -

## GAME THEORY

Game Theory is the **analysis of strategic interactions among agents**.

A **strategic interaction** is a situation in which each agent, when selecting his or her most preferred action, takes into account the likely decisions of the other agents.

**Example: War**

*“In war the will is directed at an animate object that reacts.”*

- Carl von Clausewitz, *On War*

The **objective** of game theory is **to provide predictions** about the behavior of agents (players) in strategic interactions. The more precise these predictions are, the higher their **“predictive power.”**

(1) Cf. von Clausewitz, C. (1976) *On War*, Princeton University Press, Princeton, NJ. Clausewitz lived from 1780 to 1831; for more details about his life and work, see <http://www.clausewitz.com/>. The first systematic academic treatment of game theory is von Neumann, J., Morgenstern (1944) *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

# AGENDA

## What is Game Theory?

### Building Blocks and Key Assumptions

#### Market Structure & Strategy Analysis

- Cournot Quantity Competition
- Bertrand Price Competition

#### Key Concepts to Remember

## NORMAL-FORM GAME

### Building Blocks

- **Players**,  $i \in N = \{1, \dots, n\}$
- **Action Sets (Strategy Spaces)**,  $A_i$ , with elements  $a_i \in A_i$
- **Individual Payoffs**<sup>(1)</sup>,  $u_i(a)$ , where  $a = (a_i, a_{-i}) \in A = A_1 \times \dots \times A_n$  and  $a_{-i} \in A_{-i} = A_1 \times \dots \times A_{i-1} \times A_{i+1} \times \dots \times A_n$
- **(Mixed) Strategies**,<sup>(2)</sup>  $\sigma_i \in \Delta(A_i)$  and  $\sigma_{-i} \in \Delta(A_{-i})$

**Definition: A Normal-Form Game**  $\Gamma_N$  is a collection of players, action sets, and payoffs,

$$\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$$

(1) The payoff functions  $u_i(\cdot)$  are generally taken to be **von Neumann-Morgenstern utility functions**. You can think of them as bounded measurable functions (functionals) mapping actions (probability distributions over actions) into real values. The **payoff of a mixed strategy profile** is simply the expected value over the random action profile,  $u_i(\sigma) = u_i(\sigma_i, \sigma_{-i}) = \sum_{a \in A} [\sigma_1(a_1) \cdot \dots \cdot \sigma_n(a_n)] u_i(a)$

(2) Instead of **pure strategies** (elements of  $A_i$ ) we allow "**mixed**" strategies (elements of  $\Delta(A_i)$ ). If  $A_i$  is finite containing  $m_i$  elements, then  $\Delta(A_i)$  corresponds to the  $(m_i - 1)$ -dimensional simplex over  $A_i$ .

## PRISONER'S DILEMMA Example

Two suspects, 1 and 2, are being interrogated separately about a crime

- If **both confess**, each is sentenced to five years in prison
- If **both deny** their involvement, each is sentenced to one year in prison
- If **just one confesses**, he is released but the other one is sentenced to ten years in prison

Assume that each player's payoffs are proportional to the length of time of his prison sentence.

**Formulate this game in normal form.**

## PRISONER'S DILEMMA (Cont'd) Example

### Normal-Form Representation

- **Players**,  $i \in N = \{1,2\}$
- **Action Sets**,  $A_i = \{Deny, Confess\}$
- **Individual Payoffs**,  $u(a_1, a_2)$  , defined by "**payoff matrix**"
- **(Mixed) Strategies**,  $\sigma_i = (\sigma_i(Deny), \sigma_i(Confess)) \geq 0$  , with  $\sigma_i(Deny) + \sigma_i(Confess) = 1$

### Payoff Matrix<sup>(1)</sup>

		<u>Player 2</u>	
		Confess	Deny
<u>Player 1</u>	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

## PRISONER'S DILEMMA (Cont'd) Example

### Find Prediction about Outcome of this Game

		<u>Player 2</u>	
		Confess	Deny
<u>Player 1</u>	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

- Consider player 1's "best response" when fixing player 2's strategy
- Consider player 2's "best response" when fixing player 1's strategy

Hence, each player has a **dominant strategy**: no matter what the other player does, it is optimal (i.e., payoff-maximizing) for player  $i$  to select  $a_i = Confess$ .

Note also that the outcome is **inefficient** (i.e., does not maximize social surplus).

## FUNDAMENTAL ASSUMPTIONS

**Question:** What assumptions are necessary to arrive at predictions about outcomes of normal-form games?

**Assumption 1:** All **players are rational**, i.e., they maximize (expected) payoffs.

**Assumption 2:** The players' payoff functions and action sets are **common knowledge**, i.e.,<sup>(1)</sup>

- Each player knows the rules of the game
- Each player knows that each player knows the rules
- Each player knows that each player knows that each player knows the rules
- Each player knows that each player knows that each player knows that each player knows the rules
- Each player knows that each player knows that each player knows that each player knows that each player knows the rules
- ...

**Assumptions 1 and 2 imply a unique prediction in the Prisoner's Dilemma game; we will maintain these assumptions throughout this course**

(1) For a formal definition of common knowledge, see Osborne, M.J., Rubinstein, A. (1994) *A Course in Game Theory*, MIT Press, Cambridge, MA, pp. 73—75.

# WHAT HAPPENS IF PLAYERS ARE NOT RATIONAL?

## Relaxing Assumption 1

Relaxing the rationality assumption leads to **boundedly rational agents**, which is compatible with empirical observations. Some features of real-world agents which violate the rationality assumption are:

- Overconfidence
- Sensitivity to framing of the problem
- Satisficing behavior
- Intransitive preferences over outcomes (e.g., Allais Paradox, Ellsberg Paradox)
- Limited information-processing capabilities
- Availability heuristic
- Status-quo bias (e.g., endowment effect, regret avoidance, cognitive dissonance)
- ...

There is a fast growing literature on **“behavioral game theory”** (1)

(1) See e.g., Camerer, C.F. (2003) *Behavioral Game Theory: Experiments in Strategic Interaction*, Princeton University Press, Princeton, NJ. For more on behavioral decision making, see Kahnemann, D., Tversky, A. (2000) *Choices, Values, and Frames*, Cambridge University Press, Cambridge, UK.

## UNDERSTANDING RATIONALITY

Consider the following normal-form game (for which we just provide the payoff matrix):

		<u>Player 2</u>	
		L	R
<u>Player 1</u>	U	(4,4)	(-1000,3.9)
	D	(3.9,3.9)	(4,3.8)

Player 2 has a **strictly dominant strategy**; his **dominated strategy can thus be eliminated**. This leads to a unique prediction of the outcome (U,L) in this game.

Note though that player 1 has to be **absolutely sure** of the rationality of player 2!



## PURE-STRATEGY NASH EQUILIBRIUM

**Definition:** For any normal-form game  $\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$  a **pure-strategy Nash equilibrium** is a strategy profile  $a^* = (a_i^*, a_{-i}^*)$ , such that for every  $i \in N$  :

$$a_i^* \in \arg \max_{a_i \in A_i} u_i(a_i, a_{-i}^*)$$

In other words,

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad \forall a_i \in A_i, i \in N$$

or equivalently

$$a_i^* \in B_i(a_{-i}^*) \quad \forall i \in N$$

**Examples:** The Prisoner's Dilemma game has a unique pure-strategy Nash equilibrium (NE), in the Matching Penny game such an equilibrium does not exist

## MIXED-STRATEGY NASH EQUILIBRIUM

To increase the predictive power in games such as Matching Pennies, we extend the definition of Nash Equilibrium to include mixed strategy profiles of the form  $\sigma \in \Delta(A_1) \times \dots \times \Delta(A_n)$ .

**Definition:** For any normal-form game  $\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$  a **mixed-strategy Nash equilibrium** is a strategy profile  $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ , such that for every  $i \in N$  :

$$\sigma_i^* \in \arg \max_{\sigma_i \in \Delta(A_i)} u_i(\sigma_i, \sigma_{-i}^*)$$

where

$$u_i(\sigma) = u_i(\sigma_i, \sigma_{-i}) = \sum_{a \in A} [\sigma_1(a_1) \cdot \dots \cdot \sigma_n(a_n)] u_i(a)$$

# MATCHING PENNIES

In the Prisoner's Dilemma game the assumptions of common knowledge and rationality were enough to generate a unique prediction about the outcome, the reason being that each player found it strictly dominant to confess.

As we see below, rationality and common knowledge, are generally not enough to generate a prediction for the outcome of a normal-form game.

## Example: Matching Pennies

In a game of Matching Pennies, Ann and Bert, show each other a penny with either heads (H) or tails (T) up. If they choose the same side of the penny, Ann gets both pennies, otherwise Bert gets them.

(Note that this is a **zero-sum game**, as are most games people play for leisure.)

## MATCHING PENNIES (Cont'd)

### Normal-Form Representation

$$N = \{Ann, Bert\}$$

$$A_i = \{H, T\}, i \in N$$

$u_i(\cdot)$  defined by the following **payoff matrix**

		<u>Bert</u>	
		H	T
<u>Ann</u>	H	(1,-1)	(-1,1)
	T	(-1,1)	(1,-1)

**Question: What is the outcome of this game?**

## MATCHING PENNIES (Cont'd)

Consider each player's **best-response correspondence**

$$B_i(a_{-i}) = \arg \max_{a_i \in A_i} u(a_i, a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(\hat{a}_i, a_{-i}), \forall \hat{a}_i \in A_i\}$$

$$B_{Ann}(H) = H$$

$$B_{Bert}(H) = T$$

$$B_{Ann}(T) = T$$

$$B_{Bert}(T) = H$$

		<u>Bert</u>	
		H	T
<u>Ann</u>	H	(1,-1) → (-1,1)	(-1,1)
	T	(-1,1) ← (1,-1)	(1,-1)

**Result: The players' best-response correspondences do not "intersect."**

## MATCHING PENNIES (Cont'd)

Let us try to find a mixed-strategy Nash equilibrium in the Matching Pennies game. For simplicity set Ann = Player 1 and Bert = Player 2, so that  $N = \{1,2\}$

The player's mixed-strategy spaces are

$$\Delta(A_i) = \{(\sigma_i(H), \sigma_i(T)) : \sigma_i(H), \sigma_i(T) \geq 0 \text{ and } \sigma_i(H) + \sigma_i(T) = 1\}$$

Without loss of generality, let  $\sigma_1(H) = p$  and  $\sigma_2(H) = q$ .

Then

$$\begin{aligned} u_1(\sigma) &= p(qu_1(H, H) + (1-q)u_1(H, T)) + (1-p)(qu_1(T, H) + (1-q)u_1(T, T)) \\ &= p(q - (1-q)) + (1-p)(-q + (1-q)) \\ &= p(2q - 1) + (1-p)(1 - 2q) \\ &= (1 - 2p)(2q - 1) \\ &= -u_2(\sigma) \end{aligned}$$

## MATCHING PENNIES (Cont'd)

This is a **linear optimization problem** for each player. Note that player 1 has only control over  $p$  and player 2 has only control over  $q$ .

Player 1 can make player 2 **indifferent** about any of his strategies by choosing  $p = .5$  i.e.,  $\hat{\sigma}_1 = (p, 1-p) = (0.5, 0.5)$  and thus

$$\Delta(A_2) = \arg \max_{\sigma_2 \in \Delta(A_2)} u_2(\sigma_2, \hat{\sigma}_1)$$

If player 1 chooses a different strategy, player 2 is not indifferent and strictly prefers to play either  $q = 0$  (for  $p > .5$ ) or  $q = 1$  (for  $p < .5$ ).

On the other hand, if player 2 chooses anything other than  $q = .5$ , player 1 is not indifferent about her actions and will strictly prefer to play a pure strategy.

As a result,  $\sigma^* = (\sigma_1^*, \sigma_2^*)$  with  $\sigma_i^* = (.5, .5)$  is the **unique mixed-strategy Nash equilibrium** of the Matching Pennies game.

## ROLE OF INDIFFERENCE

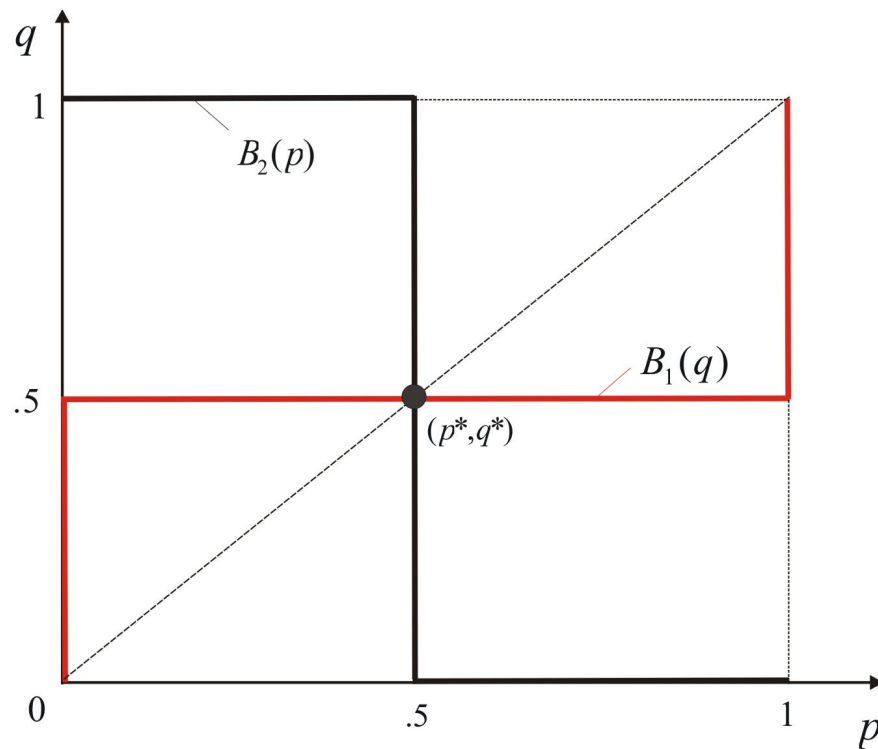
We emphasize the role that the players' indifference played in determining the NE in the Matching Pennies game. The following assumption is maintained for the rest of the course.

**Assumption: provided indifference between two or more actions in a player's (mixed-strategy) best-response correspondence, this player will select an action that is part of a (mixed-strategy) Nash equilibrium.<sup>(1)</sup>**

(1) In other words, if a player is indifferent between a strategy that is not a part of a particular Nash equilibrium and a strategy that is part of a particular Nash equilibrium, we assume that this player plays the strategy that is part of the equilibrium (i.e., he makes the equilibrium happen).

## MATCHING PENNIES (Cont'd)

It is possible to **graph the players' best-response correspondences**. The unique intersection is at  $(p^*, q^*) = (.5, .5)$ .



## AGENDA

### What is Game Theory?

### Building Blocks and Key Assumptions

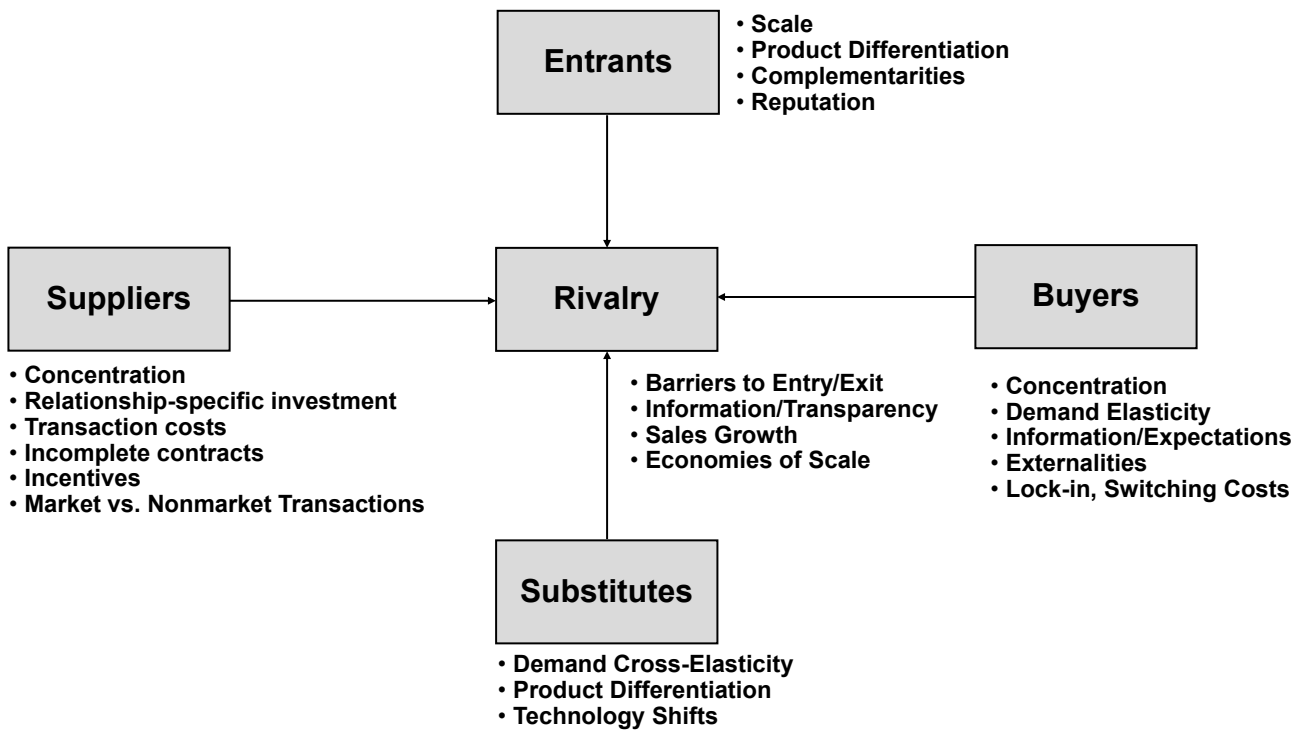
#### Market Structure & Strategy Analysis

- Cournot Quantity Competition
- Bertrand Price Competition

### Key Concepts to Remember

# PORTER'S FIVE FORCES

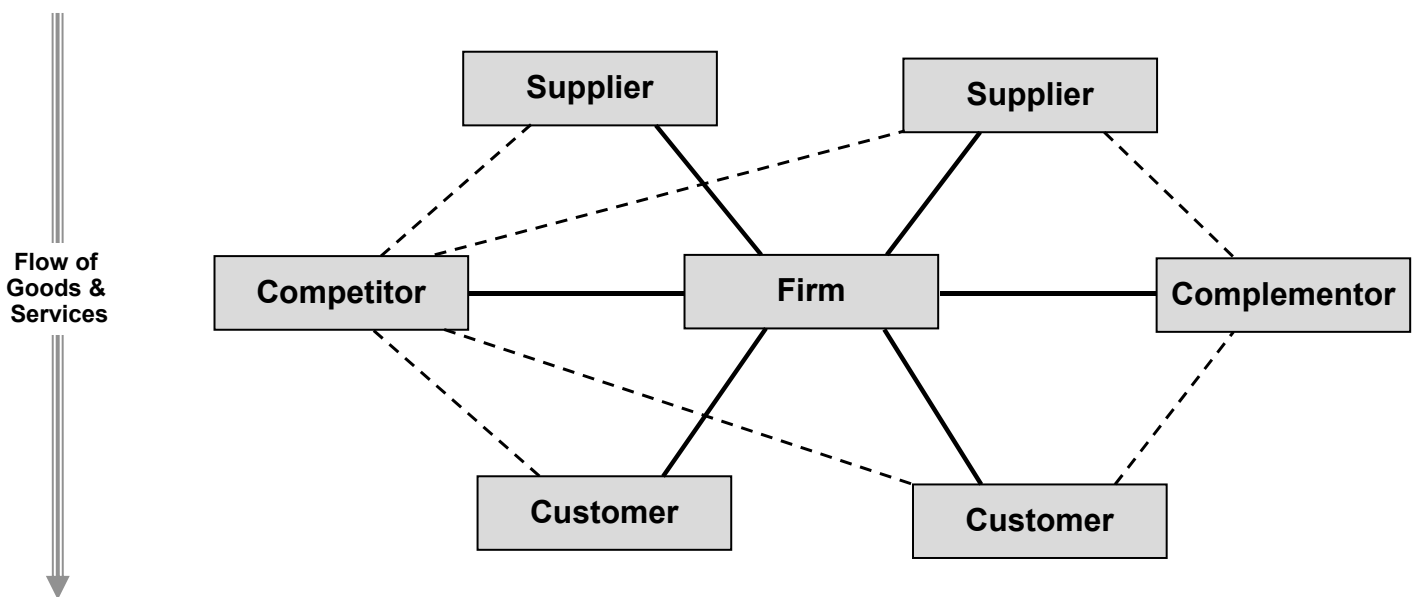
## ... and what influences them



Note: For the original presentation of the Five-Forces Model, see Porter, M.E. (1980) *Competitive Strategy*, Free Press, New York, NY.

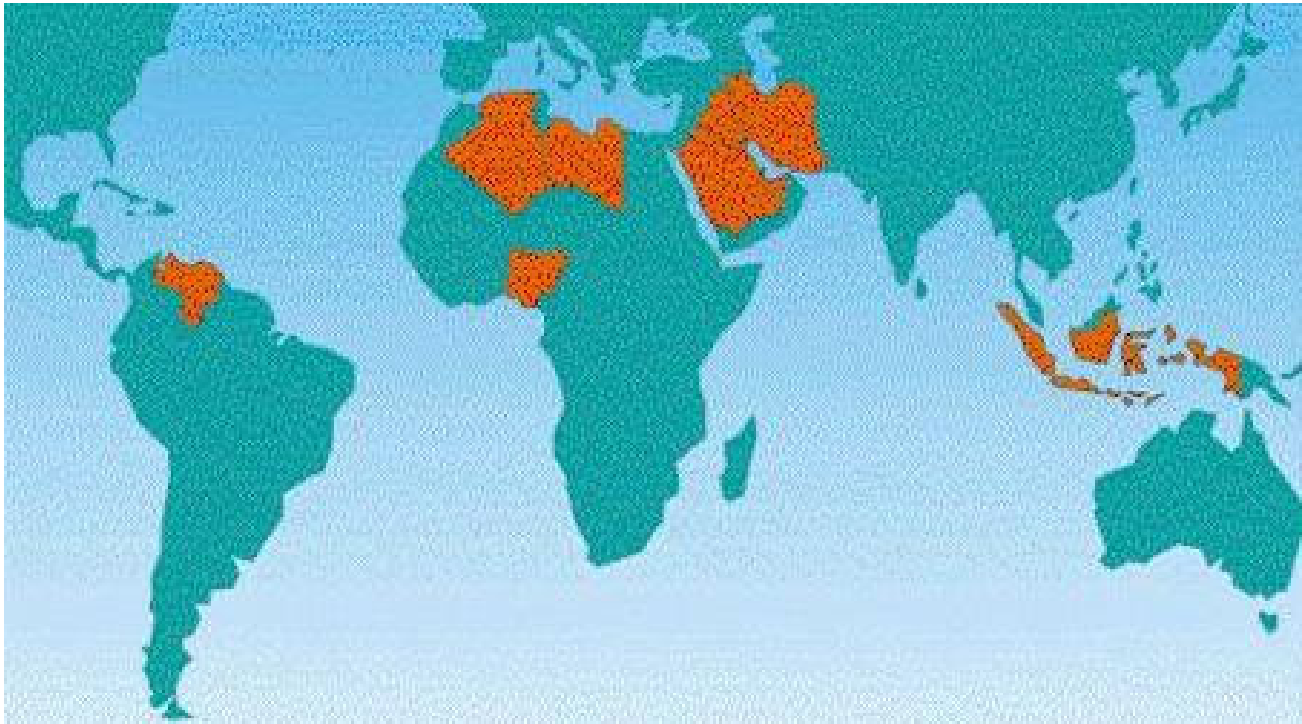
# BRANDENBURGER AND NALEBUFF'S VALUE NET

## The Firm and Its Network of Transaction Relationships



**Note that the firm and its competitors/complementors can have relationships in different markets at the same time ("multimarket contact")**

Note: For the original presentation of the value net, see Brandenburger, A.M., Nalebuff, B.J. (1995) "The Right Game: Use Game Theory to Shape Strategy," *Harvard Business Review*, Vol. 73, No. 4, pp. 57—71. The presentation here is close to the one in McAfee, R.P. (2002) *Competitive Solutions: The Strategist's Toolkit*, Princeton University Press, Princeton, NJ, p. 25.



## CHOOSING QUANTITIES: COURNOT DUOPOLY

Consider **two firms**, 1 and 2, choosing their production outputs  $q_1$  and  $q_2$  simultaneously. Each firm has a unit production cost of  $c$  (with  $0 < c < 1$ ).

- The market (inverse) demand is given by  $p(q_1, q_2) = 1 - q_1 - q_2$

**Question.** Determine a Nash equilibrium of this game.

**Solution.**

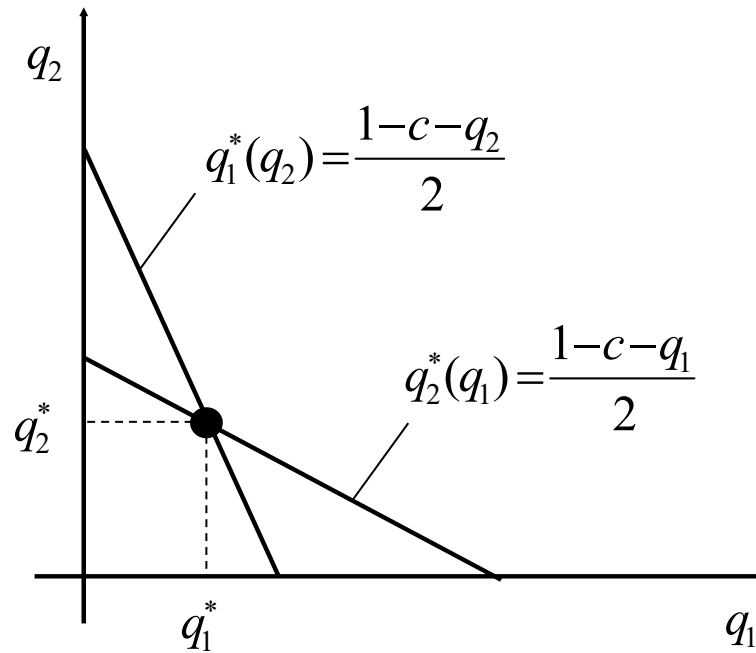
Firm  $i$ 's profit is  $\Pi_i(q_1, q_2) = (p(q_1, q_2) - c)q_i = (1 - c - q_1 - q_2)q_i$

- Its optimality condition is  $\frac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = 1 - c - 2q_i - q_j \stackrel{!}{=} 0$

- Its **best-response** to  $q_j$  is therefore  $q_i^*(q_j) = \frac{1 - c - q_j}{2}$

- Symmetry implies that at the Nash equilibrium  $q_i^* = \frac{1 - c - q_i^*}{2}$

## COURNOT DUOPOLY (Cont'd)



Unique Nash Equilibrium:

$$q_1^* = q_2^* = \frac{1-c}{3}$$

## COURNOT OLIGOPOLY Generalization of Previous Example

Consider the **symmetric linear model** and perfect substitutability, where

$$p_i(q) = p(Q) = a - bQ$$

with  $Q = q_1 + \dots + q_n$ ,  $a, b > 0$ , and

$$C_i(q_i) = cq_i$$

where  $c \in (0, a)$ .

- We find that at the unique NE **each firm produces**  $q_i^* = \frac{a-c}{(n+1)b}$
- The **total supply** is in a Cournot NE is thus  $Q^* = nq_i^* = \frac{a-c}{(1+1/n)b}$
- The **Cournot NE price** is  $p^* = a - bQ^* = \frac{a/n+c}{1+1/n}$
- The **industry Cournot profits** are  $\Pi_T^* = (p^* - c)Q^* = \frac{n}{b} \left( \frac{a-c}{n+1} \right)^2$



## COURNOT OLIGOPOLY (Cont'd)

### Comparison with Perfect Competition

The market power of each firm can be measured using the **Lerner index**  $L_i$ ,<sup>(1)</sup> which corresponds to the inverse of the **own demand elasticity**  $\varepsilon_i$  in equilibrium

$$\varepsilon_i(n) = -\left. \frac{\partial \log q_i}{\partial \log p_i} \right|_* = -\left. \frac{p_i}{q_i} \frac{dq_i}{dp_i} \right|_* = -\frac{p^*}{q_i^*} \left( -\frac{1}{b} \right) = \frac{a+nc}{a-c}$$

Let  $p^c$  and  $Q^c$  denote the price and total output in a symmetric equilibrium under perfect competition. Under perfect competition we have that necessarily  $p^c = c$  and  $Q^c$  therefore solves  $c = a - bQ^c$

We have that 
$$\lim_{n \rightarrow \infty} Q^*(n) = \lim_{n \rightarrow \infty} \frac{a-c}{(1+1/n)b} = \frac{a-c}{b} = Q^c$$

Note also: 
$$\lim_{n \rightarrow \infty} L_i(n) = \lim_{n \rightarrow \infty} \frac{a-c}{a+nc} = 0$$

(1) Lerner, A.P. (1934) "The Concept of Monopoly and the Measurement of Monopoly Power," *Review of Economic Studies*, Vol. 1, No. 3, pp. 157—175.

## STACKELBERG QUANTITY LEADERSHIP GAME

Consider firms in industries producing goods that are perfect or at least close substitutes

- Duopoly
- Oligopoly with one dominant firm
- Dominant firm and 'competitive fringe'

### Examples

- OPEC or Saudi Arabia
- Certain airlines or particular hubs

# STACKELBERG GAME: COURNOT WITH LEADER

Suppose there are **two symmetric firms**. Firm 1 is the leader and gets to choose its quantity at  $t = 0$ . Firm 2 is the **follower** and chooses its quantity at  $t = 1$ .

Any **SPNE** (the “**Stackelberg Equilibrium**”) can be found using backward induction, i.e., we start at  $t = 1$ . Firm 2 solves

$$q_2^*(q_1) = \arg \max_{\hat{q}_2 \geq 0} \{(a - c - b(q_1 + \hat{q}_2))\hat{q}_2\}$$

so that the **best-response** for firm 2 given the leader’s output choice  $q_1$  becomes

$$q_2^*(q_1) = \frac{a - c}{2b} - \frac{q_1}{2}$$

## STACKELBERG GAME (Cont’d)

Let us now examine the **leader’s optimal policy at  $t = 0$** . Firm 1’s **residual demand** is given by

$$\hat{p}(q_1) = a - b(q_1 + q_2^*(q_1)) = \frac{a + c - bq_1}{2}$$

The **elasticity of firm 1’s residual demand curve** is

$$\hat{\epsilon}_1 = -\frac{\hat{p}_1(q_1)}{q_1} \frac{dq_1}{d\hat{p}_1} = -\frac{a + c - bq_1}{2q_1} \frac{1}{\frac{d\hat{p}_1(q_1)}{dq_1}} = \frac{a + c}{bq_1} - 1 > \frac{a}{bq_1} - 1$$

**Firm 1 maximizes its profits** with respect to residual demand,

$$q_1^* = \arg \max_{\hat{q}_1 \geq 0} \{(a - c - b\hat{q}_1)\hat{q}_1 / n\} = \arg \max_{\hat{q}_1 \geq 0} \{(a - c - b\hat{q}_1)\hat{q}_1\} = \frac{a - c}{2b} = q^m$$

Hence, the **follower produces**

$$q_2^* = \frac{a - c}{2b} - \frac{q^m}{2} = \frac{a - c}{4b}$$

Demand elasticity in the absence of followers

Monopoly quantity!

## STACKELBERG GAME (Cont'd)

Total Stackelberg equilibrium output of all firms is therefore

$$Q^* = q_1^* + q_2^*(q_1^*) = \left(1 - \frac{1}{4}\right) \frac{a-c}{b}$$

and equilibrium market price is

$$p^* = a - bQ^* = c + \frac{a-c}{4}$$

The leader's equilibrium profit is

$$\Pi_1^* = \frac{(a-c)^2}{8b}$$

while the follower obtains in equilibrium

$$\Pi_2^* = \frac{(a-c)^2}{16b}$$

## BERTRAND DUOPOLY

Consider two firms selling a homogeneous product at a unit cost  $c_i \in [0,1], i \in N = \{1,2\}$

- The firms simultaneously set their prices  $p_i \in [0, \infty) = A_i$
- Let the total number of consumers be normalized to one. All consumers buy from the cheaper firm and randomize evenly between the two firms if their prices are equal
- The value of the consumers' (common) outside option is zero; their (net) value from the product if they buy from firm  $i$  is  $Y$  (Assume that  $Y > c_i$ )

**Question:** Determine the NE of this simultaneous-move game.

**Answer:** 1. Determine the firms' **payoff functions**,  $u_i(\cdot)$ :

$$u_i(p_i, p_{-i}) = \begin{cases} p_i - c_i, & p_i < p_{-i}, p_i \leq Y, \\ (p_i - c_i)/2, & p_i = p_{-i}, p_i \leq Y, \\ 0, & \text{otherwise.} \end{cases}$$

## BERTRAND DUOPOLY (Cont'd)

### 2. Determine the firms' best-response correspondences

- Assume that  $c_1 < c_2$
  - Find the set of strategies that *survive iterated deletion of strategies which are never a best response*:
    - For player  $i$ , the strategies  $p_i < c_i$  and  $p_i > Y$  are dominated by  $p_i = Y$
    - All strategies  $p_i \in [c_i, Y]$  *could* be rationalizable  $\rightarrow$  not very useful
  - Find best-response correspondences
    - Start with  $p_2 \in [c_2, Y]$ : then  $p_1 \in [p_2, Y]$  is **strictly dominated** by any  $p_1 \in (c_1, p_2)$
    - Player 1's payoffs are strictly increasing in  $p_1 \in (c_1, p_2)$ . Thus, **there is no best-response for player 1**, since the payoff from any particular strategy in  $(c_1, p_2)$  can be strictly improved upon
- However, if **increments are finite**, of arbitrarily small size  $\varepsilon > 0$ , then<sup>(1)</sup>

$$B_i(p_{-i}) = \begin{cases} Y, & p_{-i} > \max\{c_i, Y + \varepsilon\}, \\ \max\{c_i, p_{-i} - \varepsilon\}, & p_{-i} \in (c_i, Y + \varepsilon], \\ \{c_i + k\varepsilon\}_{k=0}^{\infty}, & p_{-i} = c_i, \\ \{p_{-i} + k\varepsilon\}_{k=1}^{\infty}, & p_{-i} \leq c_i. \end{cases}$$

MGT-621-Spring-2023-TAW (1) We approximate the Bertrand game at this point by a series of discrete games  $\{\Gamma_N(\varepsilon_n)\}_{n=1}^{\infty}$  with  $\varepsilon_n \rightarrow 0+$ , as  $n \rightarrow \infty$ .

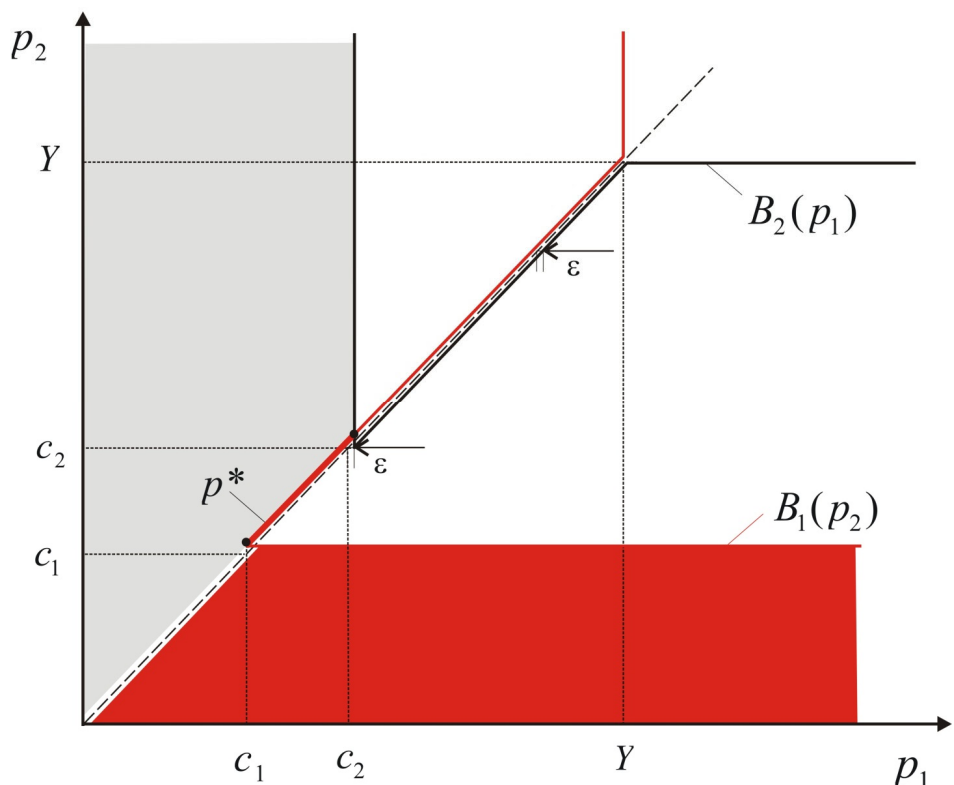
## BERTRAND DUOPOLY (Cont'd)

### 3. Find the intersection of the best-response correspondences

**Continuum of Nash equilibria:**

$p^* = (p_1^*, p_2^*)$   
with

$p_1^* \in [c_1, c_2]$   
 $p_2^* = p_1^* + \varepsilon$



**Note that all NE involve at least one player playing a weakly dominated strategy**

## BERTRAND DUOPOLY (Cont'd)

### Additional Notes

- In equilibrium with  $c_1 < c_2$ , firm 2 plays a **weakly dominated strategy**
- A **tiebreaking rule** that assigns all profits to firm 1 in case of equal prices guarantees a set of NE  $p^*$  for  $\varepsilon \rightarrow 0+$ :

“In a Bertrand equilibrium, firms charge a price between the first-  
the second-most efficient firm’s costs.”<sup>(1)</sup>

It is possible that all firms play a weakly dominated strategy in equilibrium.

## DIFFERENTIATION SOFTENS PRICE COMPETITION Generalization: Imperfect Substitutes

**Given:** Demands for products of firm 1 and firm 2:  $q_1(p_1, p_2)$  and  $q_2(p_1, p_2)$   
[& the firms’ cost functions:  $C_1(q_1)$  and  $C_2(q_2)$ ]

$$\max_{p_2} \{ p_2 q_2(p_1, p_2) - C_2(q_2(p_1, p_2)) \}$$

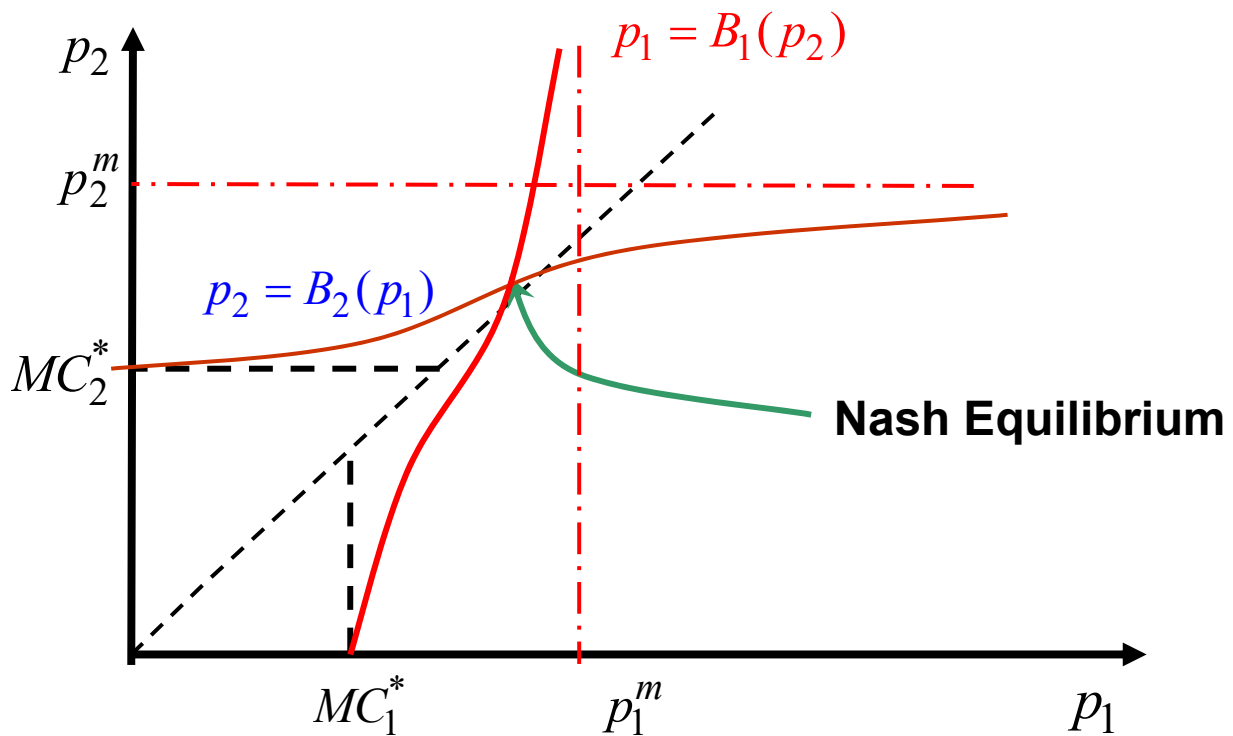
$$FOC: \quad MR_2(q_2) = MC_2(q_2) \quad \Rightarrow \quad p_2 = B_2(p_1)$$

$$\max_{p_1} \{ p_1 q_1(p_1, p_2) - C_1(q_1(p_1, p_2)) \}$$

$$FOC: \quad MR_1(q_1) = MC_1(q_1) \quad \Rightarrow \quad p_1 = B_1(p_2)$$

$$\left[ p_1 - \frac{dC_1(q_1)}{dq_1} \right] = - \frac{q_1(p_1, p_2)}{\frac{\partial q_1(p_1, p_2)}{\partial p_1}}$$

## BERTRAND WITH IMPERFECT SUBSTITUTES (Cont'd)



## AGENDA

**What is Game Theory?**

**Building Blocks and Key Assumptions**

**Market Structure & Strategy Analysis**

- Cournot Quantity Competition
- Bertrand Price Competition

**Key Concepts to Remember**

## **KEY CONCEPTS TO REMEMBER**

- **Predictive Power**
- **Payoff Matrix**
- **Pure/Mixed Strategy**
- **Dominant Strategy**
- **Best-Response**
- **Nash Equilibrium**
- **Cournot and Bertrand Game**
- **Stackelberg Sequential-Move Games**