

ECOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
College of Management of Technology
MGT 621 – MICROECONOMICS

Thomas A. Weber *

Autumn 2026

Course Notes

This Version: September 1, 2026

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8. General Equilibrium
9. Markets and Intermediaries

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

**SYLLABUS
(Version: 01/09/26)**

MGT 621 – Microeconomics

Autumn 2026

**September 1/2
9 am – 1 pm
September 11/14/15/22/25
3 pm – 7 pm**

(ODY -1.0019.2)

Instructor

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Course Website

<http://econspace.net/MGT-621.html>

Summary. This course presents a first introduction to microeconomic theory and its applications. It lays the foundation for more advanced courses. The main *objectives* of the course are:

- to provide a calculus-based introduction to models for the rational behavior of individual units of an economy and their interactions;
- to give students an appreciation of how economic models can be used to develop an understanding of real-life phenomena;
- to enable students to address a simple microeconomic problem by structuring it as a formal model, the analysis of which yields useful predictions and insights.

Topics to be covered include the theory of choice, the theory of the firm, partial and general equilibrium, market failure, regulation, and welfare economics.

Reading. The material in this course is largely contained in the following book. The references therein point to additional literature on the subject.

[PR] PINDYCK, R.S., RUBINFELD, D.L. (2017). *Microeconomics (9th Edition)*, Pearson.

This course is taught at a level that is slightly above [PR] in terms of the mathematical tools used. However, PR is an excellent introductory text written by two outstanding economists. I had the pleasure using an earlier version of this book in the first economics graduate course that I took at MIT a while back.

In addition, I recommend that you take a look at the following survey paper on pricing: WEBER, T.A. (2012) “Price Theory in Economics,” in: Özer, Ö., Phillips, R. (Eds.) *Oxford Handbook of Pricing Management*, Oxford University Press, pp. 281—339. A pre-publication version of this chapter is available at <http://ssrn.com/abstract=1270297>

Assessment. Students need to complete a number of homework assignments (problem sets). There will be a final exam as well as an individual course project, including a short oral defense.

The final examination covers the whole of the course. The course grade is computed as follows:

$$\text{Grade} = 0.2 (\text{Problem Sets}) + 0.4 (\text{Exam}) + 0.4 (\text{Project}).$$

For all *problem sets* I strongly encourage cooperation. Since some of the analysis can be demanding in terms of the new intuition required, discussing the problems with others is very important. Solutions need to be written up and handed in individually.

Course Website. Go to <http://econspace.net/MGT-621.html>.

Problem Sets. Homework will be assigned about every other class in the form of a problem set. Homework is due at the beginning of class on its due date. **No late homework will be accepted and there is no makeup homework!** Graded homework assignments will be returned in class.

Exams. There will be a *final exam* on September 25, in class. The exam is open-book. **There is no makeup exam!** If you have a justified scheduling conflict, you will have to make an arrangement with the teaching staff by September 21 in class. Any requests made after that day (other than documented emergencies) may or may not be granted.

Class Participation. Your presence and participation in class are essential for gaining mastery of the material.

Project. The project is an integral part of the course and complements the course material. Each student should identify a concrete real-world economic phenomenon and develop a small formal model showing how concepts from the course can be used to

understand it. The project should contain actual analysis and computations and should emphasize economic reasoning rather than merely describing the chosen application. Students may use computational and generative-AI tools, but remain responsible for all assumptions, calculations, factual statements, and references, and must briefly disclose their use of such tools in the final report. Detailed instructions are provided in the separate Course Project handout.

A *one-page proposal* is due in Session 6, with instructor feedback provided in Session 7. The final report, of at most 10 pages excluding references, a possible appendix, and the statement on computational and AI tools, is due four weeks after the end of the course, on Friday, October 23, before 5 pm, by email to the instructor. Each student will subsequently complete an individual oral defense of approximately 15 minutes, to be scheduled separately. The written report and oral defense will be evaluated jointly according to relevance and originality, clarity of expression and economic interpretation, the quality of synthesizing course concepts for the chosen application, academic rigor, and the student's demonstrated command of the project.

Honor Code.¹ The Honor Code is the EPFL's statement on academic integrity. It articulates the school's expectations of students and faculty in establishing and maintaining the highest standards in academic work:²

"EPFL has the mission of generating, sharing and utilizing knowledge. It encourages the development of a critical mind and the spirit of innovation within a climate of collaboration, respect and mutual confidence amongst all its members. It respects an ethical charter which, based on concerns shared by all cultures, is in keeping with the international character of EPFL.

In my capacity as a student of EPFL, I actively undertake to:

- *Adhere to the principles of the EPFL ethical charter;*
- *Acknowledge and respect the regulations governing school life;*
- *Respect the EPFL community as a whole and the work accomplished, in particular by favouring constructive dialogue amongst members;*
- *Strive for quality and integrity in my own work, refusing any cheating or plagiarism;*
- *Contribute to the efficiency of teaching and knowledge acquisition by way of a positive attitude;*
- *Respect the institution's infrastructure and its environment;*
- *Use resources provided by EPFL, particularly IT facilities, in an appropriate manner.*

Through my actions and commitment to this Honor Code, I contribute to the impact and reputation of EPFL and thus to respect and value associated with its diplomas."

Specifically, adhering to the EPFL Honor Code implies that all work in exams and quizzes must be done individually. For homework, students may consult with the teaching assistant and with other students, but must write up solutions independently

¹ For more information, see [here](#) (under 2.3.1, in French)

² http://polylex.epfl.ch/files/content/sites/polylex/files/recueil_pdf/ENG/2.3.1_ch_code_honor_en.pdf

based on their own understanding. All references and sources (e.g., in the course project) must be clearly identified and properly referenced. Lastly, *if you work with other students on a problem set, you must acknowledge their names on the front page of your submission.*

Topics. Following is a tentative schedule of topics for this semester.

Tentative Schedule

1. Theory of Choice (PR: Chapters 3 – 5)

- Introduction to Individual Decision Making
- Preferences and Utility Representation
- Consumer Choice
- Demand Theory
- Choice under Uncertainty

2. Theory of the Firm (PR: Chapters 6 – 8)

- Production Sets
- Profit Maximization and Cost Minimization
- Market Power

3. Markets (PR: Chapters 1 – 2; 9 – 13; 16)

- Competitive
- Monopoly
- Oligopoly
- General Equilibrium

4. Market Failure (PR: Chapters 17 – 18)

- Asymmetric Information
- Externalities and Public Goods

FINAL EXAM (Friday, September 25, from 3 to 6 pm): Covers all material.

Background Reading. Although we use PR as our main reference, I recommend a couple of other useful books for occasional background reading if you would like to flesh out the material using complementary texts. *These books are completely optional. You might want to refer to this list after the course has finished, as you become interested in studying specific aspects of economic analysis in greater detail.*

O. General Microeconomics

MAS-COLELL, A., WHINSTON, M.D., GREEN, J.R. (1995). *Microeconomic Theory*, Oxford University Press, Oxford, UK. [Standard reference in economics doctoral programs; encyclopedic in approach and coverage; you should purchase it, if you become more interested in economics.]

KREPS, D.M. (1990). *A Course in Microeconomic Theory*, Princeton University Press, Princeton, NJ. [Good chatty text, which you may want to use as a complement to V and MWG.]

VARIAN, H.R. (1992). *Microeconomic Analysis*, Norton, New York, NY. [Excellent introductory text which contains a small number of practice problems with solutions.]

I. Theory of Choice

INGERSOLL, J.E. (1987). *Theory of Financial Decision Making*, Rowman & Littlefield, Savage, ML. [Classic treatment of financial decision making, including the portfolio problem, and intertemporal choice in finance. Definitely a text that is beyond the scope of MS&E 241, but may serve as a useful reference for those of you interested in finance. It is a standard reference for introductory courses in finance PhD programs.]

KREPS, D.M. (1988). *Notes on the Theory of Choice*, Westview Press, Boulder, CO. [Rigorous introduction to the representation of preferences, which has been derived from notes for a course taught at the Stanford GSB.]

II. Theory of the Firm

CHAMBERS, R.G. (1988). *Applied Production Analysis: A Dual Approach*, Cambridge University Press, Cambridge, UK. [Contains a very detailed discussion of production and cost functions, including the important case of multi-output technologies.]

CHAMBERS, R.G., QUIGGIN, J. (2000). *Uncertainty, Production, Choice, and Agency*, Cambridge University Press, Cambridge, UK. [This text examines the effects of uncertainty on a firm's production choices. I would recommend it only if you happen to have a special interest in this topic.]

III. Partial and General Equilibrium, Market Structure

DEBREU, G. (1959). *Theory of Value*, Yale University Press, New Haven, CT. [One of the very best books you can ever lay your hands on. A classic!]

MAGILL, M., QUINZII, M. (1996). *Theory of Incomplete Markets*, MIT Press, Cambridge, MA. [This book establishes the connection between general equilibrium theory and financial markets. Consider it as a reference, which is beyond this class.]

MCKENZIE, L.W. (2002). *Classical General Equilibrium Theory*, MIT Press, Cambridge, MA. [Rigorous summary of the main results. Covers a lot of material we do not even touch. A reference by a leader in the field.]

SALANIÉ, B. (2000). *Microeconomics of Market Failures*, MIT Press, Cambridge, MA. [Here you find material on why markets can fail. There is not much time for this during our course, but having a look at this helps you better appreciate the assumptions under which markets exist and are efficient.]

SCHERER, F.M. (1996). *Industry Structure, Strategy, and Public Policy*, Harper Collins, New York, NY. [This book contains nine case studies of different industries. The discussion is excellent. There is no theory, though. I recommend this book to see how economic reasoning can be connected to the real world.]

IV. Welfare Economics

ARROW, K.J. (1963). *Social Choice and Individual Values*, Yale University Press, New Haven, CT. [Classic introduction to social choice theory by the founder of the field.]

ARROW, K.J., SEN, A.K., SUZUMURA, K. (EDS.) (2002). *Handbook of Social Choice and Welfare, Vol. 1*, Elsevier, Amsterdam, NL. [Reference book. Contains many excellent chapters on various aspects of social choice. Very good to get started on research.]

KAPLOW, L., SHAVELL, S. (2002). *Fairness versus Welfare*, Harvard University Press, Cambridge, MA. [In this nontechnical book, which does not contain a single formula, you find an interesting discussion of fairness and welfare as it relates to institutions and in particular to aspects of legal systems. Steven Shavell is a leading thinker in the area of “law and economics.”]

MOULIN, H.J. (2003). *Fair Division and Collective Welfare*, MIT Press, Cambridge, MA. [Excellent recent introduction to notions of fairness at the undergraduate level. Nice read.]

SEN, A. (1973/1997). *On Economic Inequality*, Clarendon Press, Oxford, UK. [This is a classic book by a winner of the 1998 Nobel Memorial Prize in Economics. The augmented 1997 edition (with James Foster) contains a very accessible introduction to notions of economic inequality.]

V. Regulation

VISCUSI, K., VERNON, J.M., HARRINGTON, J.E. (2000). *Economics of Regulation and Antitrust*, MIT Press, Cambridge, MA. [Comprehensive textbook on regulation often used in undergraduate economics programs. Contains virtually no formulas.]

LECTURE NOTES

MGT 621 – MICROECONOMICS

Thomas A. Weber

1. Theory of Choice

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

Administrivia & Course Overview

Preferences and Utility Representation

Some Properties

Utility Representation (Cont'd)

Demand Theory: Basics

A Little Refresher on Constrained Optimization

Key Concepts to Remember

INFRASTRUCTURE

My Coordinates

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INFRASTRUCTURE (Cont'd)

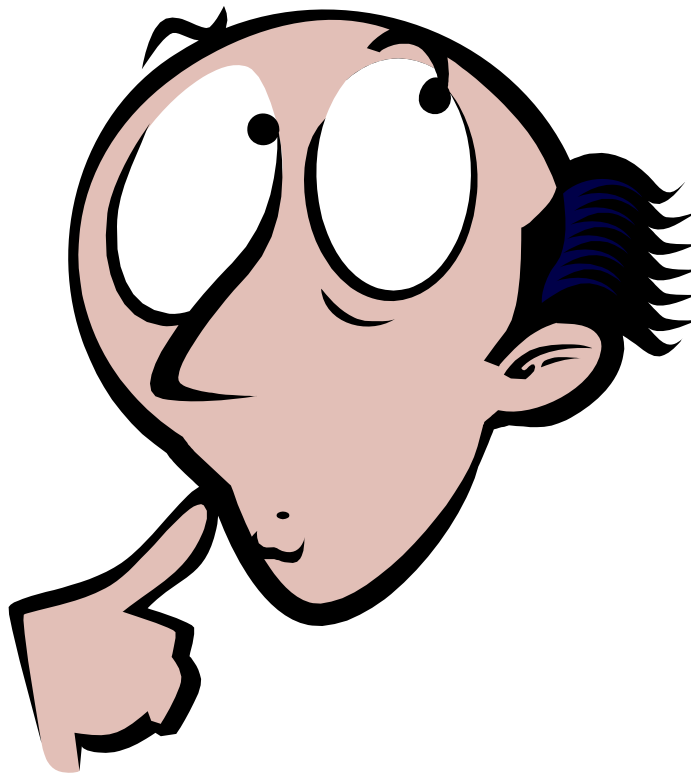
Course Material & Information

- Course website: <http://econspace.net/MGT-621.html>
- Required Text:
 - [PR] Pindyck, R.S., Rubinfeld, D.L. (2017). *Microeconomics (9th Edition)*, Pearson
 - All notes & additional readings are posted
- Solid knowledge in calculus required
- Access to spreadsheet & math software (e.g., MS Excel, Matlab, Maple) may be useful for some homework and the course project
- AI use needs to be acknowledged (... it will not help you in the final exam)
- Links to general information on course website

Honor Code(!)

ADMINISTRIVIA

Did we forget anything?



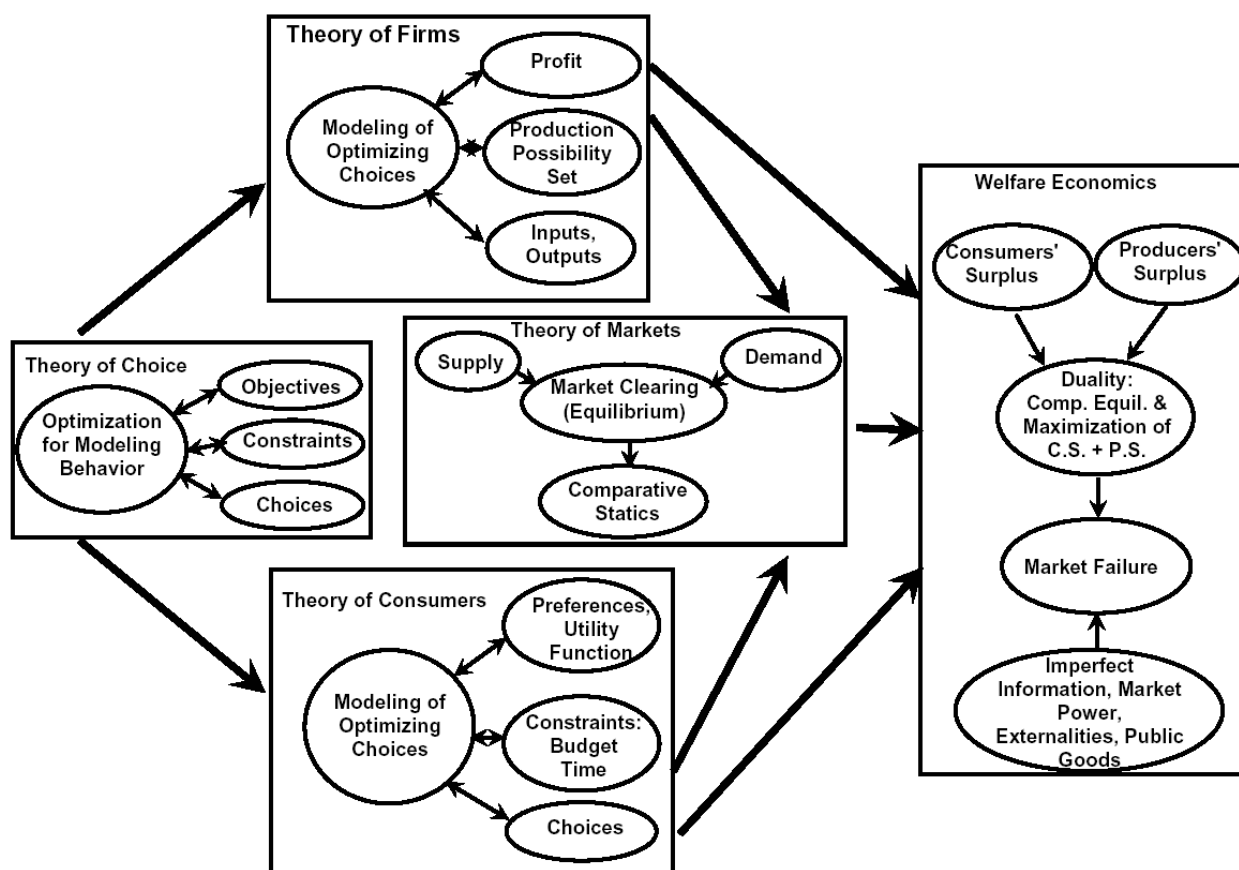
ASSESSMENT

- **PROBLEM SETS (20%)**
 - Reproductive & productive questions
 - Cooperation ok!!
 - Assignments need to be written up & turned in individually

- **FINAL EXAM (40%)**
 - Held on Friday, September 25, 2026; there is no makeup
 - *Any arrangements by September 18*
 - 3 hours (open book)
 - Covers everything discussed in the course

- **COURSE PROJECT (40%)**
 - Report due on October 23 (before 5 pm; by email to the instructor)
 - Oral assessment, to be scheduled individually, necessary because of AI presence

MGT 621 MICROECONOMICS – OVERVIEW



MGT-621-Spring-2026-TAW

- 7 -

TOPICS IN THIS COURSE Tentative List

- I. Theory of Choice
 - Individual Decision Making
 - Preferences and Utility Representation
 - Consumer Choice (+ under uncertainty)
 - Aggregate Demand
- II. Theory of the Firm
 - Production Sets
 - Profit Maximization and Cost Minimization
 - Aggregation
- III. Market Equilibrium
- IV. Market Failure

MGT-621-Spring-2026-TAW

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TOPICS IN THIS COURSE (cont'd)

Tentative List

- I. Theory of Choice**
- II. Theory of the Firm**
- III. Market Equilibrium**
 - **Competitive Markets**
 - **Profit Maximization and Cost Minimization**
 - **Aggregation**
- IV. Market Failure**
 - **Monopoly**
 - **Externalities**
 - **Public Goods**
 - **Regulation & Taxation**

AGENDA

Administrivia & Course Overview

Preferences and Utility Representation

Some Properties

Utility Representation (Cont'd)

Demand Theory: Basics

A Little Refresher on Constrained Optimization

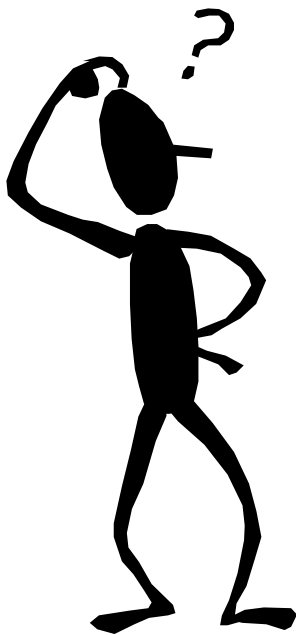
Key Concepts to Remember

AN “ECONOMIC CHOICE PROBLEM”: BUYING A CAR



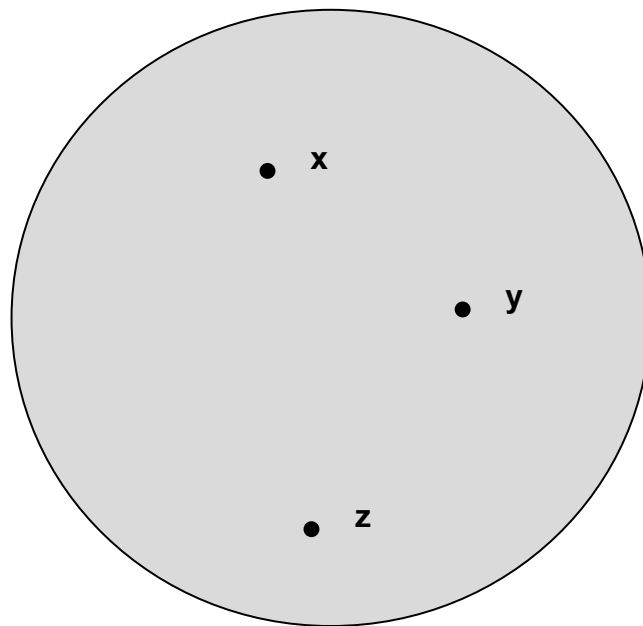
What are your goals? ... alternatives? ... selection criteria?

PREFERENCES



Homo Economicus : “Joe”

(You? Me? Everybody else?)



Choice Set $X = \{x,y,z\}$

Contains all potentially feasible
(mutually exclusive) alternatives.

PREFERENCES (Cont'd)

To decide which alternative to choose Joe needs to be able to rank them, i.e., he needs to have a preference ordering of all elements of his choice set X .

Definition. A **preference relation on X** is a binary relation " $\preceq$ " that for any two elements x, y in X compares them so that

(i) $x \preceq y$: y is (weakly) preferred to x ,

or

(ii) $y \preceq x$: x is (weakly) preferred to y .

If both (i) and (ii) hold, then we say that there is **indifference** between x and y , denoted by $x \sim y$ or, equivalently, $y \sim x$. If (i) but not (ii), then $x \prec y$, and we say that y is **strictly preferred** to x .

Only if Joe has a preference relation on X , is he able to establish a preference ordering of all elements of his choice set X

PREFERENCES (Cont'd)

Potential problem:

If Joe has a preference relation on $X = \{x, y, z\}$, he might have the following preference ordering between pairs of elements:

1. $x \prec y$
2. $y \prec z$
3. $z \prec x$



Example:

- x : apple
- y : banana
- z : orange

Problem?

Lack of transitivity!

(i.e., $x \prec y, y \prec z \Rightarrow x \prec z$ does not hold)

PREFERENCES (Cont'd)

Lack of transitivity can be generated through aggregation of preferences of individuals with transitive preference relations on X : **Arrow's Impossibility Theorem**

Example: Consider *three alternatives* $x, y, z \in X$ and three agents ($N = \{1, 2, 3\}$). Let the agents' preferences be as follows:

- **Agent 1:** $x \succ_1 y, y \succ_1 z$
- **Agent 2:** $z \succ_2 x, x \succ_2 y$
- **Agent 3:** $y \succ_3 z, z \succ_3 x$

If we use pairwise majority voting to aggregate the agents' preferences, then we obtain that socially $y \succ z, x \succ y, z \succ x$; in other words, social preferences would be intransitive

Arrow's Impossibility Theorem generalizes this result, and shows that dictatorship (or outside imposition) is required for a consistent aggregation of (at least 3 agents') preferences over (at least 3) independent alternatives.

MORE GENERALLY: QUASI-ORDER ON SETS

Consider a **set** X of **alternatives** (outcomes), which has at least two elements.

Definition: A **quasi-order** R on X is a **binary relation** that is **complete**, **reflexive**, and **transitive**, i.e.,

- For all $x, y \in X$: xRy or yRx (**Completeness**)
- For all $x \in X$: xRx (**Reflexivity**)
- For all $x, y, z \in X$: $xRy, yRz \Rightarrow xRz$ (**Transitivity**)

If xRy , we say that x is "weakly preferred" to y .

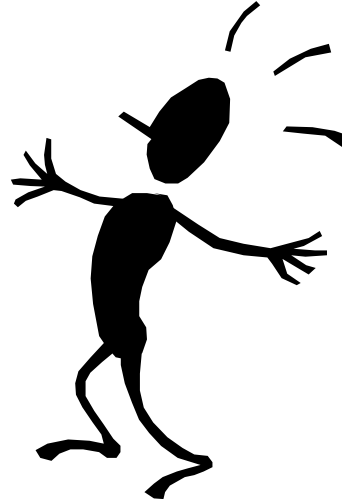
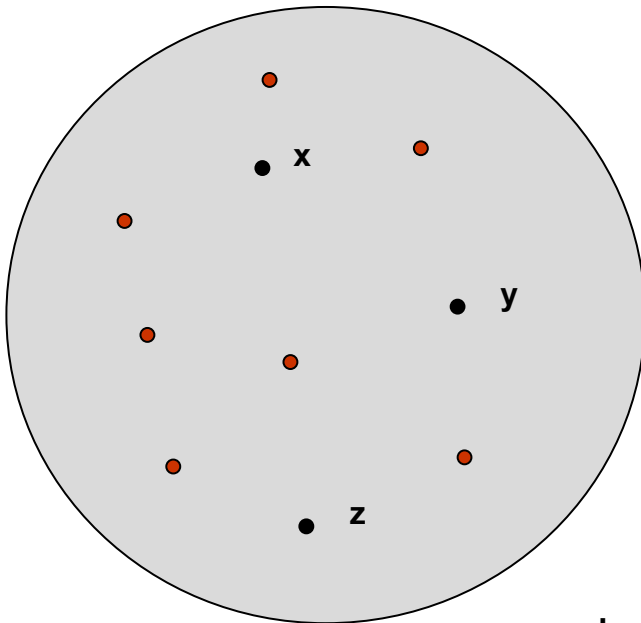
Definition: A **strict partial order** P is a **binary relation** that is **irreflexive** and **transitive**. For any $x, y \in X$ and quasi-order R on X , we define xPy as (not yRx). If xPy , we say that x is "**(strictly) preferred**" to y .⁽¹⁾

(1) In the same spirit we define xIy as (xRy and yRx) and say, if xIy , that x is "indifferent" to y .

Remark: For more on ordering alternatives and the representation of preferences, see Fishburn, P.C. (1970) *Utility Theory for Decision Making*, Wiley, New York, NY. Another useful reference is Kreps, D.M. (1988) *Notes on the Theory of Choice*, Westview Press, Boulder, CO.

RATIONAL PREFERENCE ORDER

Definition. A **preference relation on X** is **rational** if it is a quasi-order on X , i.e., if it is complete, reflexive, and transitive.



Joe can now make 'rational' choices ...

... could they depend on the whole set X ?

RATIONAL PREFERENCES – WHERE DO THEY COME FROM?

Mainstream economic theory does not try to explain preferences, but typically takes preferences as data, that is, as fixed for the economic agent.

Preferences in fact result from many forces, e.g.,

- National culture
- Advertising
- Social institutions and norms
- Parental influence
- Education
- Religion
- Personal tastes

Preferences can be rational – that is, complete & transitive – and still be the result of the various forces. And they can be rational and change over time (e.g., under the influence of advertising)

A CLASS EXPERIMENT

1. You have been given \$200 and have a choice between the following two options

- A: Win \$150 with certainty
- B: Win \$300 with probability .5
Win \$0 with probability .5

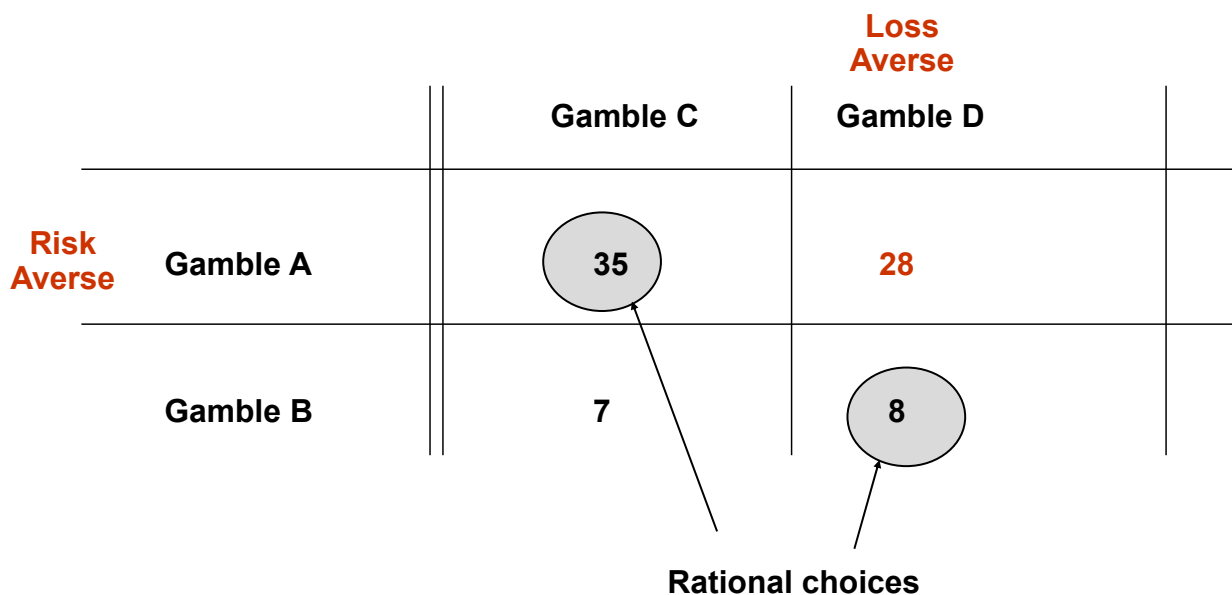
- Do you prefer A or B?

2. You have been given \$500 and have a choice between the following two options

- C: Lose \$150 with certainty
- D: Lose \$300 with probability .5
Lose \$0 with probability .5

- Do you prefer C or D?

RESULT: FRAMING GENERALLY DOES MATTER



Since, $A = C$ and $B = D$, a rational agent's choice should be such that if A is preferred to B then C is preferred to D and vice versa.

However, the “modal choices” are (i.e., “most people prefer”) A and D to avoid losses.

Remark: The *reflection effect* refers to the observation that preferences tend to reverse when the lottery is “reflected” from the domain of gains (with respect to the *status quo*) to the domain of losses. For details, see Kahneman, D., Tversky, A. (1979) “Prospect Theory: An Analysis of Decision Under Risk,” *Econometrica*, Vol. 47, No. 2, pp. 263—291.

ANOTHER EXAMPLE: ELLSBERG PARADOX

An urn is known to contain 90 balls of which 30 are red and the other 60 black or yellow in unknown proportions. (Neither you nor the person with the urn knows the actual proportions.) One ball is to be drawn at random from the urn and your “reward” depends on the color of the ball drawn. You must choose between the following two bets, which have consequences as indicated.

	<u>Red</u>	<u>Black</u>	<u>Yellow</u>
a. Bet on red	\$100	\$0	\$0
b. Bet on black	\$0	\$100	\$0

Now under the same general conditions which bet would you choose in this second situation?

	<u>Red</u>	<u>Black</u>	<u>Yellow</u>
c. Bet on red and yellow	\$100	\$0	\$100
d. Bet on black and yellow	\$0	\$100	\$100

ELLSBERG PARADOX: CLASS RESULTS

	Gamble c	Gamble d
Gamble a	7	58
Gamble b	1	12

‘consistent’ choices

If you prefer a to b then you should prefer c to d because yellow ball is irrelevant for each pair of decisions.

The “modal choices” are (i.e., “most people prefer”) a and d to avoid ambiguity (→ ambiguity aversion) ... we will deal with choice under uncertainty later.

UTILITY REPRESENTATION OF PREFERENCES

Idea: Joe's rational preference relation on a nonempty choice set X could be represented by dots on the real line if there is a “**utility function**” u that maps every element x of X to a real number $u(x)$, such that preferred elements get always mapped to larger real numbers.

Then instead of making a pairwise comparison between elements of X , Joe could ‘simply’ maximize his utility function u on X .

Definition. A function $u : X \rightarrow \mathbb{R}$ is a **utility function** that **represents the preference relation** $\preceq$ on X if for any x, y in X :

$$x \preceq y \iff u(x) \leq u(y)$$

UTILITY REPRESENTATION (Cont'd)

For a utility representation of a preference relation to exist, the preference relation must necessarily be rational!

Proposition. If the function $u : X \rightarrow \mathbb{R}$ represents the preference relation $\preceq$ on X , then $\preceq$ is rational.

Proof (in 2 Steps)

1. Consider any x, y in X . Then, either $u(x) \leq u(y)$ or $u(y) \leq u(x)$. Since u represents $\preceq$, it is therefore either $x \preceq y$ or $y \preceq x$, so that $\preceq$ is **complete**. It is also **reflexive** (trivial).
2. Consider any x, y, z in X , such that $x \preceq y$ and $y \preceq z$. Thus, $u(x) \leq u(y) \leq u(z)$, which implies that $x \preceq z$. Hence, the preference relation $\preceq$ is also **transitive**.

QED

UTILITY REPRESENTATION MAY NOT EXIST!

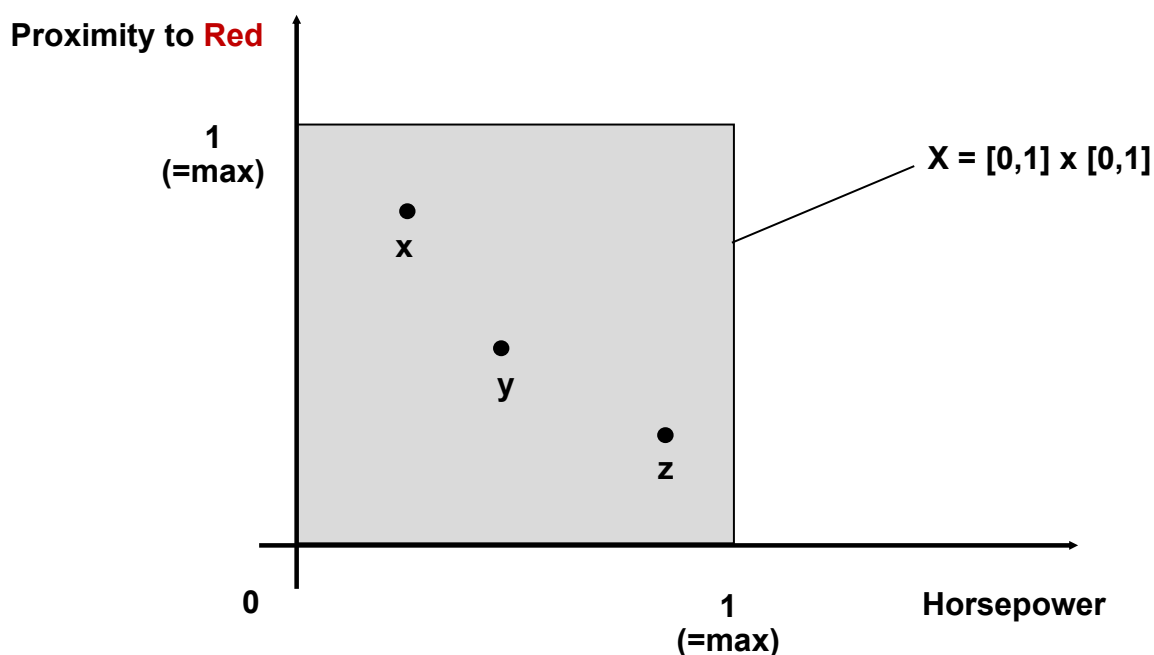
Example: Preferences for a used car.



Joe would like to buy a **Ford Mustang**. He cares about two attributes: horsepower and color. Of two given models, he would always prefer the more powerful one. If they have the same power, then he would take the one that has a color closest to **red**.

—————> Lexicographic Preferences

PREFERENCES FOR USED CAR (Cont'd)



Question. What is Joe's preference ordering?

Right, $x \prec y \prec z$!

AGENDA

Administrivia & Course Overview

Preferences and Utility Representation

Some Properties

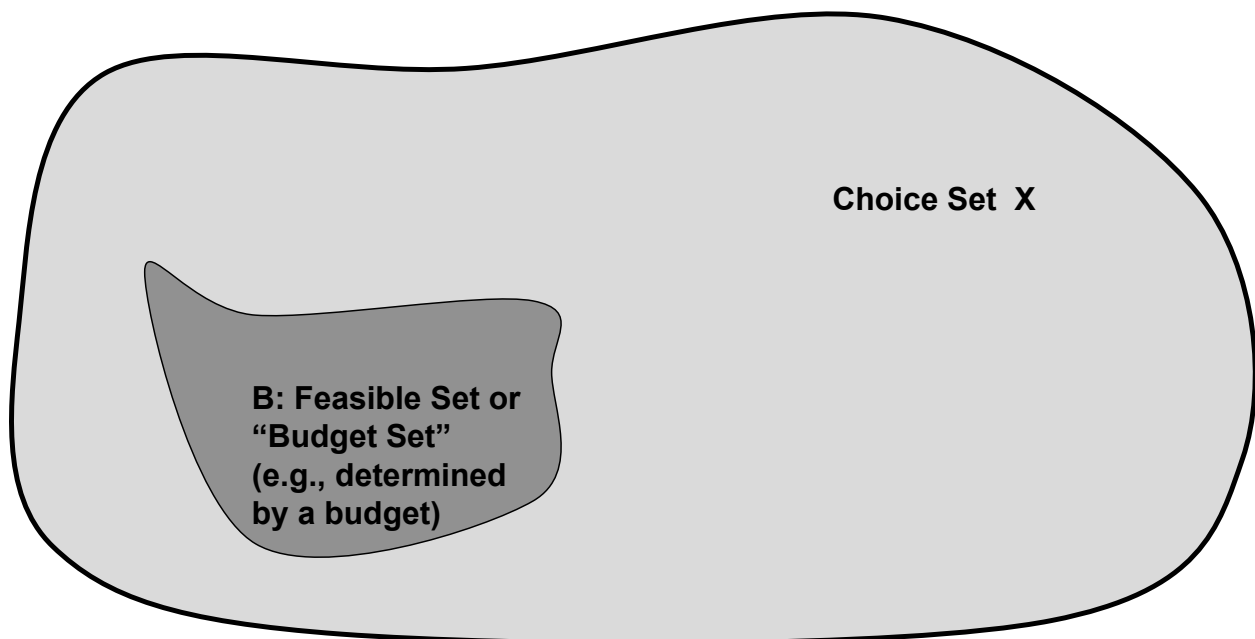
Utility Representation (Cont'd)

Demand Theory: Basics

A Little Refresher on Constrained Optimization

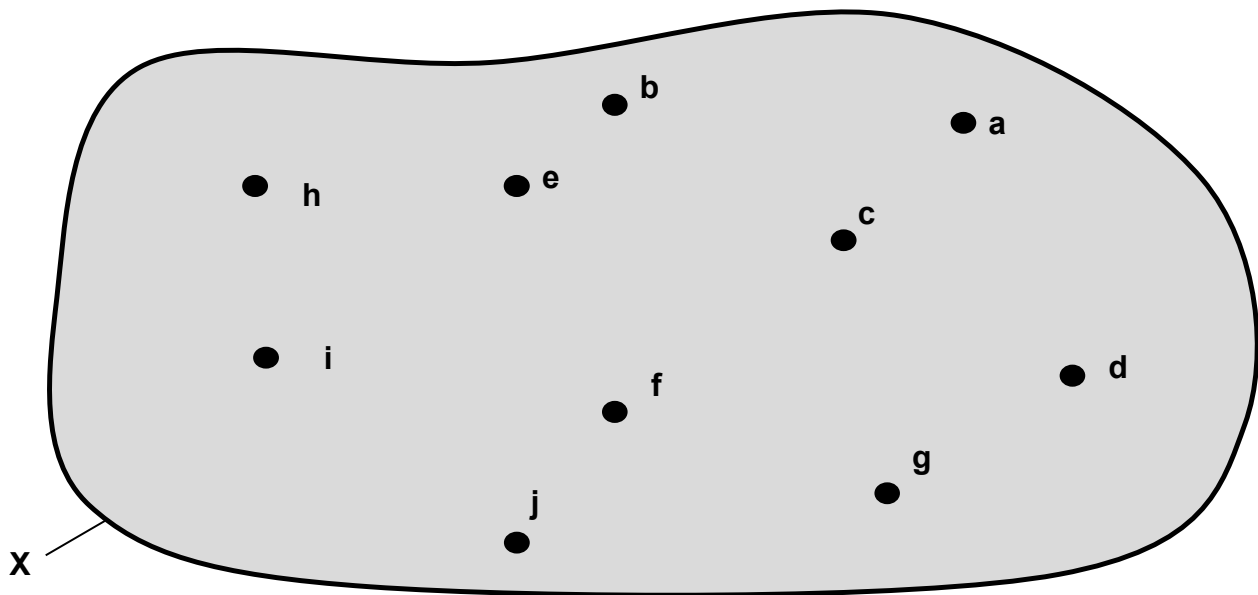
Key Concepts to Remember

ONLY A SUBSET OF THE CHOICE SET MAY BE FEASIBLE



$$B \subset X$$

COMPLETE PREFERENCE RELATION ...



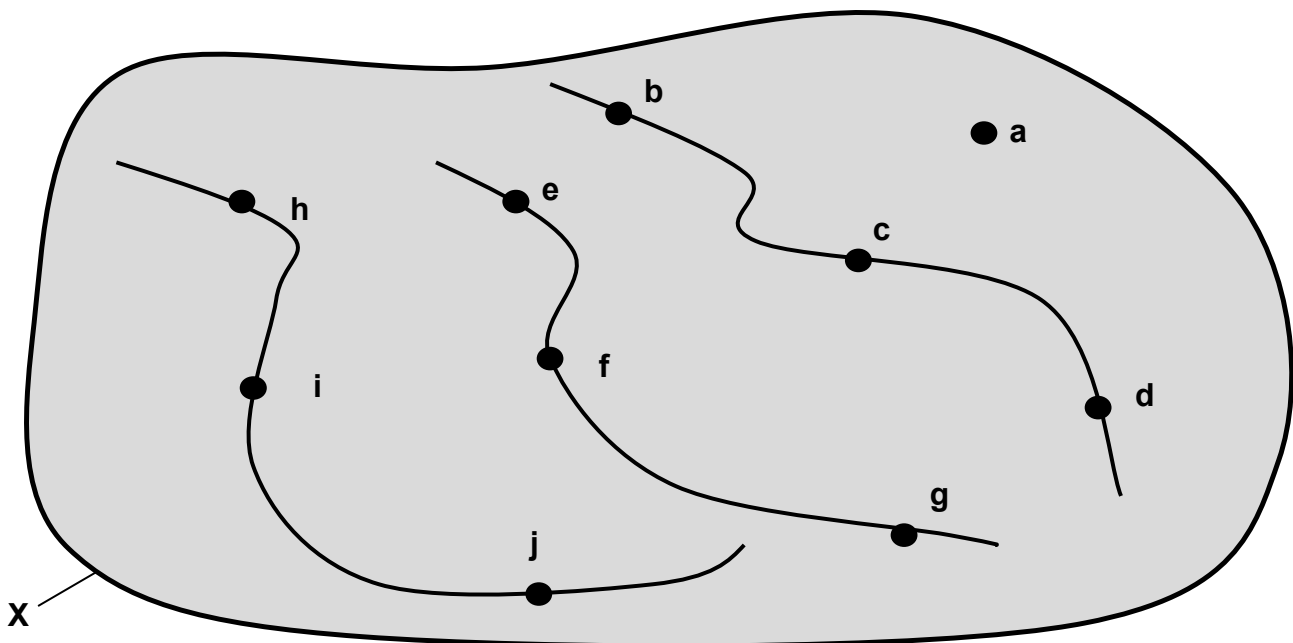
$$a \succeq c \succeq f \succeq j$$

$$b \sim c \sim d$$

$$e \sim f \sim g$$

$$h \sim i \sim j$$

... ALLOWS TO DEFINE “EQUIVALENCE CLASSES” OR “INDIFFERENCE CURVES” WHEN THERE IS A UTILITY FUNCTION



$$a \succeq c \succeq f \succeq j$$

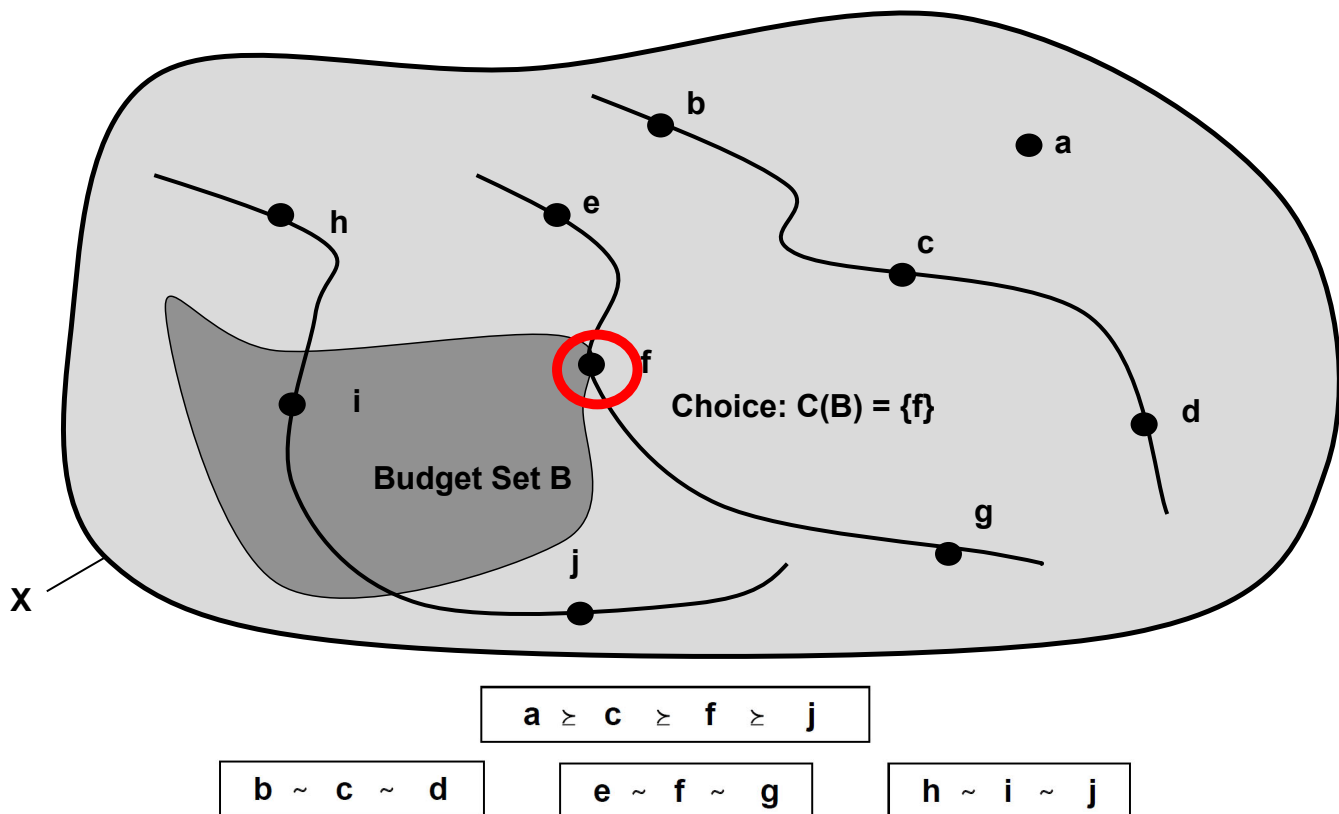
$$b \sim c \sim d$$

$$e \sim f \sim g$$

$$h \sim i \sim j$$

Remark: It is not really necessary to be able to connect all the points of a choice set that an agent is indifferent about by “continuous” indifference curves. These “curves” might not look like curves at all, and do not really have to. What is important, is that different indifference curves do not intersect!

RATIONAL CHOICE: MOST PREFERRED ALTERNATIVE (HERE AT POINT f)



SOME PROPERTIES OF CHOICE

Independence of irrelevant alternatives

- If we reduce the budget set, eliminating points that are not chosen, then the optimal point – the choice point – will not change

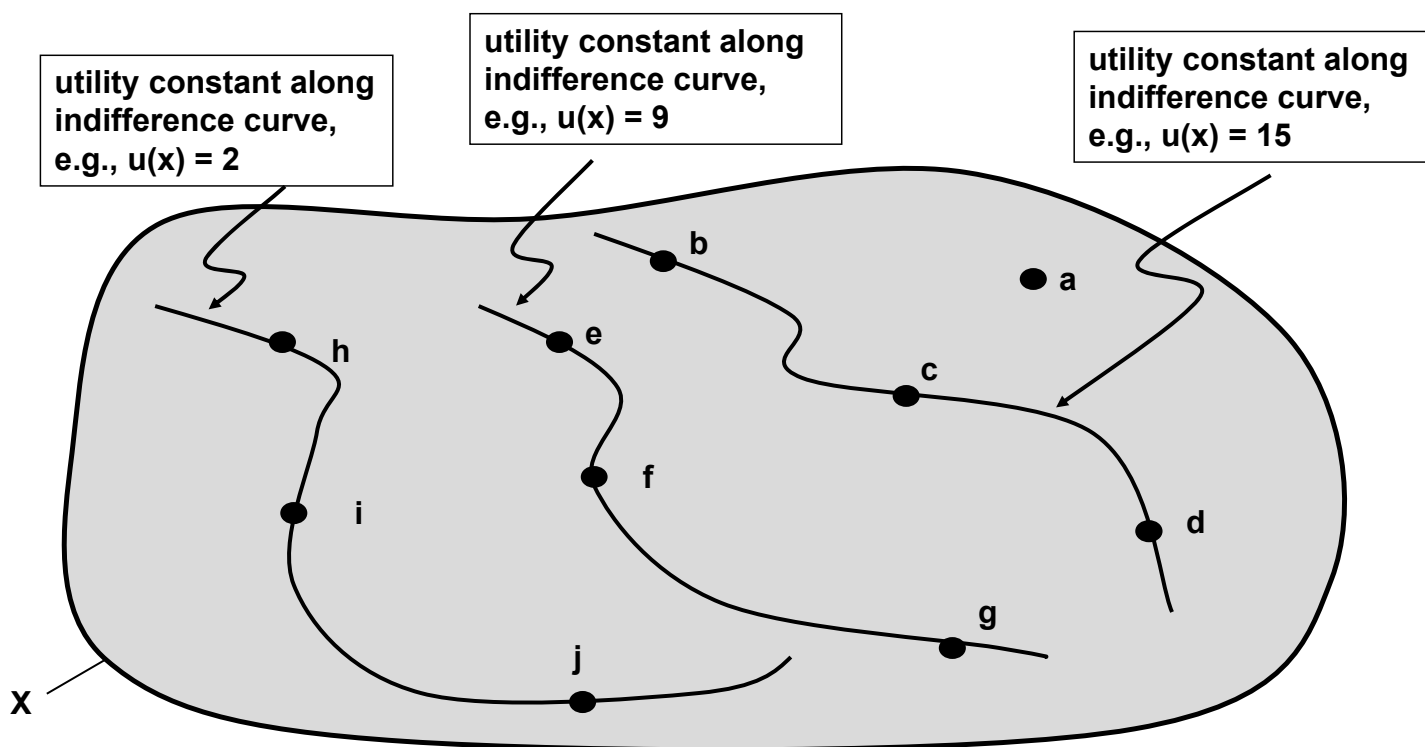
Intensity of preferences is irrelevant to choice

- Saying that 'C is MUCH preferred to F' or that 'C is slightly preferred to F' has no relevance to what point will be chosen

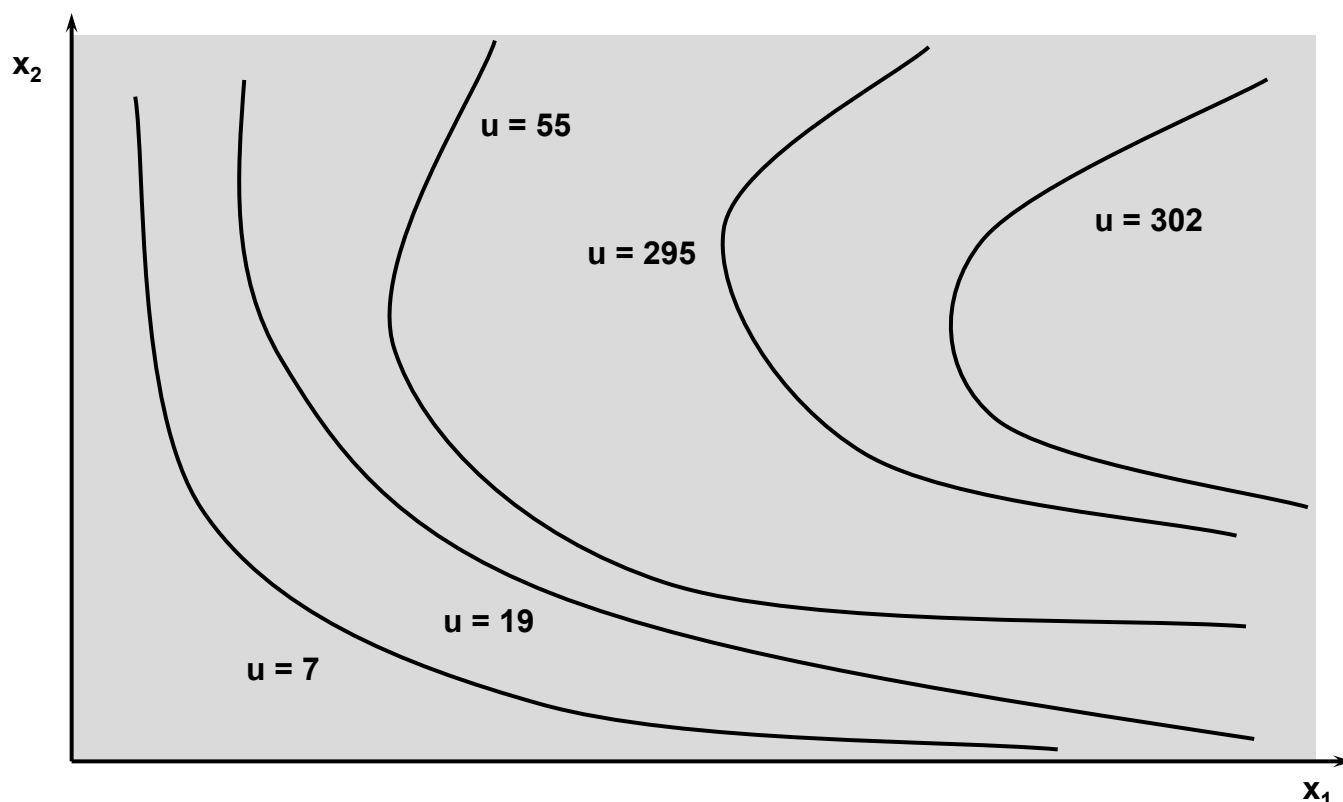
Choice is invariant with respect to changes that leave feasible set unchanged

- Expansion or contraction of choice set (X) has no impact on choice if expansion or contraction does not impact feasible set
- Rescaling of problem parameters that leave the feasible set unchanged will not impact choice

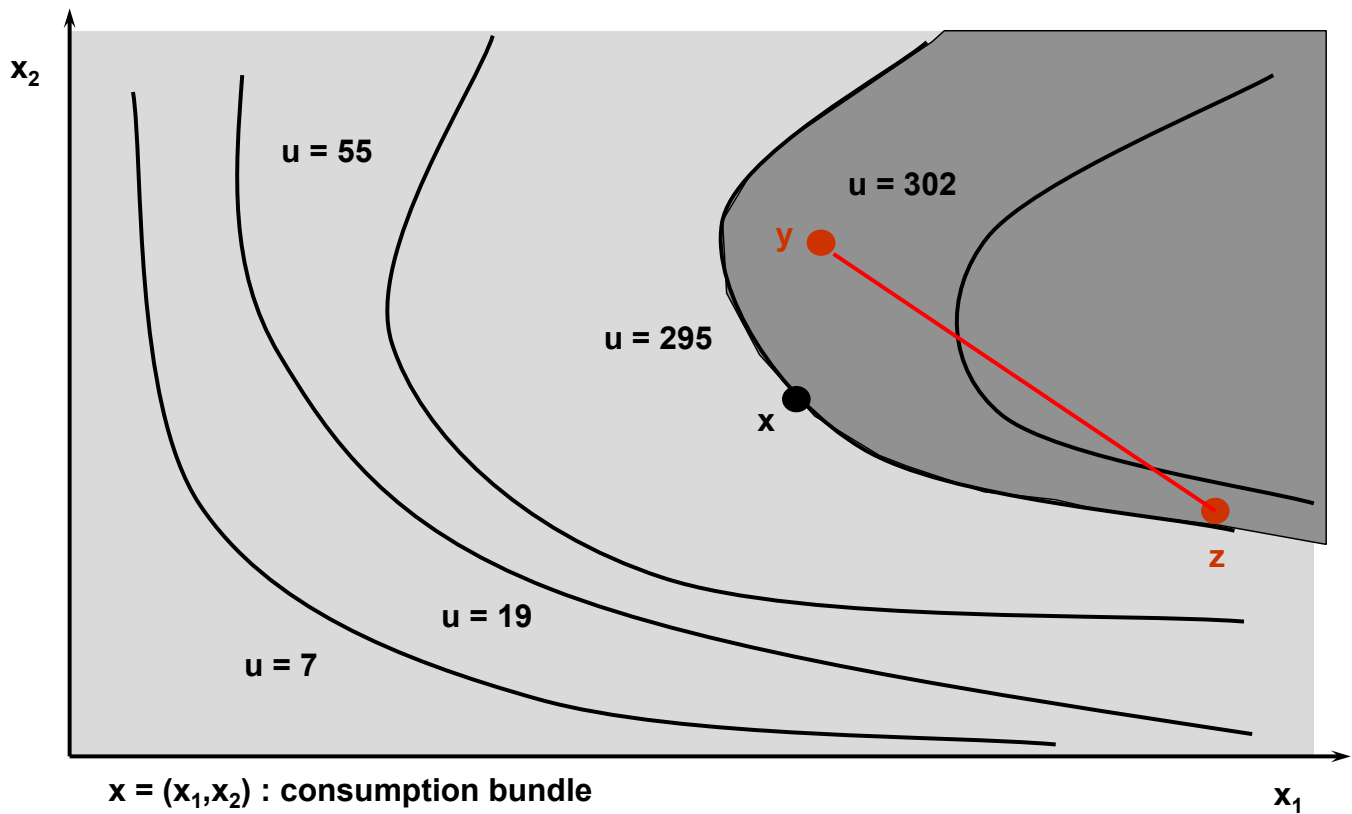
REPRESENTATION OF PREFERENCE RELATION BY UTILITY FUNCTION



EXAMPLE: CONSUMPTION SET WITH INDIFFERENCE CURVES



CONVEXITY OF PREFERENCES



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Some Properties

Utility Representation (Cont'd)

Demand Theory: Basics

A Little Refresher on Constrained Optimization

Key Concepts to Remember

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THEORY OF THE CONSUMER: PREFERENCES

All consumer preferences assumed to be **rational**

- Complete
- Reflexive
- Transitive

Preferences also assumed to be **continuous**

- Preference order does not jump around discontinuously
 $\{x: x \succeq y\}$ and $\{x: y \succeq x\}$ are both closed sets
- Exclude situation: consumer prefers $x(n)$ to y for sequence of $x(n)$ converging to limit $x(\infty)$, but strictly prefers y to $x(\infty)$ ⁽¹⁾

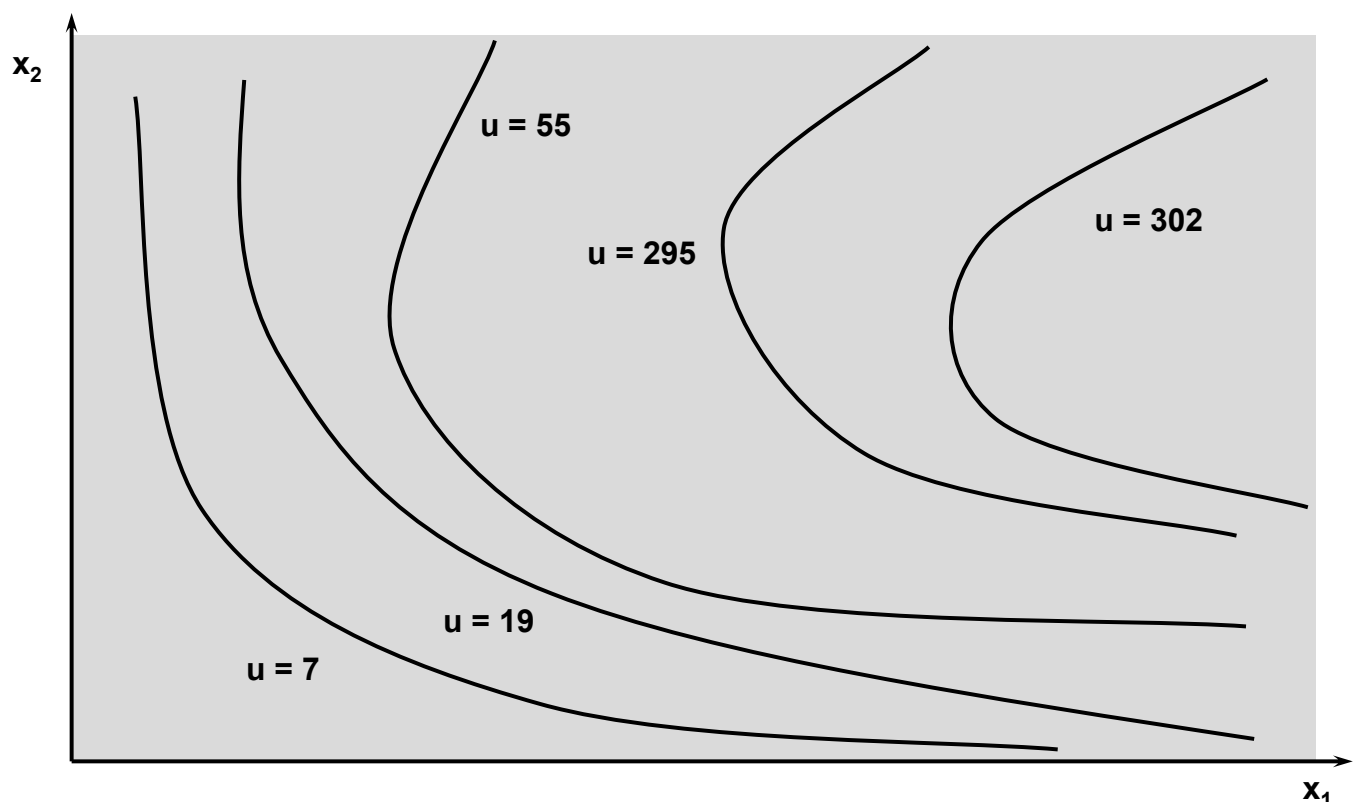
Theorem. If preferences are rational and continuous, then there exists a continuous utility function $u(x)$ that describes preferences.

(1) Example: Take lexicographic preferences on the square $X = [0,1] \times [0,1]$ (as in the first lecture) and let $x(n) = (1/n, 1/n)$, $y = (0,1)$.

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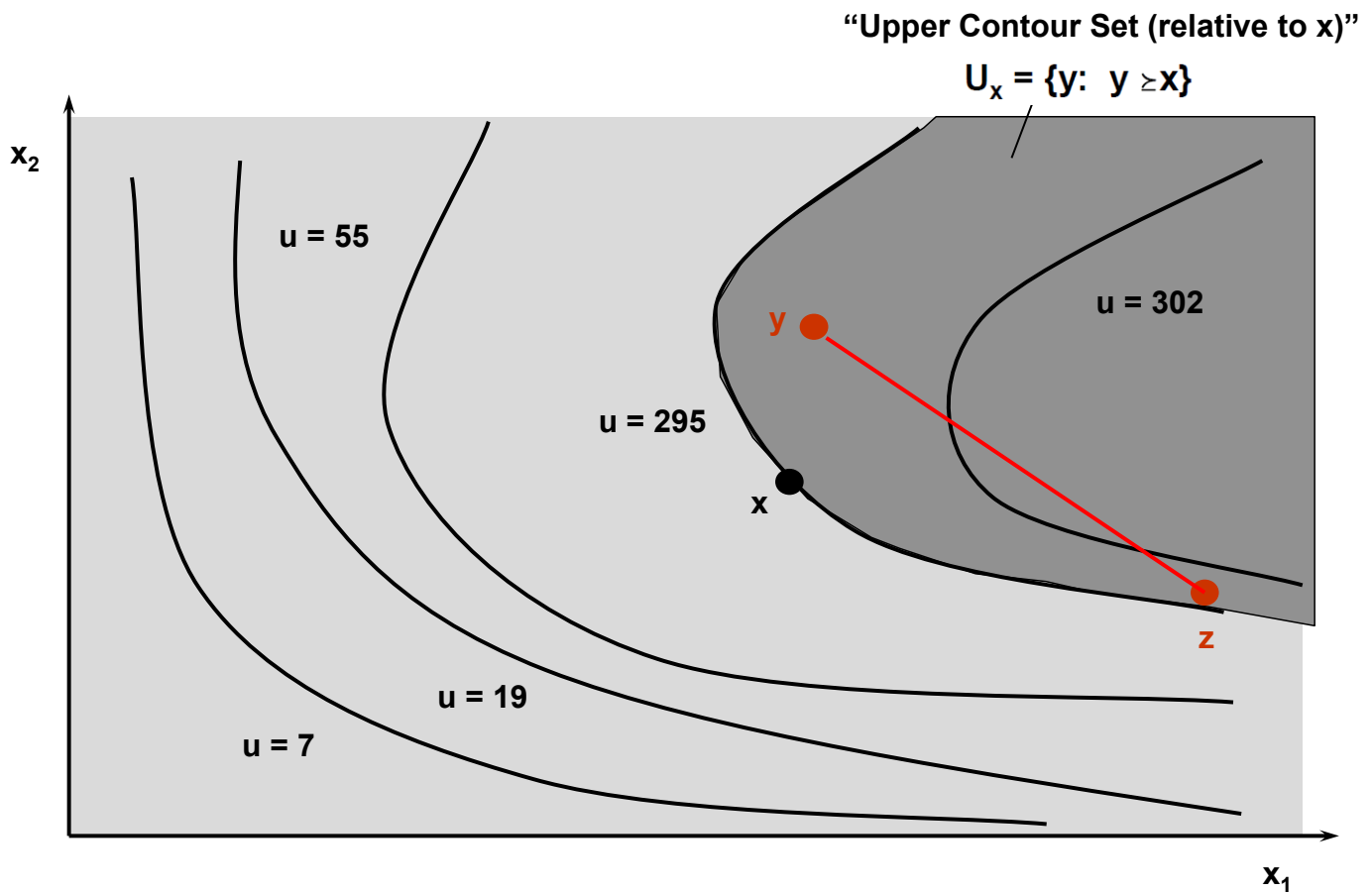
TYPICAL CONSUMPTION SET WITH INDIFFERENCE CURVES



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CONVEXITY OF PREFERENCES



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CONVEXITY

Definition. A rational preference relation $\preceq$ on X is **convex** if the upper contour set $U_x = \{y: x \preceq y\}$ is convex for any x in X , i.e.,

$$y, z \in U_x \Rightarrow \theta y + (1 - \theta)z \in U_x \quad \forall \theta \in (0, 1)$$

Proposition. A utility representation of a convex preference relation is quasi-concave (i.e., single-peaked).

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ORDINAL VS. CARDINAL PROPERTIES

A utility representation $u(x)$ for a given rational preference relation $\succeq$ on X is generally *not unique*.

The preference relation $\succeq$ fixes only ordering of elements of the choice X , and is therefore called **ordinal**.

Given the utility representation $u(x)$ of $\succeq$ on X , the function $v(x) = \phi(u(x))$ is also a utility representation of $\succeq$ on X , as long as the (real-valued) transformation ϕ is increasing.

Each specific utility representation of $\succeq$ on X is called **cardinal**.

Thus, while the ordinal properties of utility functions are invariant with respect to increasing transformations, their cardinal properties are not!

UTILITY FUNCTIONS RELEVANT FOR CONSUMERS AND FIRMS

Theory of Consumers

- Need assumptions about preferences to ensure utility function exists.
- Normally only ordinal properties (which express ordering of options) of utility functions are important.
- For theories of consumer choice under uncertainty, cardinal properties are important. (Cardinal properties express how much better one option is than another.)

Theory of Firms

- Preferences assumed for firms – profit – can always be written as a function, a profit function.
- Profit function plays the same role as utility function.
- For theory of firm behavior under uncertainty, we can use utility function with cardinal properties.

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THEORY OF THE CONSUMER

Choice Set = Consumption Set

Consumer modeled as choosing among **bundles** of commodities (“market baskets”)

Example: x_1 shirts, x_2 lbs of beef, x_3 gallons of gasoline

$x = (x_1, x_2, x_3)$ is vector of these quantities (L-dimensional vector if there are L commodities)

Choice set X (= “Consumption Set”) contains all feasible (not necessarily affordable!) bundles $x \in X$

Standard Assumptions

- We typically include $x = 0$
 - Sometimes $x=0$ may include necessary commodities for survival
 - Sometimes choice set may be discrete, e.g., when only integer amounts of consumption are possible
- We often assume that preferences are **(locally) nonsatiated**. This means that a bigger bundle (if available to the consumer) is always strictly preferred.

PROBLEM IN REALITY?

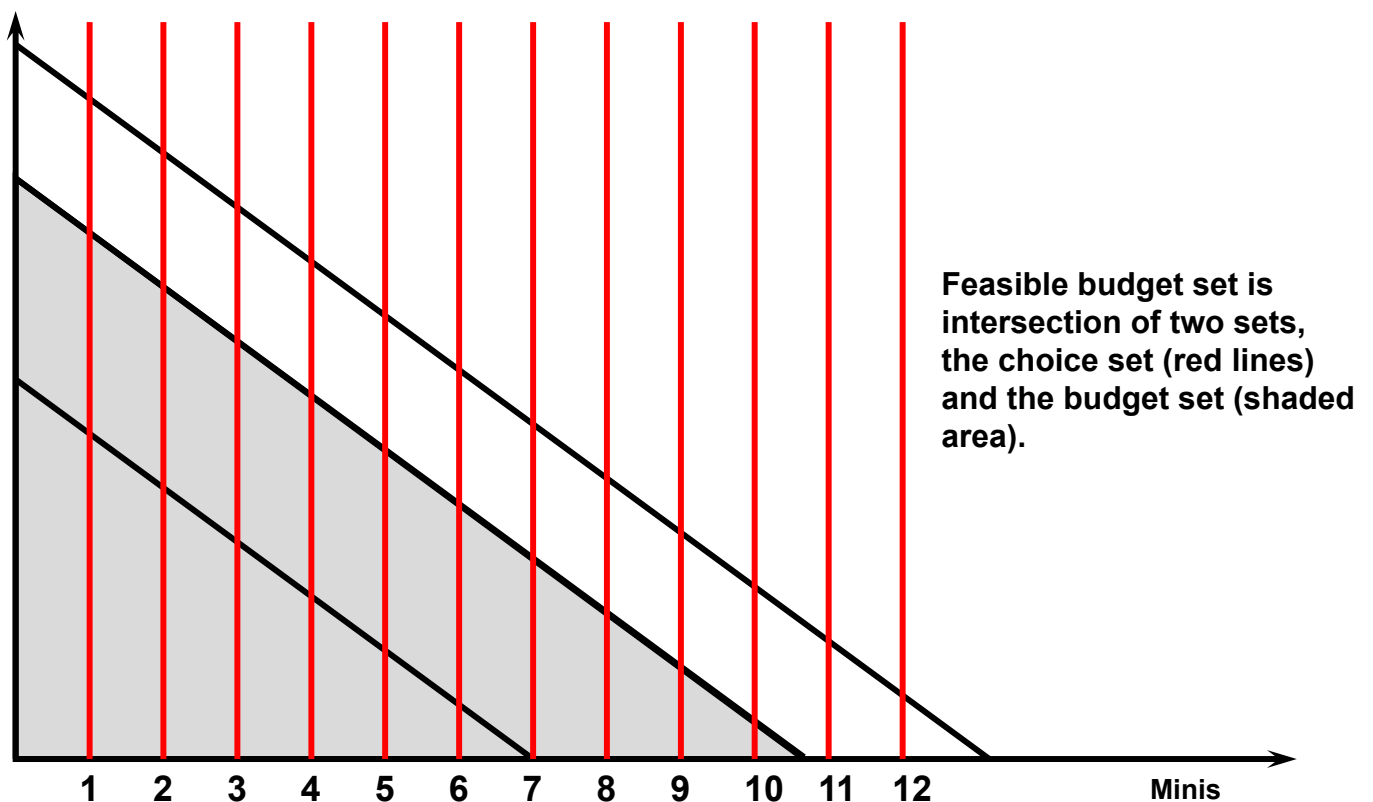


Indivisibilities ... !

BUDGET SET FOR DISCRETE COMMODITIES

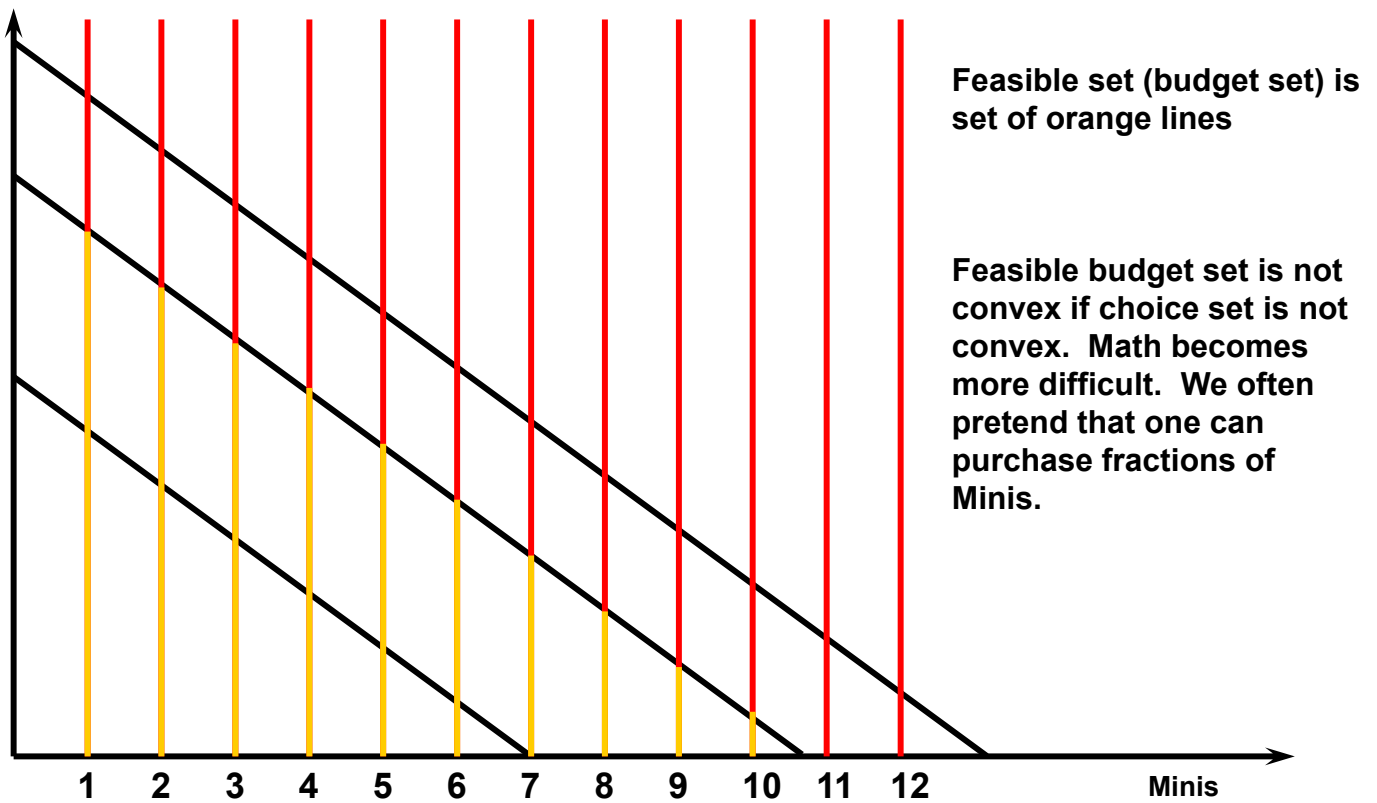
Aggregate of Other
Commodities

Choice set consists of points
on the red lines



BUDGET SET FOR DISCRETE COMMODITIES (Cont'd)

Aggregate of Other
Commodities



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PROBLEM IN REALITY?



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THEORY OF THE CONSUMER

Budget Set

A consumer's choices are constrained to consumption bundles s/he can afford.

- Commodities traded at prices $p_1, p_2, \dots, p_L$
- Prices represented by an L-dimensional price vector $p = (p_1, p_2, \dots, p_L)$
- Assume that consumer cannot influence prices

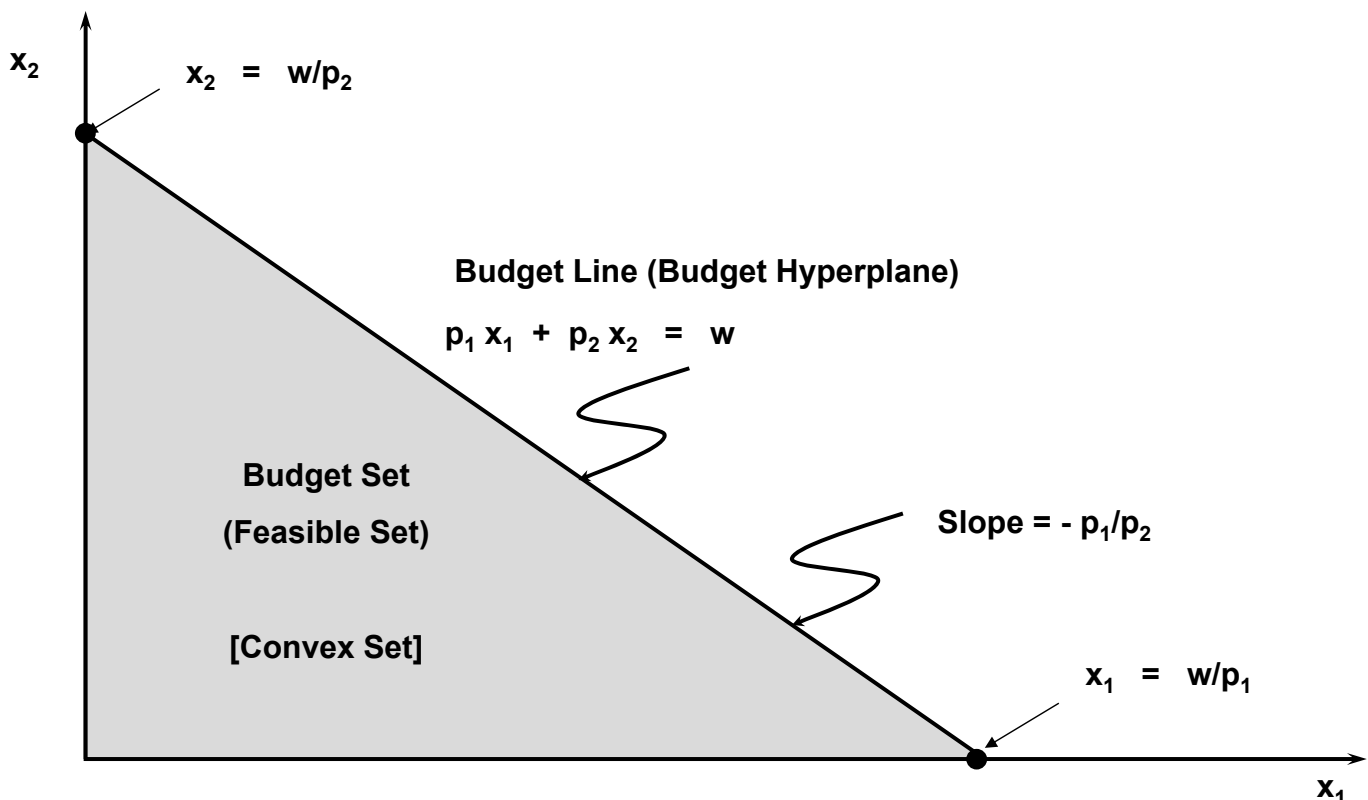
Consumption bundle is affordable if total cost does not exceed the consumer's nonnegative wealth (income), represented by w .

$$p \cdot x \leq w$$

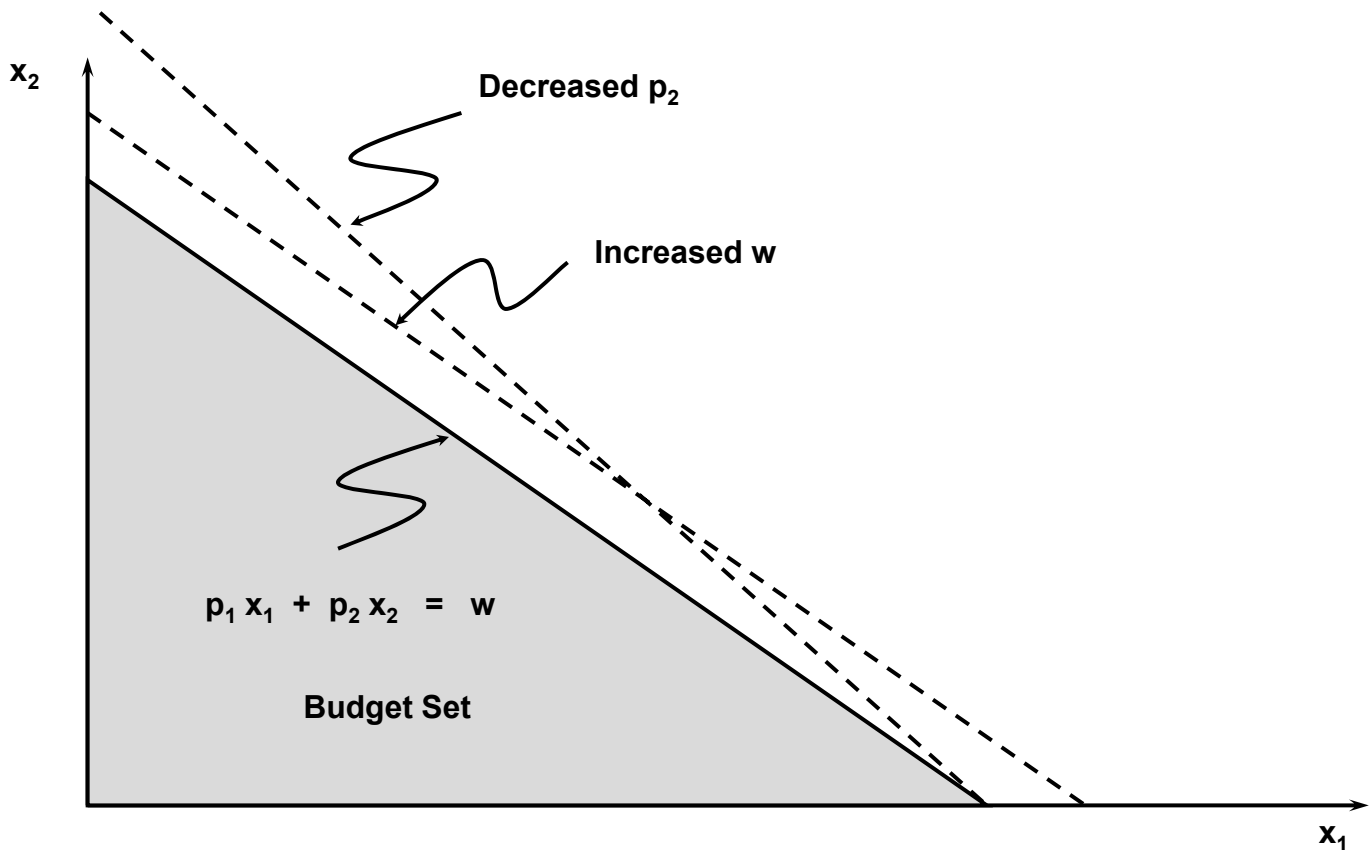
$$p_1 x_1 + p_2 x_2 + p_3 x_3 + \dots + p_L x_L \leq w$$

Set of bundles x in X that satisfy this constraint are known as the **budget set $B(p,w)$** .

BUDGET SET



BUDGET SET DEPENDS ON PRICES AND WEALTH



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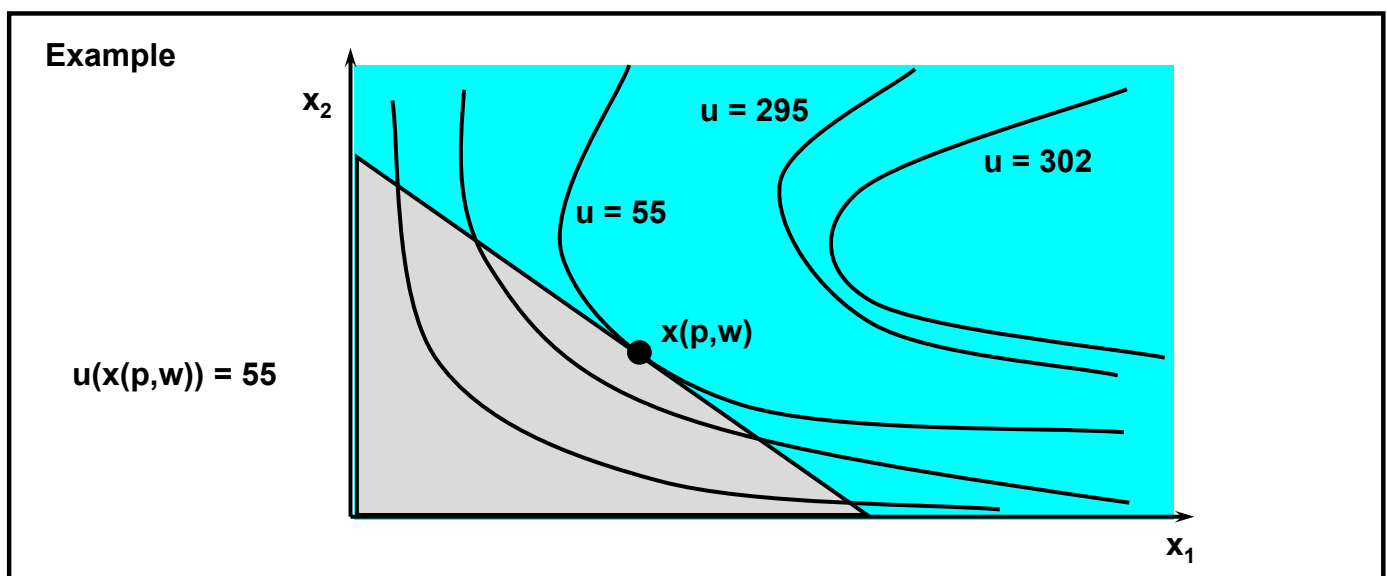
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RATIONAL CHOICE = UTILITY MAXIMIZATION PROBLEM

Consumer chooses by maximizing utility over all alternatives in budget set, i.e., s/he solves

$$\begin{array}{ll} \text{Maximize} & u(x) \\ \text{such that} & p \cdot x \leq w \end{array}$$

$x(p, w)$ denotes the optimal choice, and is referred to as the demand function



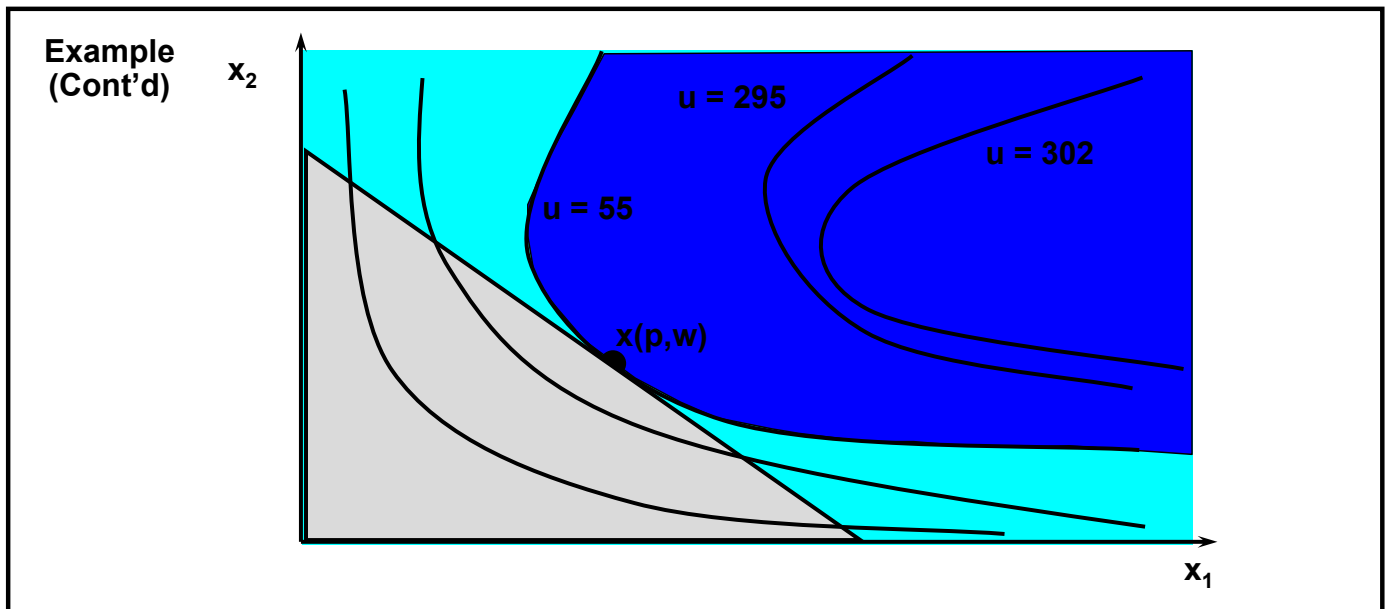
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UTILITY MAXIMIZATION PROBLEM (Cont'd)

$x(p,w)$ denotes the **optimal choice**, or (“**Walrasian**”) **demand function**.

- No feasible point (x) has $u(x) > 55$
- No feasible point strictly preferred to $x(p,w)$.
- Set of preferred points (= upper contour set) and feasible set have no common interior points.

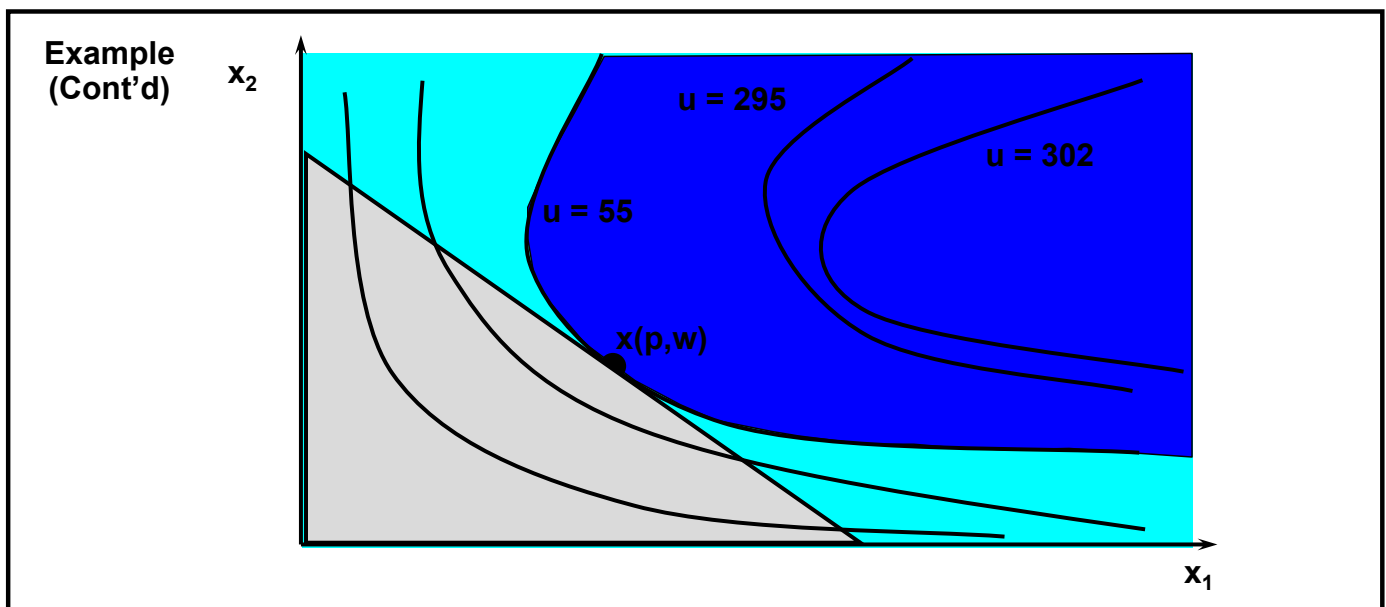


EQUIVALENT EXPENDITURE MINIMIZATION PROBLEM (Cont'd)

Equivalently, $x(p,w)$ solves a second problem:

$$\begin{aligned} &\text{Minimize } p \cdot x \\ &\text{such that } u(x) \geq 55 \end{aligned}$$

That is, minimize expenditure, under constraint that $u(x) \geq 55$



PROPERTIES OF THE CONSUMER DEMAND FUNCTION

Homogeneity of degree zero in p, w :

$$x(\alpha p, \alpha w) = x(p, w) \quad \text{for any } \alpha > 0$$

Walras' Law:

$$p \cdot x(p, w) = w$$

(holds if preferences are locally nonsatiated)

Convexity:

If preferences are convex, then $x(p, w)$ is a convex set

Uniqueness:

If preferences are strictly convex, then $x(p, w)$ is a single point

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Utility Representation (Cont'd)

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A Little Refresher on Constrained Optimization

Key Concepts to Remember

CONSTRAINED OPTIMIZATION

Let $f : X \rightarrow \mathbb{R}$ be a real-valued function, where X is a nonempty, compact subset of $\mathbb{R}^n$,⁽¹⁾ where n is a positive integer.

We would like to maximize $f(x)$ on X , i.e., solve the problem

$$\max_{x \in X} f(x)$$

Objective Function (Maximand)
Constraint Set

(Constrained Optimization Problem)

Remark:

The utility maximization problem is of this form, with $f(x) = u(x)$ and $X = \{x \in \mathbb{R}_+^L : p \cdot x \leq w\}$ for some price vector $p = (p_1, \dots, p_L) \gg 0$, wealth $w > 0$, and number of commodities $L > 0$.

(1) A subset of a finite-dimensional Euclidean space is *compact* if it is closed and bounded. A set is *closed* if the limit of any (sub-)sequence constructed from elements of the set also lies in the set. A solution to a constrained optimization problem on a compact set exists, provided the objective function f is continuous.

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OPTIMALITY CONDITIONS WHEN CONSTRAINT NOT BINDING One-Dimensional Case

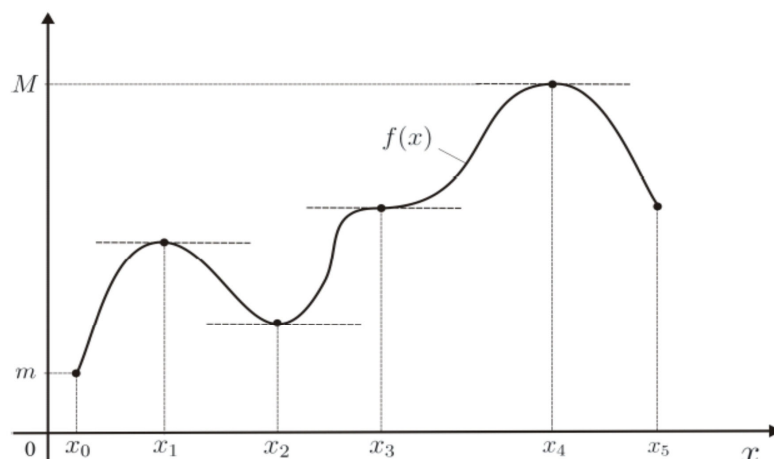
Fermat's Rule (= First-Order Necessary Optimality Condition ["FOC"])

$$x \text{ local extremum} \Rightarrow f'(x) = 0$$

Example:

$$X = [x_0, x_5] \subset \mathbb{R}$$

f differentiable



Note that the FOC is satisfied for **local** maxima **and** local minima in the **interior** of X .

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OPTIMALITY CONDITIONS (Cont'd) One-Dimensional Case

In order to guarantee that one has arrived at an interior maximizer (= optimal x that maximizes the objective function and does not lie on the boundary of X), one can use additional optimality conditions: second-order optimality conditions

$$x \text{ local maximizer} \Rightarrow f''(x) \leq 0$$

$$x \text{ local minimizer} \Rightarrow f''(x) \geq 0$$

Second-Order **Necessary**
Optimality Condition

$$f''(x) < 0 = f'(x) \Rightarrow x \text{ local maximizer}$$

$$f''(x) > 0 = f'(x) \Rightarrow x \text{ local minimizer}$$

Second-Order **Sufficient**
Optimality Condition ["SOC"]

Examples (Gap Between Necessary and Sufficient Second-Order Optimality Conditions):

(a) The function $f(x) = 1 - x^4$ has a maximum at $x = 0$ which does not satisfy SOC.

(b) The function $f(x) = 1 - x^3$ has no extremum even though it satisfies $f''(0) \leq 0 = f'(0)$.

OPTIMALITY CONDITIONS WHEN CONSTRAINT NOT BINDING Multidimensional Case

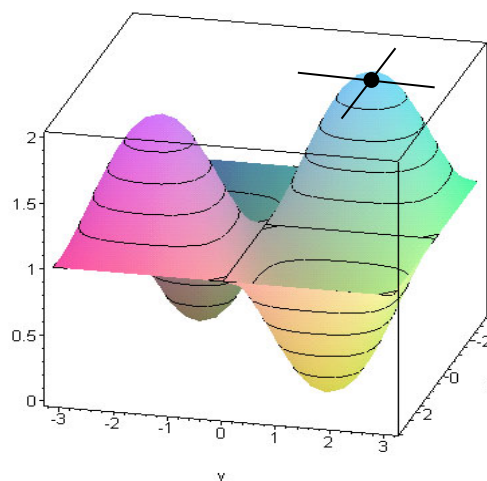
Fermat's Rule generalizes to the case of multiple dimensions ($n > 1$)

$$x \text{ local extremum} \Rightarrow Df(x) = (\partial f(x) / \partial x_1, \dots, \partial f(x) / \partial x_n) = 0$$

Example:

$$X = [-\pi, \pi]^2 \subset \mathbb{R}^2$$

$$f(x) = 1 - \sin(x_1) \cos(x_2)$$



Again, the FOC is satisfied for **local** maxima **and** local minima in the **interior** of X .

CONSTRAINED OPTIMIZATION

Let $f: X \rightarrow \mathbb{R}$ be a real-valued objective function, $X = \{x \in \mathbb{R}^n : g(x) \leq 0\}$ be a nonempty compact constraint set, where $g: X \rightarrow \mathbb{R}^k$ is a vector-valued function.

The **standard constrained optimization problem** is then often written in the form

$$\max_{x \in \mathbb{R}^n} f(x), \quad \text{s.t.} \quad g(x) \leq 0$$

Idea:

Relax this problem by introducing k additional variables (“Lagrange Multipliers”), one for each constraint component.

Then find critical points (= points that satisfy FOC) of “Lagrangian” L (= relaxed objective function), where

$$L(x; \lambda) = f(x) - \lambda \cdot g(x)$$

CONSTRAINED OPTIMIZATION: INTUITION

Interpret $Dg(x)$ as a vector perpendicular to frontier (determined $g(x) = 0$), pointing in direction of increasing $g(x)$.

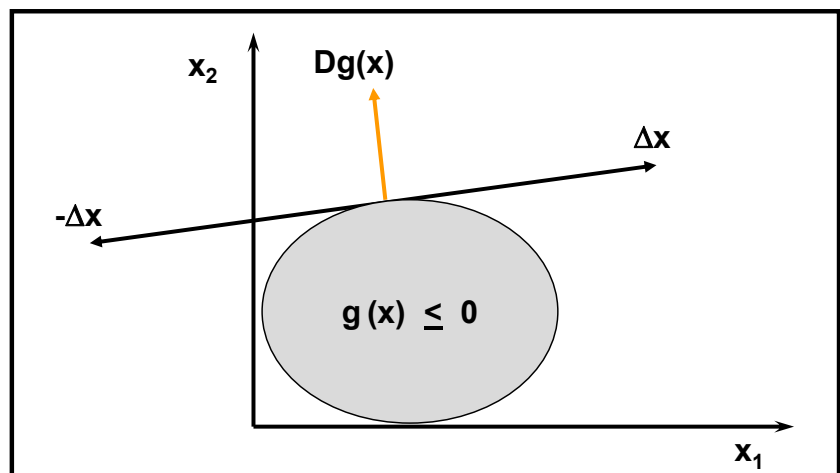
Choose Δx that is tangent to frontier. For tiny movements along Δx or along $-\Delta x$, the function $g(x)$ does not change in value. Thus,

$$\Delta x_1 \frac{\partial g}{\partial x_1} + \dots + \Delta x_n \frac{\partial g}{\partial x_n} = 0 \quad \text{or} \quad \Delta x \cdot Dg(x) = 0$$

Any two vectors whose inner product is zero must be perpendicular to each other. Thus, Δx and $Dg(x)$ are perpendicular to each other.

Now take $\Delta x = Dg(x)$ (assumed nonzero). Then, $\Delta g = \Delta x \cdot Dg(x) > 0$, because all components of Δg are $(\partial g / \partial x_i)^2 > 0$.

Hence, $g(x)$ is increasing in direction of $Dg(x)$.



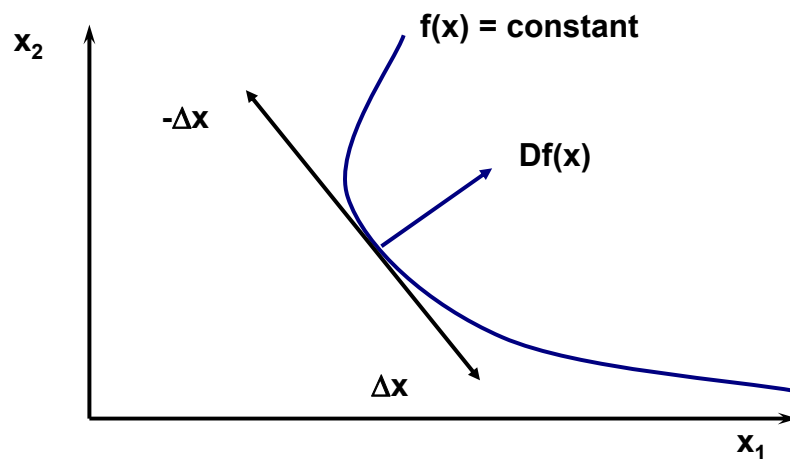
CONSTRAINED OPTIMIZATION: INTUITION (Cont'd)

Interpret $Df(x)$ as a vector perpendicular to the level set $\{y : f(x) = f(y)\}$

This vector is pointed in the direction of increasing value of $f(x)$.

Choose Δx that is tangent to the level set, $f(x) = \text{constant}$. For tiny movements along Δx or along $-\Delta x$, $f(x)$ does not change in value. Thus $\Delta x_1 \frac{\partial f}{\partial x_1} + \dots + \Delta x_n \frac{\partial f}{\partial x_n} = 0$ or $\Delta x \cdot Df(x) = 0$.

Thus, Δx and $Df(x)$ are perpendicular to each other.



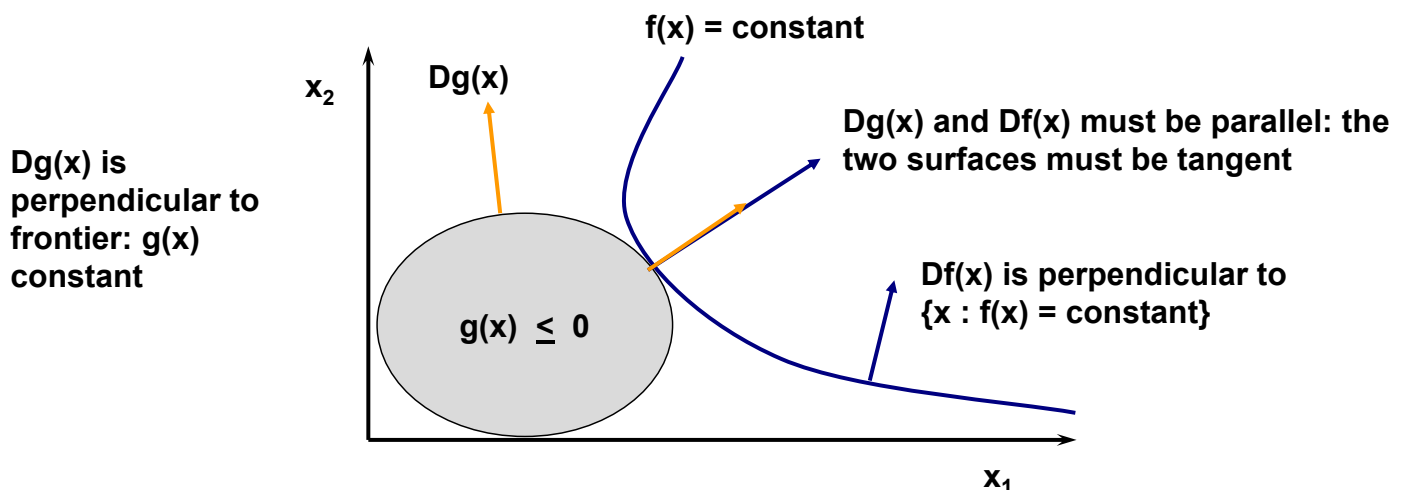
INTUITION OF FIRST-ORDER CONDITION

First-Order Necessary Optimality Condition for Constrained Optimization:

If the constraint $g(x)=0$ is binding, then a level set of f must be tangent to the constraint set at an extremal point. This implies that the gradient of f and the gradient of the constraint function g need to be parallel.

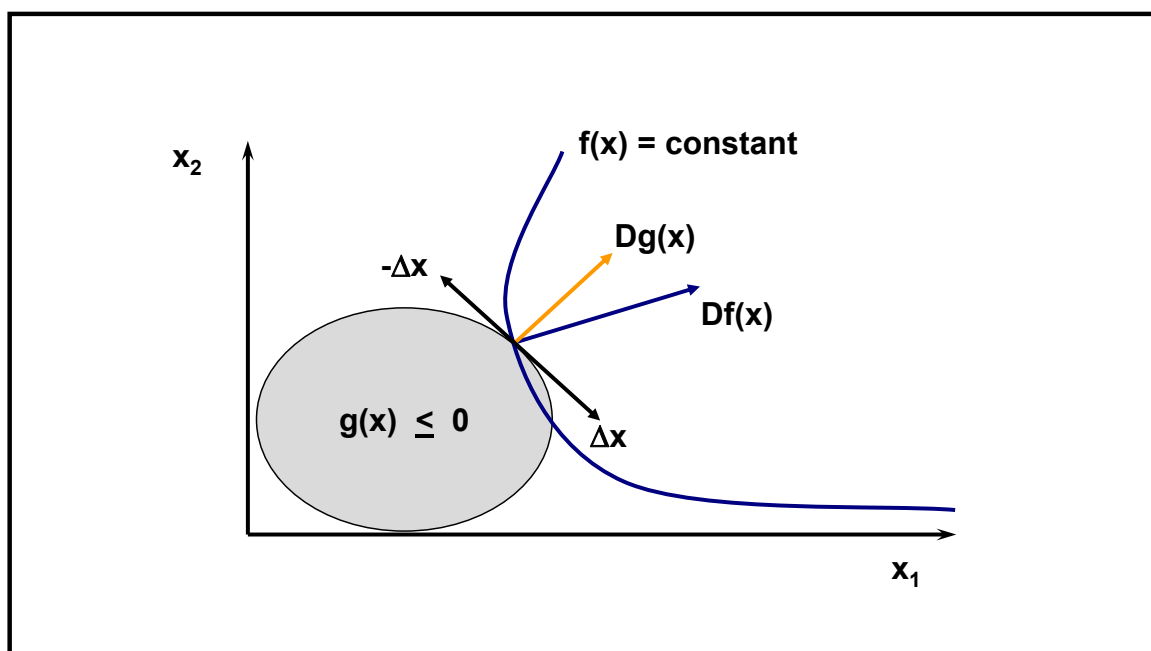
$$Df(x) = \lambda Dg(x) \quad \text{for some } \lambda \geq 0 \quad (\lambda \text{ is a scalar})$$

If constraint is binding (i.e., if $g(x) = 0$), then $\lambda > 0$



INTUITION OF FIRST-ORDER CONDITION

If $Dg(x)$ and $Df(x)$ are not parallel, there are feasible points with greater $f(x)$. They can be found by moving tiny distance in direction Δx or $-\Delta x$.



FIRST-ORDER NECESSARY OPTIMALITY CONDITION FOR CONSTRAINED OPTIMIZATION PROBLEM Formal Statement

Let $L(x; \lambda) = f(x) - \lambda \cdot g(x)$ be the Lagrangian associated with the constrained optimization problem

$$\max_{x \in \mathbb{R}^n} f(x), \quad \text{s.t.} \quad g(x) \leq 0$$

Necessary Optimality Conditions (Kuhn-Tucker Conditions):⁽¹⁾

$$x \text{ is local extremum of } f(x) \text{ in } \{x: g(x) \leq 0\} \Rightarrow \begin{aligned} D_x L(x; \lambda) &= 0, \\ \lambda_i g_i(x) &= 0, i \in \{1, \dots, k\} \end{aligned}$$

The k relations $\lambda_i g_i(x) = 0, i \in \{1, \dots, k\}$ are also referred to as **complementary slackness conditions**. The variables λ_i are called **Lagrange multipliers** or **dual variables**.

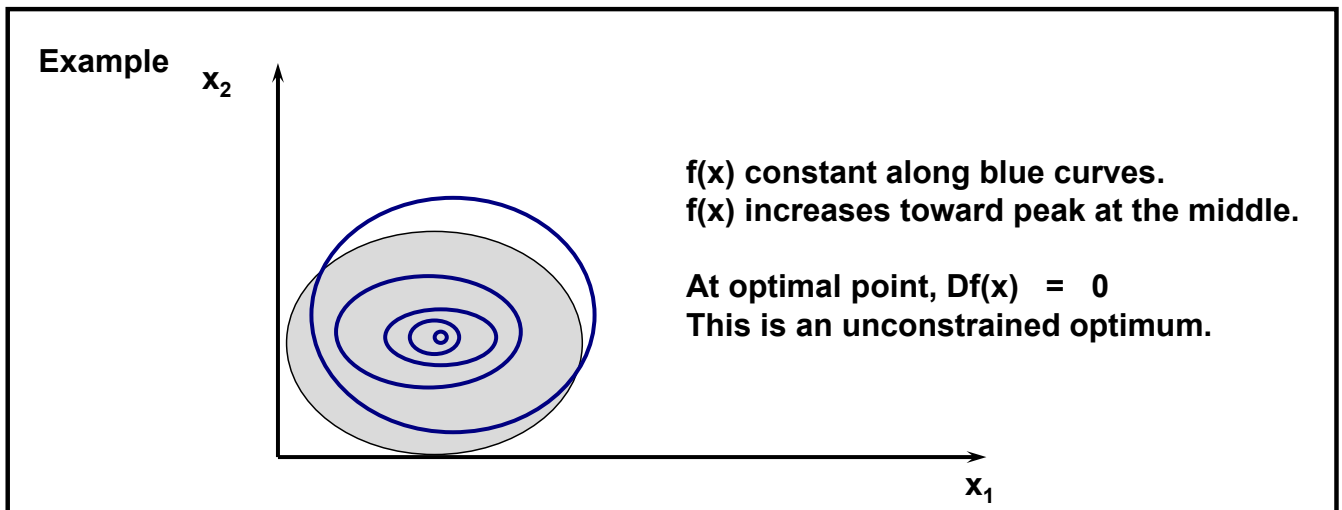
⁽¹⁾ The validity of these conditions depends on whether the constraint function g satisfies a "constraint qualification" in a neighborhood of the extremum, which means that in this neighborhood the constraints are nonredundant, so that $Dg(x)$ is of full rank (i.e., of rank k) there.

RELATION BETWEEN FIRST-ORDER CONDITIONS FOR CONSTRAINED AND UNCONSTRAINED OPTIMIZATION PROBLEMS

Consider the constrained optimization problem

$$\max_{x \in \mathcal{R}^n} f(x), \quad \text{s.t.} \quad g(x) \leq 0$$

If, at an extremum x , the constraint is not binding, i.e., if $g(x) < 0$, then complementary slackness implies that all Lagrange multipliers vanish.



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CONSTRAINED OPTIMIZATION WITH MULTIPLE CONSTRAINTS Intuition

Consider the following problem:

$$\max_x f(x), \quad \text{s.t.} \quad g_1(x) \leq 0 \quad \text{and} \quad g_2(x) \leq 0$$

First Order Necessary condition:

$$Df(x) = \lambda_1 Dg_1(x) + \lambda_2 Dg_2(x) \quad [Df(x) \text{ lies between } Dg_1(x) \text{ and } Dg_2(x)]$$

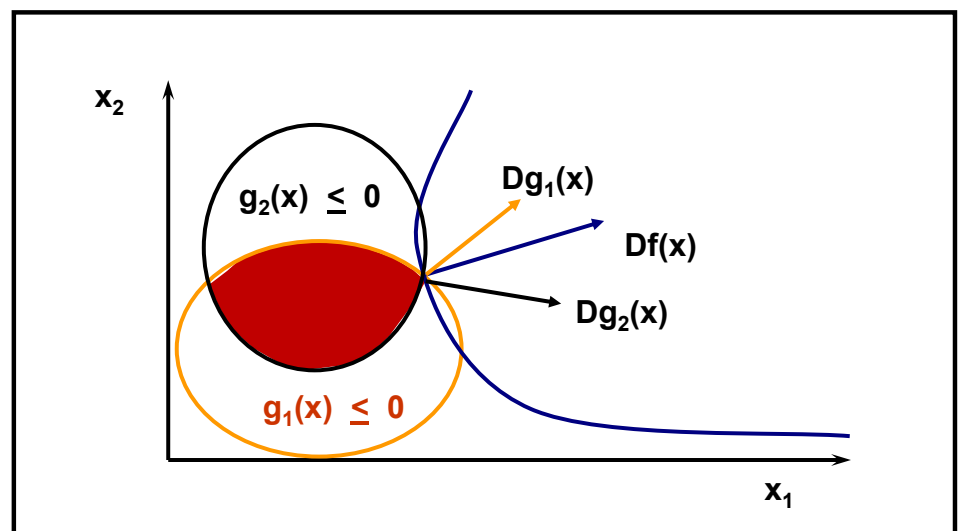
$$g_1(x) \leq 0 \quad \lambda_1 \geq 0$$

$$g_2(x) \leq 0 \quad \lambda_2 \geq 0$$

$$\lambda_1 g_1(x) = 0$$

$$\lambda_2 g_2(x) = 0$$

**Red area is feasible
set with two constraints**



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INTERPRETATION OF THE DUAL VARIABLES

λ corresponds to an increase in the optimized objective function f per unit relaxation of the constraint $g(x) \leq 0$. (Relaxation means $g(x) \leq b$, for very small vector $b \gg 0$)

Show: $\Delta f = \lambda \cdot b$

$$\Delta f = \frac{\partial f}{\partial x_1} \Delta x_1 + \dots + \frac{\partial f}{\partial x_n} \Delta x_n$$

$$\Delta g = \frac{\partial g}{\partial x_1} \Delta x_1 + \dots + \frac{\partial g}{\partial x_n} \Delta x_n = b$$

Calculate Δu , remembering first-order necessary optimality condition, $\frac{\partial f}{\partial x_i} = \lambda \frac{\partial g}{\partial x_i}$

$$\Delta f = \lambda \frac{\partial g}{\partial x_1} \Delta x_1 + \lambda \frac{\partial g}{\partial x_2} \Delta x_2 + \dots = \lambda \Delta g$$

The dual variables (Lagrange multipliers) are equal to the value of being able to relax the constraints. They are often called the **shadow prices** of the problem.

CONSUMER SOLVES UTILITY MAXIMIZATION PROBLEM

Choice is the maximum utility alternative from feasible set, in this case, the budget set

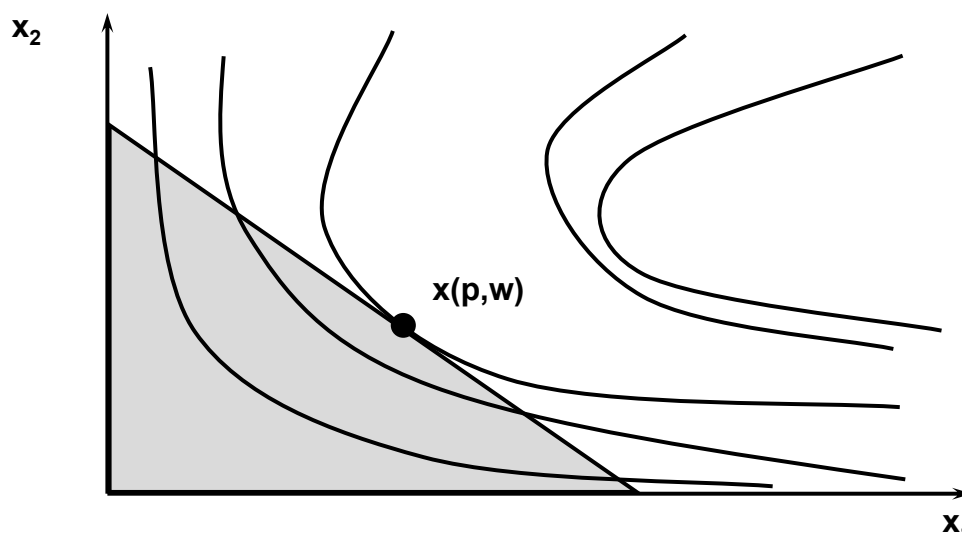
Maximize $u(x)$

such that:

$$p \cdot x \leq w$$

$$x \geq 0$$

$x(p, w)$ denotes the optimal choice, or Walrasian demand function, given p and w .



CHARACTERIZING OPTIMAL CHOICE

$$\begin{array}{ll} \text{Maximize } u(x) \\ \text{such that:} & p \cdot x \leq w \\ & x \geq 0 \end{array}$$

Constraints can be written in standard form:

$$\begin{array}{ll} p \cdot x - w \leq 0 & \text{becomes } g_0(x) = p \cdot x - w \leq 0 \\ -x \leq 0 & \text{becomes } g_i(x) = -x_i \leq 0 \text{ for each } i \text{ in } \{1, \dots, L\} \end{array}$$

First-order necessary optimality conditions:

$$Du(x) = \sum_{i=0}^L \lambda_i Dg_i(x) = \lambda_0 p - \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_L \end{bmatrix}$$

and

$$\begin{array}{l} \lambda_0 (p \cdot x - w) = 0 \\ \lambda_i x_i = 0, i \in \{1, \dots, L\} \end{array}$$

INTERPRETATION OF LAGRANGE MULTIPLIERS

$$Du(x) = \lambda_0 p - (\lambda_1, \dots, \lambda_L)$$

$$(p \cdot x - w) \lambda_0 = 0$$

with local nonsatiation, $p \cdot x = w$ and $\lambda_0 > 0$

$$(\lambda_1, \dots, \lambda_L) \cdot x = 0$$

if $x_i > 0$, then $\lambda_i = 0$

Thus:

$$\begin{array}{ll} \partial u / \partial x_i = \lambda_0 p_i & \text{if } x_i > 0 \\ \partial u / \partial x_i \leq \lambda_0 p_i & \text{if } x_i = 0 \end{array}$$

The marginal utility of each good that is purchased is equal to its price multiplied by the shadow price on wealth. If a good is not purchased, its marginal utility is smaller than its price multiplied by the shadow price on wealth.

INTERPRETATION (Cont'd)

$$\begin{array}{ll} \partial u / \partial x_i = \lambda_0 p_i & \text{if } x_i > 0 \\ \partial u / \partial x_i \leq \lambda_0 p_i & \text{if } x_i = 0 \end{array}$$

For two goods that are both purchased (that is $x_i > 0$) :

$$\frac{\partial u / \partial x_j}{\partial u / \partial x_i} = \frac{\lambda p_i}{\lambda p_j} = \frac{p_i}{p_j}$$

Interpret:

$$\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = - \partial x_j / \partial x_i \mid_{u \text{ constant}}$$

Marginal Rate of Substitution of good i for good j and is denoted MRS_{ij} . MRS_{ij} is the amount of good j the consumer would need to receive in order to exactly be compensated for a unit loss of good i.

INTERPRETATION (Cont'd)

For two goods that are both purchased (that is $x_i > 0$) :

$$\frac{\partial u / \partial x_j}{\partial u / \partial x_i} = \frac{\lambda p_i}{\lambda p_j} = \frac{p_i}{p_j}$$

Thus

$$MRS_{ij} = p_i / p_j$$

The price ratio is equal to the MRS. Amount of good j the consumer would need to receive to exactly be compensated for a unit loss of good i is equal to the price ratio.

Dollar value of good i lost is p_i . Dollar value of good j gained to exactly compensate is $p_j MRS_{ij}$

INTERPRETATION (Cont'd)

$$\begin{array}{ll} \partial u / \partial x_i = \lambda p_i & \text{if } x_i > 0 \\ \partial u / \partial x_i < \lambda p_i & \text{if } x_i = 0 \end{array}$$

Assume good j is not purchased, but good i is. Then take ratio of two sides of equation:

$$\frac{\partial u / \partial x_j}{\partial u / \partial x_i} > \frac{\lambda p_i}{\lambda p_j} = \frac{p_i}{p_j}$$

Thus $MRS_{ij} > p_i / p_j$

If good j is not purchased, but good i is purchased

to be exactly compensated for a unit loss of good i, the person would need to get **more** than p_i/p_j units of good j.

AN INTERESTING OBSERVATION

Different consumers have different preferences. Thus, different consumers generally choose different consumption bundles.

But at the optimal consumption bundle, **each consumer has the same MRS_{ij} as any other consumer**

- Relative value of two goods (subjective sense) is identical among all people who buy positive quantities of both
- Everyone who buys i and j have same rate at which they are willing to substitute one product for another product

EXAMPLE: CONSUMER WITH COBB-DOUGLAS UTILITY FUNCTION

Consider a **consumer** with **Cobb-Douglas utility function**

$$u(x_1, x_2) = (x_1)^\alpha (x_2)^{1-\alpha}$$

where $\alpha \in (0,1)$ is a given constant.

Given a price vector $p = (p_1, p_2)$, the consumer's **utility maximization problem** yields (using the Lagrangian methods described earlier) the Walrasian demand vector $x(p,w)$ as a function of price and wealth:

$$x(p, w) = \arg \max_{(x_1, x_2) \in B(p, w)} \{ \alpha \log x_1 + (1 - \alpha) \log x_2 \} = \underbrace{\left(\frac{\alpha w}{p_1}, \frac{(1 - \alpha) w}{p_2} \right)}_{\text{"Walrasian Demand"}}$$

AGENDA

Administrivia & Course Overview

Preferences and Utility Representation

Some Properties

Utility Representation (Cont'd)

Demand Theory: Basics

A Little Refresher on Constrained Optimization

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Choice Set & Quasi-Order on Sets**
- **Preference Relation & Rational Preferences**
- **Utility Function**
- **Framing**
- **Ellsberg Paradox**
- **Properties of Choice**
- **Continuity & Convexity of Preferences**
- **Cardinal vs. Ordinal Properties**
- **Choice Set vs. Budget Set**
- **Utility Maximization Problem**
- **Walrasian Demand / Walras' Law**
- **Constrained Optimization / Necessary Optimality Conditions**
- **Lagrange Multipliers (Dual Variables, Shadow Prices)**
- **Cobb-Douglas Utility Function**

APPENDIX

(Optional)

LEXICOGRAPHIC PREFERENCES CANNOT BE REPRESENTED BY A UTILITY FUNCTION (1/3)

Example 3 Consider a decision maker with *lexicographic preferences* $\preceq$, defined on the set $\mathcal{S} = [0, 1] \times [0, 1]$, such that

$$(s_1, s_2) \preceq (\hat{s}_1, \hat{s}_2) \stackrel{\text{def}}{\iff} s_1 < \hat{s}_1 \text{ or } (s_1 = \hat{s}_1 \text{ and } s_2 \leq \hat{s}_2).$$

The decision maker thus prefers an improvement in s_1 more than *any* improvement in s_2 . Let us for a moment assume that there exists a utility function $u : \mathcal{S} \rightarrow \mathbb{R}$ that represents these preferences according to (4). We now show that this inevitably leads to a contradiction. Note first that $(s_1, 0) \prec (s_1, 1)$ and therefore $u((s_1, 0)) < u((s_1, 1))$ for all $s_1 \in [0, 1]$. If we let

$$\Delta(s_1) = u((s_1, 1)) - u((s_1, 0)),$$

LEXICOGRAPHIC PREFERENCES CANNOT BE REPRESENTED BY A UTILITY FUNCTION (2/3)

then $\Delta(s_1) > 0$ for all $s_1 \in [0, 1]$. As a result, the range $[0, 1]$ of the first coordinate can be written as a union of the subsets $\mathcal{S}_{1k} = \{s_1 : \Delta(s_1) \geq 1/k\}$,

$$[0, 1] = \bigcup_{k=1}^{\infty} \mathcal{S}_{1k}.$$

Since the interval $[0, 1]$ is uncountable, some of the sets $\mathcal{S}_{1k}$ have to be uncountable as well. Let $\mathcal{S}_{1\bar{k}}$ be such an uncountable set for an appropriate $\bar{k} \in \{1, 2, \dots\}$. Let $\bar{\Delta} = u((1, 1)) - u((0, 0))$ be the largest possible utility difference between any two elements in $\mathcal{S}$ and let $K > \bar{k}\bar{\Delta} + 1$ be an integer. Then for any K elements $\sigma_1, \dots, \sigma_K \in \mathcal{S}_{1\bar{k}}$ with $\sigma_1 < \sigma_2 < \dots < \sigma_K$ we have

$$u((\sigma_k, 0)) - u((\sigma_{k-1}, 0)) > u((\sigma_{k-1}, 1)) - u((\sigma_{k-1}, 0)) > 1/\bar{k}$$

for any $k \in \{2, \dots, K\}$.

LEXICOGRAPHIC PREFERENCES CANNOT BE REPRESENTED BY A UTILITY FUNCTION (3/3)

Hence,

$$\begin{aligned}\bar{\Delta} &= u((1, 1)) - u((0, 0)) \\ &= [u((1, 1)) - u((\sigma_K, 0))] + [u((\sigma_K, 0)) - u((\sigma_{K-1}, 0))] + \cdots \\ &\quad + [u((\sigma_2, 0)) - u((\sigma_1, 0))] + [u((\sigma_1, 0)) - u((0, 0))] \\ &> 0 + 1/\bar{k} + \cdots + 1/\bar{k} + 0 = (K - 1)/\bar{k} > \bar{\Delta},\end{aligned}$$

i.e., a contradiction. A utility representation of lexicographic preferences is therefore not possible. The intuition is that because of the nonseparability of the choice set with respect to $\preceq$, any finite difference in the first attribute must yield an unbounded utility difference, which results from adding up the uncountably many finite utility differences (generated by variations in s_2 for each fixed s_1) in between. $\square$

MGT 621 – MICROECONOMICS

Thomas A. Weber

2. Demand Theory

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

Some Special Utility Functions

Wealth Effects

Price Effects

Demand Aggregation

Standard Welfare Measures

Welfare Changes

Key Concepts to Remember

SOME SPECIAL UTILITY FUNCTIONS

Let $\alpha_i, \rho > 0$ and $k: \mathbb{R} \rightarrow \mathbb{R}$ be an increasing function.

1. Cobb-Douglas:

$$u(x) = k \left(\prod_{i=1}^L x_i^{\alpha_i} \right)$$

Preferences are strongly monotonic and strictly convex on $\mathbb{R}_{++}^L$

2. Constant-Elasticity-of-Substitution (CES)

$$u(x) = k \left(\left[\sum_{i=1}^L \alpha_i x_i^\rho \right]^{1/\rho} \right)$$

CES preferences are strongly monotonic and strictly convex on $\mathbb{R}_{++}^L$. Cobb-Douglas is a special case for $\rho = 1$.⁽¹⁾

3. Fixed-Coefficient (Leontief)

$$u(x) = k \left(\min_{i \in \{1, \dots, L\}} \{ \alpha_i x_i \} \right)$$

Preferences are continuous, monotonic (not strongly), convex (but not strictly)

(1) The “elasticity of substitution” (introduced by John Hicks) between good 1 and 2 is $E_{12} = - d \ln(x_1/x_2) / d \ln(u_{x1}/u_{x2})$ ($= E_{21}$).

AGENDA

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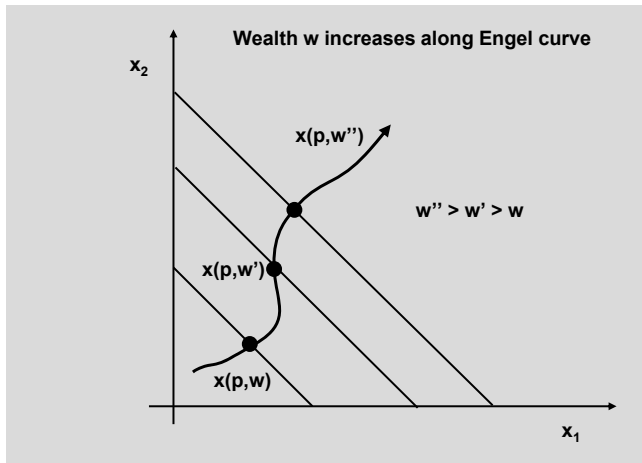
Key Concepts to Remember

COMPARATIVE STATICS: WEALTH EFFECT

Definition. Let $x(p,w)$ be a consumer's Walrasian demand function.

For any fixed price p , the function $f(w) = x(p,w)$ is called a wealth expansion function; its graph is a **wealth expansion path** (also known as **Engel curve**).

The derivative of $x(p,w)$ with respect to wealth, $f'(w) = D_w x(p, w) = \left[\frac{\partial x_i(p, w)}{\partial w} \right]_{i=1}^L$, is called the **wealth (or income) effect** (on demand).



Good i is called **normal** if $\frac{\partial x_i(p, w)}{\partial w} \geq 0$, otherwise it is called **inferior** (at (p,w)).

Demand is called **normal** if all goods are normal at any (p,w) .

INCOME ELASTICITY

Income elasticity of demand for good i :

- differentiate Walrasian demand x_i with respect to w , then multiply by (w/x_i) :

$$e_i = (w/x_i) \partial x_i / \partial w$$

AVERAGE INCOME ELASTICITY

$$\Sigma \{ p_i x_i \} = w$$

Differentiate with respect to w

$$\Sigma \{ p_i \partial x_i / \partial w \} = 1$$

Multiply each term by $(x_i w / x_i w)$ and regroup

$$\Sigma \left[\underbrace{(p_i x_i) / w}_{\text{Fraction of income spent on good } i} \right] \left[\underbrace{(\partial x_i / \partial w) (w / x_i)}_{\text{Income elasticity of demand for good } i} \right] = 1$$

Fraction of
income spent
on good i

Income elasticity
of demand for
good i

Weighted average of income elasticities equals 1. Weights are fractions of income spent on various goods

AGENDA

Some Special Utility Functions

Wealth Effects

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Key Concepts to Remember

PRICE ELASTICITY

(Own-)Price elasticity of demand for good i

- differentiate Walrasian demand for good i with respect to p_i , then multiply by (p_i/x_i) :

$$\varepsilon_{ii} = (p_i/x_i) (\partial x_i / \partial p_i)$$

Cross-Price elasticity of demand for good i with respect to changes in the price of good j

- differentiate Walrasian demand for good i with respect to p_j , then multiply by (p_j/x_i) :

$$\varepsilon_{ij} = (p_j/x_i) (\partial x_i / \partial p_j)$$

PRICE / INCOME ELASTICITIES FOR DIFFERENT COMMODITIES

Commodity	Price Elasticity	Income Elasticity
Electricity	- 1.2	+ 0.2
Beef (Meat)	- 0.9	+ 0.4
Women's hats	- 3.0	
Sugar	- 0.3	
Corn	- 0.5	
Potatoes	- 0.3	
Movies	- 3.7	
Flour		- 0.4
Restaurant Meals		+ 1.5
Margarine		- 0.2
Butter		+ 0.4
Furniture		+ 1.5
Milk		+ 0.1

ELASTICITIES FOR DIFFERENT COMMODITIES (Cont'd)

Cross-Price Elasticities

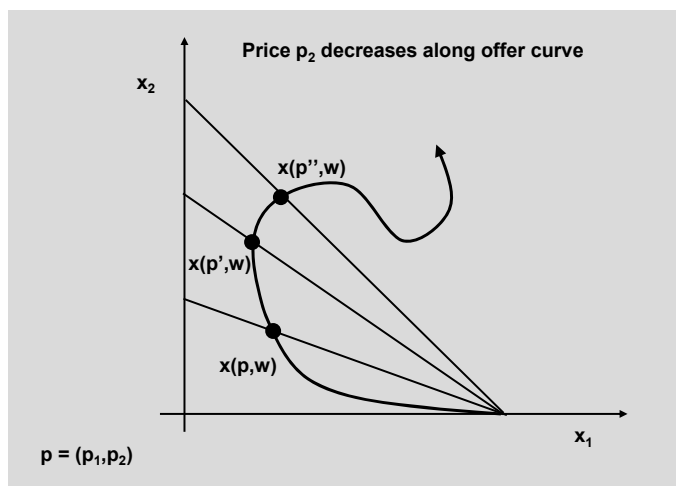
Commodity	With Respect to Price of	Cross-Price Elasticity
Electricity	Natural Gas	+ 0.2
Natural Gas	Fuel Oil	+ 0.4
Beef	Pork	+ 0.3
Pork	Beef	+ 0.1
Margarine	Butter	+ 0.8
Butter	Margarine	+0.7
Gasoline	Automobiles	Negative
Solar Panels	Electricity	Positive
Software	Computers	Negative
Hotel Rooms	Airline travel	Negative

COMPARATIVE STATICS: PRICE EFFECT

Definition. Let $x(p,w)$ be a consumer's Walrasian demand function.

For any fixed wealth w , the graph of the function $g(p) = x(p,w)$ is called the consumer's **offer curve**.

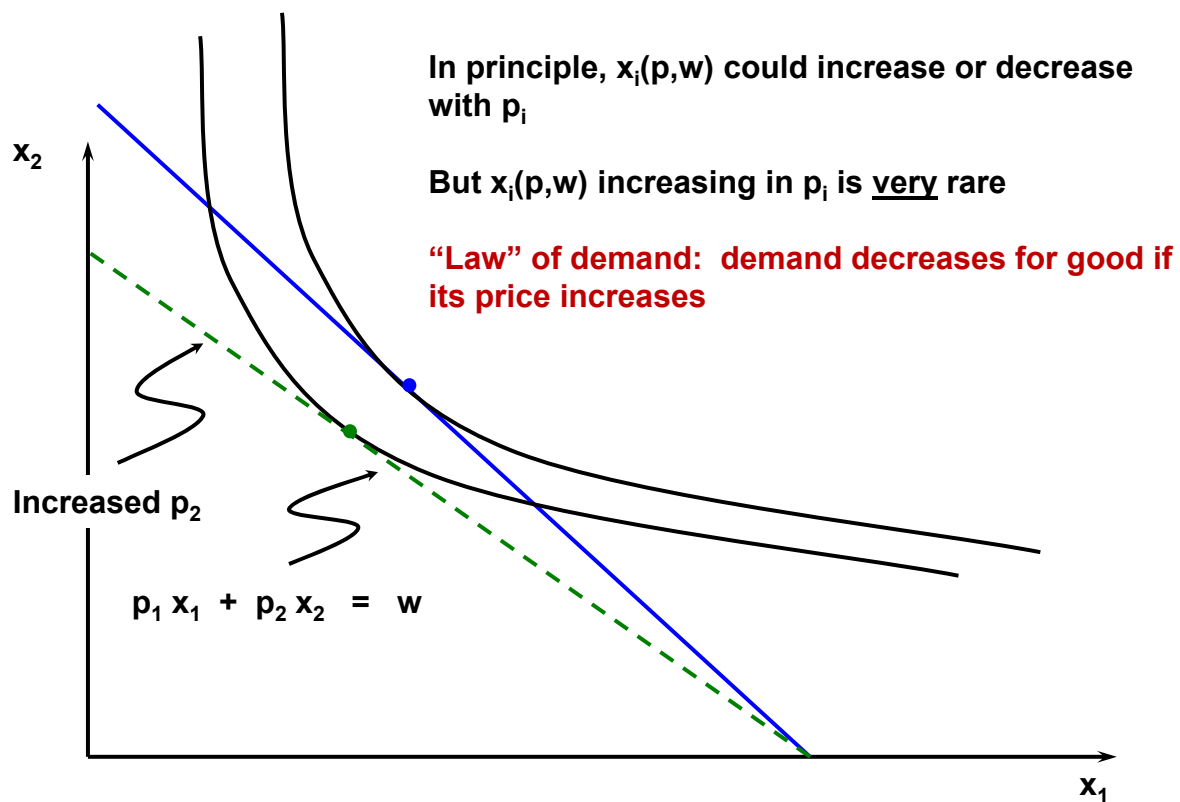
The derivative of $x(p,w)$ with respect to price, $Dg(p) = D_p x(p,w) = \left[\frac{\partial x_i(p,w)}{\partial p_j} \right]_{i,j=1}^L$, is called the **price effect** on demand.



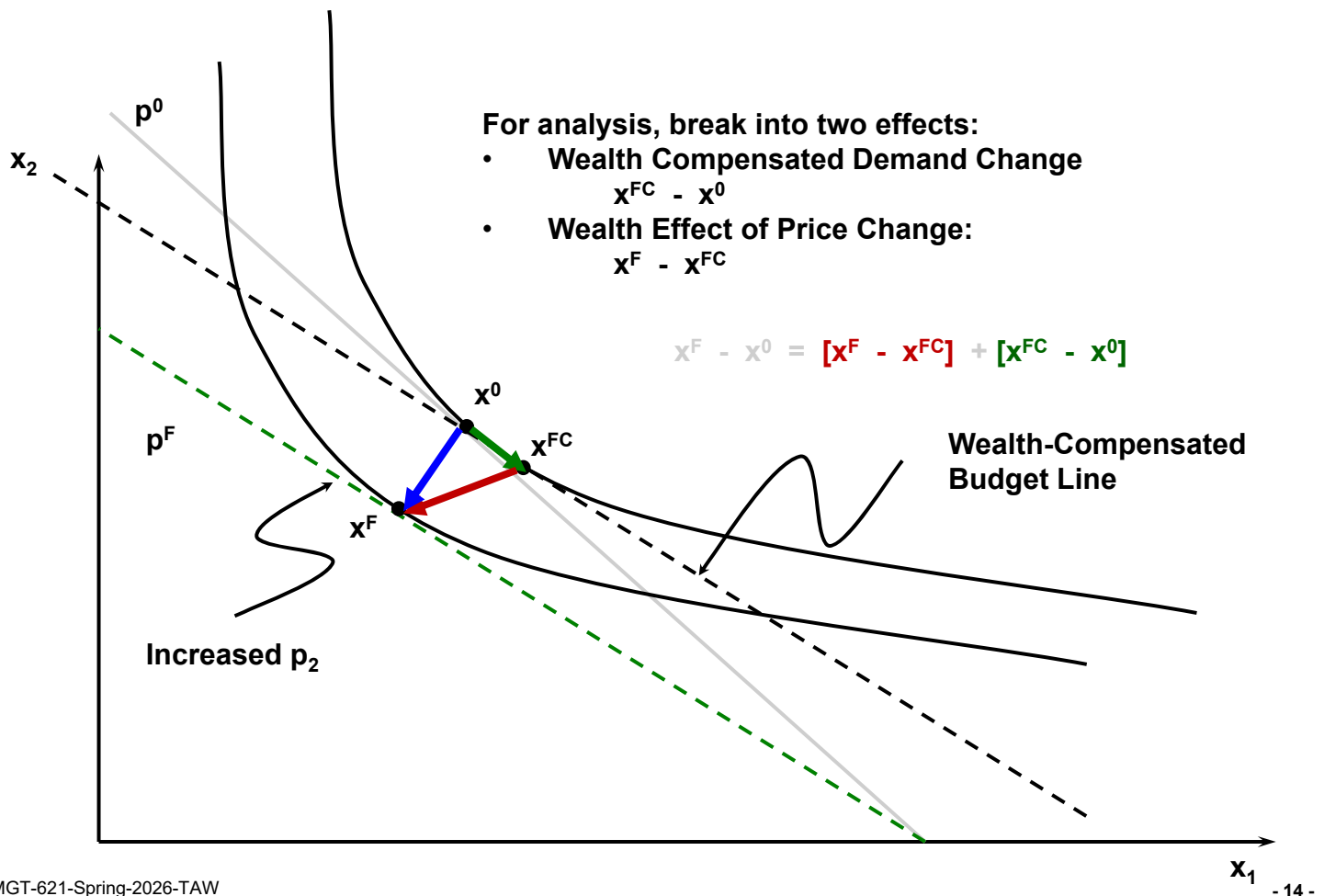
Good i is called a **Giffen good (at (p,w))** if $\frac{\partial x_i(p,w)}{\partial p_i} > 0$, otherwise it is called a **non-Giffen good (= standard case)**.

Demand exhibits **own-price effects** (of price of good i on demand of good i), and **cross-price effects** (of price of good i on demand of good j).

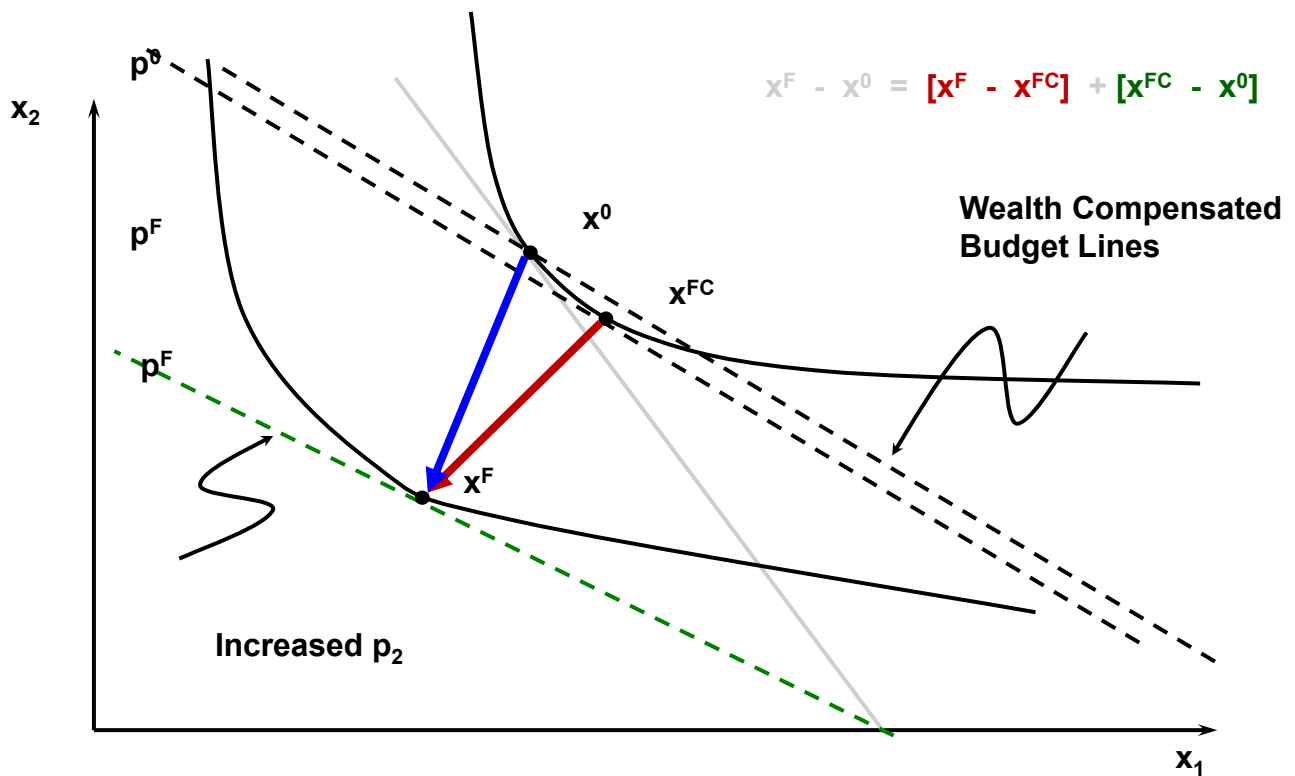
COMPARATIVE STATICS WITH RESPECT TO PRICES



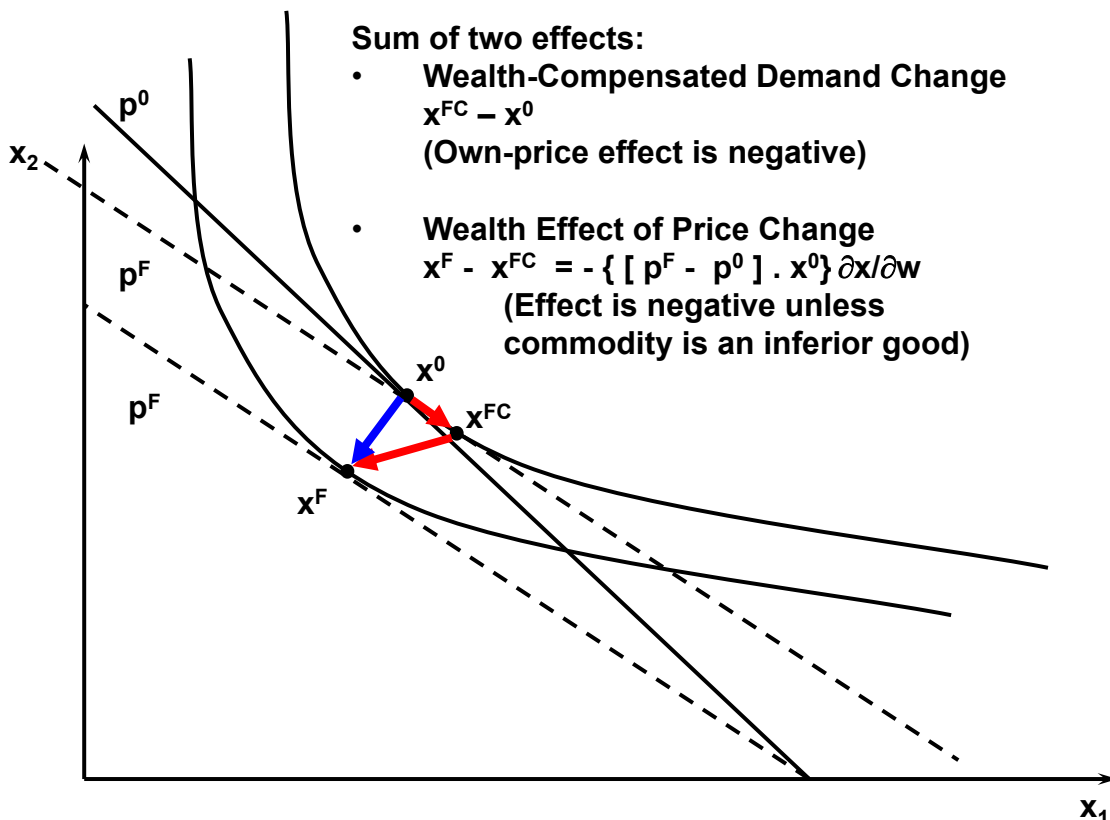
RESPONSE TO A PRICE INCREASE



RESPONSE TO A PRICE INCREASE (Cont'd)



RESPONSE TO A PRICE INCREASE (Cont'd)



INDIRECT UTILITY AND EXPENDITURE FUNCTION

Definition. A consumer's **indirect utility function** $v(p,w)$ corresponds to her maximized utility over her Walrasian budget set $B(p,w)$,

$$v(p,w) = \max_{x \in B(p,w)} u(x)$$

Hence, if $x(p,w)$ is the consumer's Walrasian demand function, then $v(p,w) = u(x(p,w))$. Note that $v(p,w)$ is strictly increasing in w , as long as the preferences are locally nonsatiated; hence, $v(p,w)$ for a fixed p , has an inverse with respect to w .

Now, given any utility level U , we can define the **expenditure function** $e(p,U)$ in terms of the consumer's indirect utility implicitly, by setting,

$$v(p, e(p,U)) = U$$

i.e., the expenditure function defines the minimum expenditure necessary to achieve a given utility level U . In other words,

$$e(p,U) = \min_{x \in \{\hat{x} \in \mathcal{R}_+^L : u(\hat{x}) \geq U\}} p \cdot x$$

HICKSIAN DEMAND FUNCTION

Definition. The **Hicksian demand function** $h(p,U)$ for a given price p and utility level U is given by the Walrasian demand function evaluated at the price p and the minimum wealth necessary to achieve U , i.e.,

$$h(p,U) = x(p, e(p,U)) \in \arg \min_{x \in \{\hat{x} \in \mathcal{R}_+^L : u(\hat{x}) \geq U\}} p \cdot x \quad (*)$$

From (*) we can conclude that

$$\frac{\partial h_j(p,U)}{\partial p_i} = \frac{\partial x_j(p, e(p,U))}{\partial p_i} + \frac{\partial x_j(p, e(p,U))}{\partial w} \underbrace{\frac{\partial e(p,U)}{\partial p_i}}_{h_i(p,U)}$$

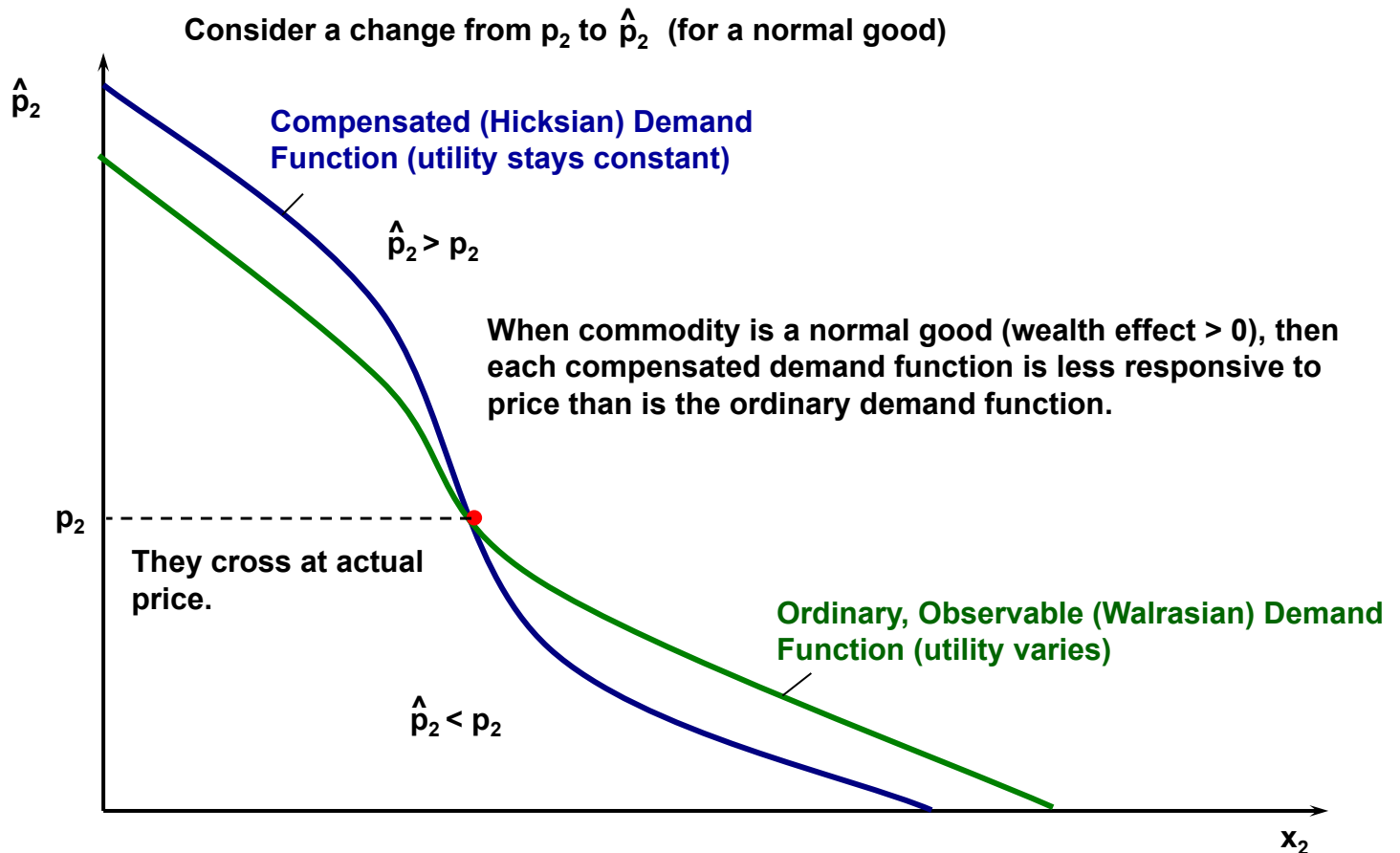
Roy's Identity
(follows directly from application of envelope theorem)

whence

$$\frac{\partial x_j(p, e(p,U))}{\partial p_i} = \underbrace{\frac{\partial h_j(p,U)}{\partial p_i}}_{\text{Price Effect}} - \underbrace{\frac{\partial x_j(p, e(p,U))}{\partial w} h_i(p,U)}_{\text{Wealth-Compensated Demand Change}} \quad \underbrace{\hspace{1cm}}_{\text{Wealth Effect}}$$

Slutsky Equation

WALRASIAN VS. COMPENSATED (HICKSIAN) DEMAND FUNCTION



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PROOF OF ROY'S IDENTITY

Differentiate expenditure function with respect to one price component,

$$\frac{\partial e(p, U)}{\partial p_i} = \frac{\partial}{\partial p_i} \left[\min_{x \in \{\hat{x} \in \mathcal{R}_+^L : u(\hat{x}) \geq U\}} p \cdot x \right] = \frac{\partial}{\partial p_i} [p \cdot x - \lambda(U - u(x))] \Big|_{x=x(p, e(p, U))} = x_i(p, e(p, U)) = h_i(p, U)$$

Envelope Theorem

This is **Roy's identity**.

QED

We conclude that

$$\frac{\partial h_j(p, U)}{\partial p_i} = \frac{\partial^2 e(p, U)}{\partial p_i \partial p_j}$$

The matrix $S(p, U) = \left[\frac{\partial h_j(p, U)}{\partial p_i} \right]_{i, j=1}^L = \left[\frac{\partial^2 e(p, U)}{\partial p_i \partial p_j} \right]_{i, j=1}^L$ is called **Slutsky matrix**.

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PROPERTIES OF THE SLUTSKY MATRIX

Proposition. The Slutsky matrix $S(p,U)$ is symmetric, negative semidefinite, and satisfies $S(p,U)p = 0$.

Proof.

(i) S is symmetric as long as expenditure function is twice continuously differentiable (theorem by Cauchy [sometimes attributed to H.A. Schwarz]).

(ii) The negative semi-definiteness of S (i.e., the fact that $D_p h(p,U) \leq 0$) follows from the “**law of compensated demand**”, which states that

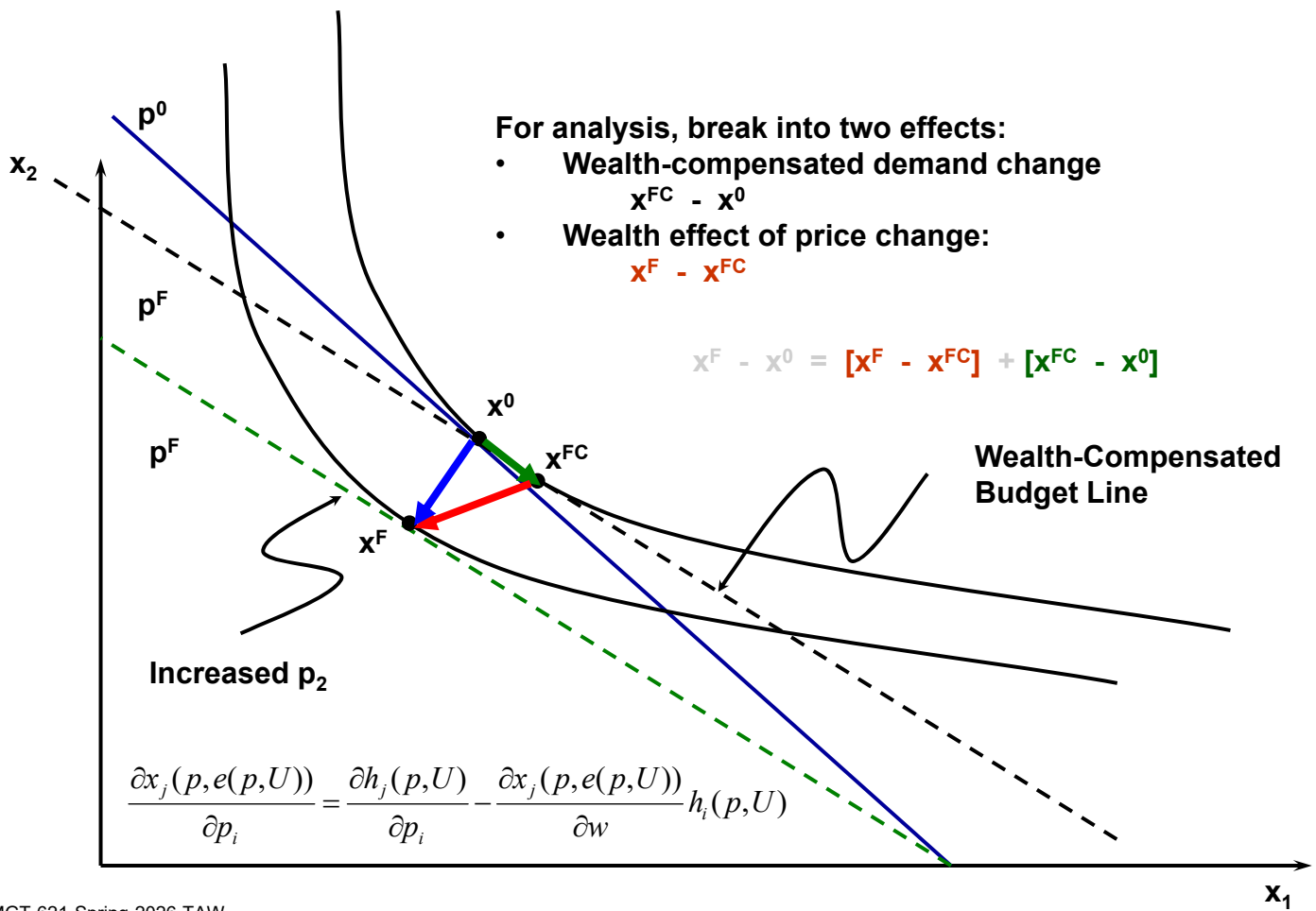
$$(p' - p)(h(p', U) - h(p, U)) \leq 0 \quad (\#)$$

Relation (#) holds, because $p' \cdot h(p', U) \leq p' \cdot h(p, U)$ and $p \cdot h(p, U) \leq p \cdot h(p', U)$.

(iii) Note that Hicksian demand $h(p,U)$ is homogeneous of degree zero in p (prove this as an exercise!), so that

$$\left. \frac{\partial h(\alpha p, U)}{\partial \alpha} \right|_{\alpha=1} = D_p h(p, U)p = S(p, U)p = 0 \quad \text{QED}$$

RESPONSE TO A PRICE INCREASE



INTERPRETATION OF THE SLUTSKY EQUATION

Decompose change of demand: $x^F - x^0 = [x^{FC} - x^0] + [x^F - x^{FC}]$

For very small price changes Δp , obtain:

$$\begin{aligned} [x^{FC} - x^0] &= S(p, w) \cdot \Delta p \\ [x^F - x^{FC}] &= -\partial x / \partial w \{x^0 \cdot \Delta p\} \end{aligned}$$

$$\Delta x = S(p, w) \cdot \Delta p - \partial x / \partial w \{x \cdot \Delta p\}$$

This is the Slutsky Equation. Most often it is written in terms of partial derivatives of x_i with respect to p_j (for small change $x = x^0$):

$$\partial x_i / \partial p_j = S_{ij} - \partial x_i / \partial w x_j$$

S_{ij} is (i,j)-th element of Slutsky substitution matrix, derivative of wealth compensated demand x^{FC}_i with respect to p_j . Referred to as “**substitution effect**” of a price change.

$-\partial x_i / \partial w x_j$ is referred to as the “**income effect**” of a price change.

INTERPRETATION OF THE SLUTSKY EQUATION (Cont'd)

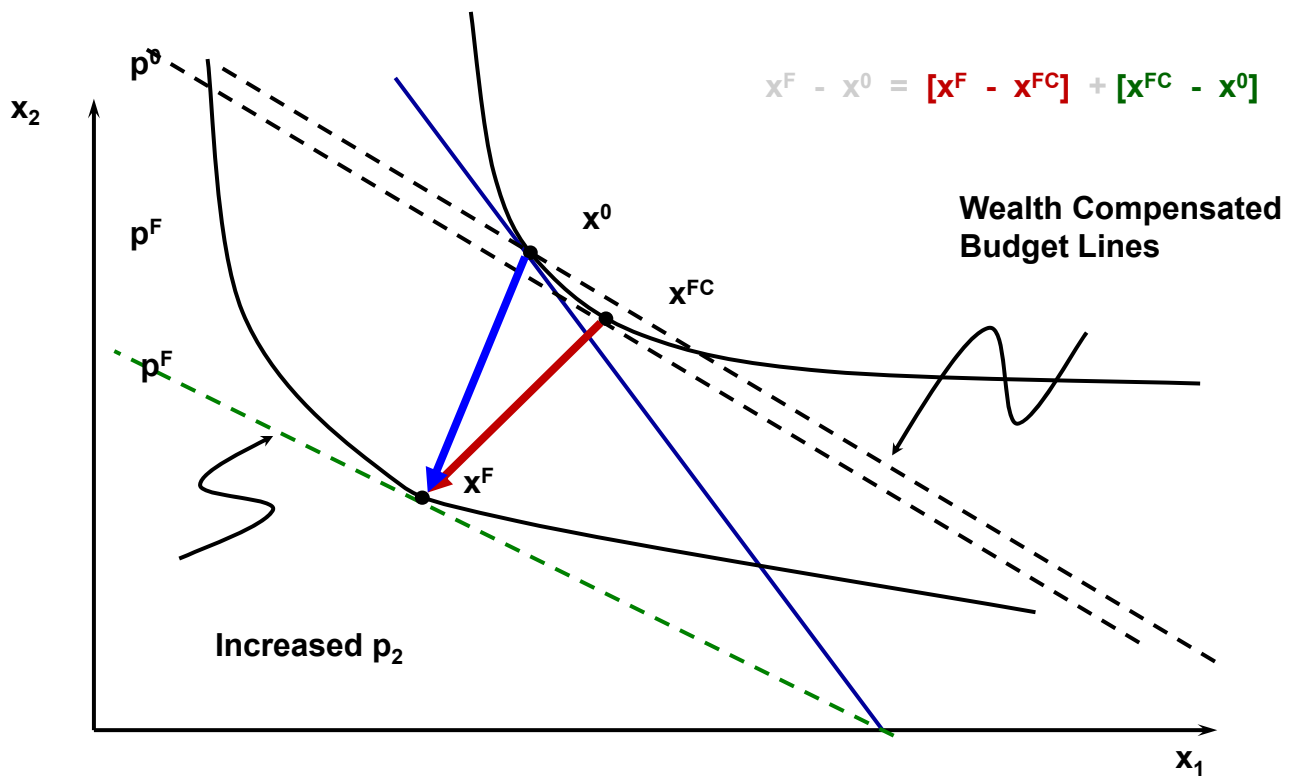
$$D_p x(p, w) = \underbrace{S(p, w)}_{\text{“Substitution Effect”}} - \underbrace{\frac{\partial x(p, w)}{\partial w} x(p, w)}_{\text{“Income Effect”}}$$

Note that (compared to earlier slides)

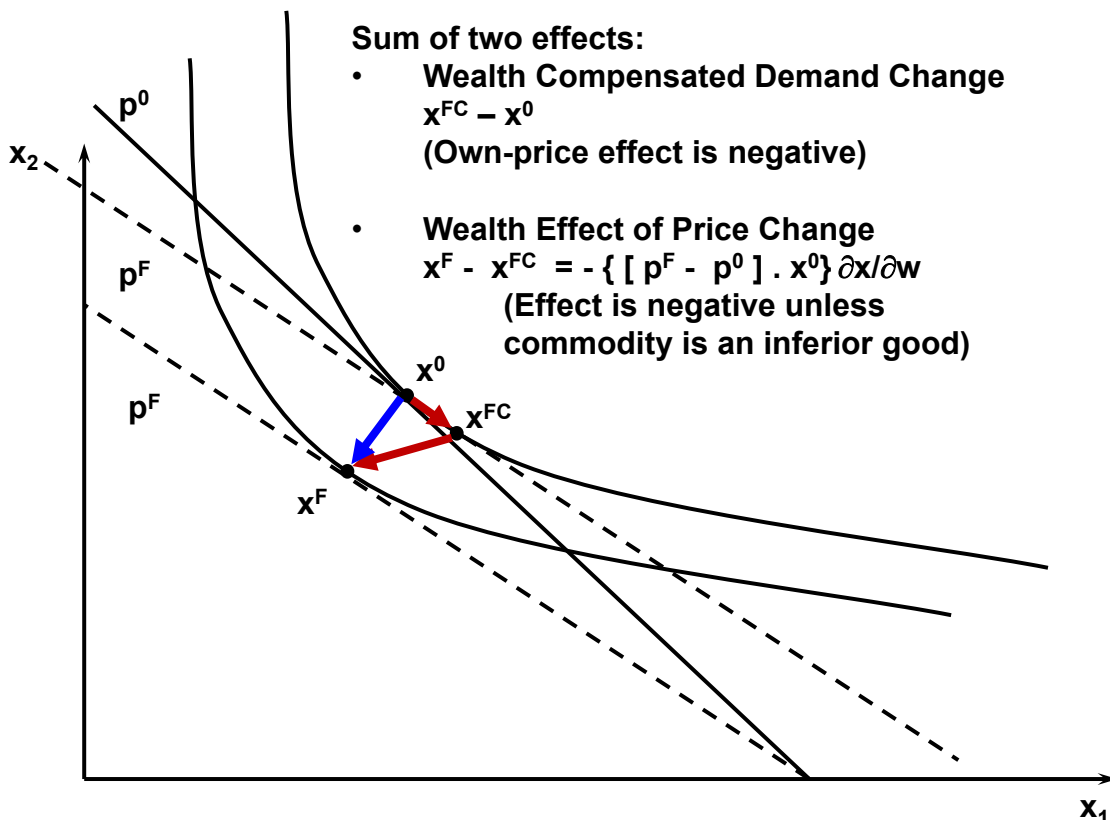
$$w = e(p, U)$$

$$x(p, w) = h(p, U)$$

RESPONSE TO A PRICE INCREASE (Cont'd)



RESPONSE TO A PRICE INCREASE (Cont'd)



AGENDA

Some Special Utility Functions

Wealth Effects

Price Effects

Demand Aggregation

Standard Welfare Measures

Welfare Changes

Key Concepts to Remember

MARKET DEMAND FUNCTION

Demand aggregated over (finitely) many individuals: **Market Demand Function**

If following properties hold for each individual demand function:

- **Continuity**
- **Homogeneity of degree zero**
- **Walras' Law**

Then they also hold for the market demand function

MARKET DEMAND FUNCTIONS (Cont'd)

Is it possible to find **aggregate demand function** $D(p, w)$ (for n individuals), such that

$$D(p, w) = \sum_{k=1}^n x^k(p, w^k) \quad \text{for} \quad w = \sum_{k=1}^n w^k \quad ?$$

In general, if everyone faces the **same** price vector p , then aggregate demand can be written as a function of p , but NOT necessarily also as a function of aggregate income w , unless

$$\sum_{k=1}^n \frac{\partial x^k}{\partial w^k} dw^k = 0 \quad \text{for any small wealth change } dw \text{ that leaves aggregate wealth the same, i.e., for which} \quad dw = (dw^1, \dots, dw^n) \quad \sum_{k=1}^n dw^k = 0$$

In other words, all the $\partial x^k / \partial w^k$ have to be the same across all consumers.

Wealth effects must compensate each other in the aggregate, no matter how the wealth is redistributed among the individuals!

MARKET DEMAND FUNCTIONS (Cont'd)

Proposition. A (necessary and) sufficient condition for demand aggregation to be possible is for preferences to be such that each consumer k 's indirect utility v^k is quasilinear ("of the Gorman form"), i.e.,

$$v^k(p, w^k) = a^k(p) + b(p)w^k$$

Proof: (sufficiency only)

By the definition of indirect utility it is $v^k(p, e^k(p, u)) = u$.

Thus,

$$\begin{aligned} v_p^k(p, w^k) + v_{w^k}^k(p, w^k) e_p^k(p, u^k(x^k(p, w^k))) &= v_p^k(p, w^k) + v_{w^k}^k(p, w^k) h^k(p, u^k(x^k(p, w^k))) \\ &= \underbrace{v_p^k(p, w^k)}_{a_p^k(p) + b'(p)w^k} + \underbrace{v_{w^k}^k(p, w^k)}_{b(p)} x^k(p, w^k) \\ &= 0 \quad (= \partial u / \partial p) \end{aligned}$$

And therefore, $\frac{\partial x^k(p, w^k)}{\partial w^k} = -\frac{b'(p)}{b(p)}$ is the same for any consumer k , no matter what his or her wealth level w^k .

QED

MARKET DEMAND FUNCTIONS (Cont'd)

A market demand function is useful for making statements about consumer response to changes in price and/or aggregate income.

Example.

Sometimes a market demand function is useful to explain other aggregate effects, such as the “bandwaggon effect,” under which the demand for a good depends on the collective *expectation* about how many consumers will purchase the product.

A LITTLE DETOUR: **NETWORK EXTERNALITIES**

Externalities exist when the action of one agent directly affects the environment of another agent; *network externalities* are externalities between participants of a common network

“How much would you pay for the first fax machine?”

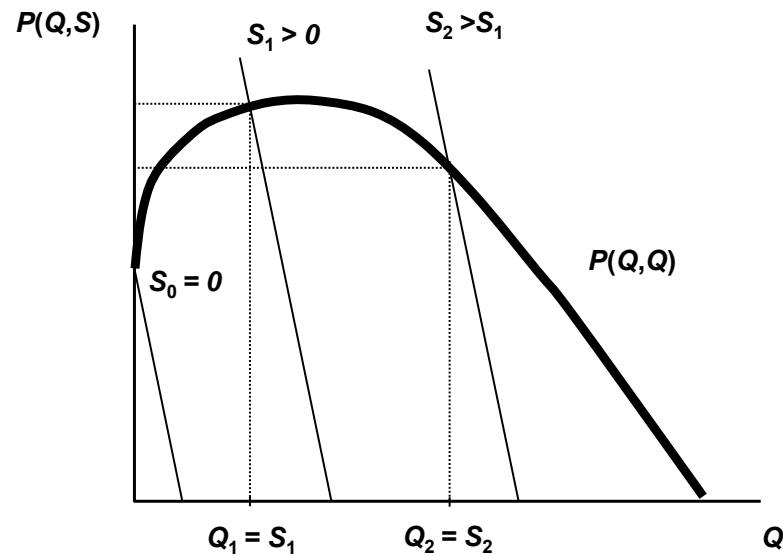
Complementarity

- *Direct* (e.g., in 2-way networks, “exchange transactions”)
- *Indirect* (e.g., *Microsoft Word*)
- Necessary conditions:
 - Compatibility (= ability to connect, usually to some hardware)
 - Interoperability (= ability to exchange and make use of information)

Aggregate demand depends on the expected demand.

GENERATING **FULFILLED-EXPECTATIONS DEMAND CURVE**

Demand in the Presence of Network Externalities



Bandwagon Effect

DEMAND CURVE SHIFTS DUE TO NETWORK EXTERNALITIES

Fulfilled-Expectations Demand

Quantitative Approach

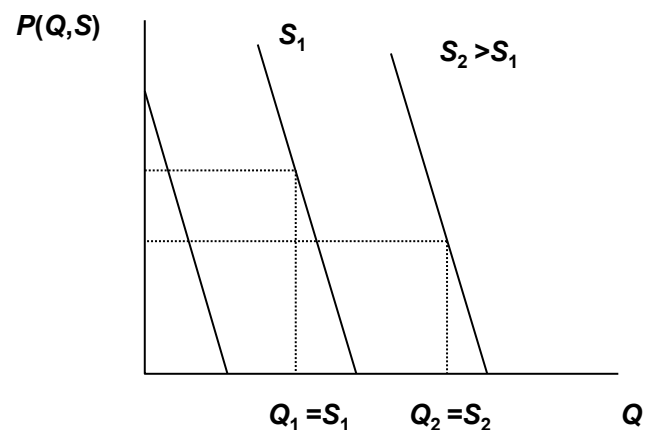
$$P(Q,S) = P(Q,0) + f(S)$$

P Willingness-to-pay

f(S) Externality function, $f(0) = 0$, $f'' < 0 < f'$

Q Output

S Expected demand / installed base



NETWORK EXPANSION PATH CAN HAVE SEVERAL FULFILLED EXPECTATIONS EQUILIBRIA

'Chicken and Egg' Paradox

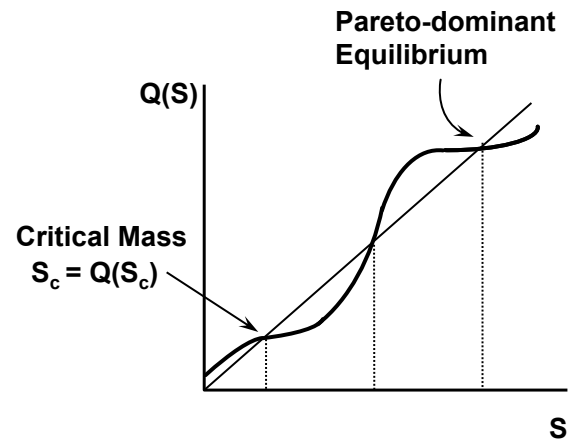
If perceived installed base is too small, customers may not be willing to purchase the product.

For certain products, small networks are not observed: "*Discontinuous Network Expansion*" (at least seemingly)

Existence of *minimum* feasible network ("critical mass") does not depend on the market structure (i.e., a demand-side phenomenon)

Equilibrium network sizes resulting from perfect competition / oligopoly / monopoly generally different (e.g., Economides/Himmelberg 1994)

Network Expansion Path



WHAT IS THE CRITICAL MASS? Let's compute it!

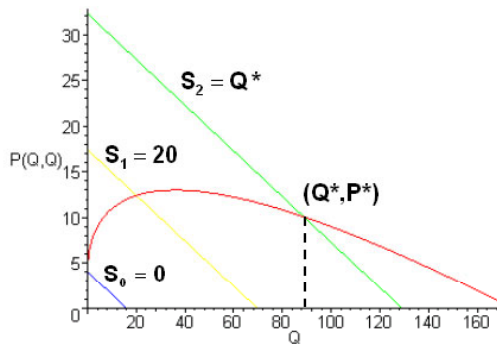
Example⁽¹⁾

Willingness to Pay:	$P(Q, S) = 4 - Q/4 + 3\sqrt{S}$
Profit in Fulfilled-Expectations Equilibrium ($S=Q$):	$\Pi(Q) = Q \cdot P(Q, Q) - 2Q = (4 - Q/4 + 3\sqrt{Q})Q - 2Q$
Maximization yields:	$Q^* = S^* = 88.8, \quad P^* = P(Q^*, S^*) = 10.1$
Network Expansion Path:	$P^* = P(Q, S) = 4 - Q/4 + 3\sqrt{S} \Rightarrow S(Q) = (P^* - 4 + Q/4)^2$
Critical Mass:	$Q_c = \min\{Q': S(Q') = Q' \geq 0\} \Rightarrow Q_c = 6.6$

Note that $Q_c/Q^* = 6.6/88.8 = 7.4\%$ is significant. Thus, in order to have a chance to achieve the optimum, the firm has to instill the belief that in equilibrium more than the critical mass of users (i.e., more than 6.6 Million) will eventually adopt.

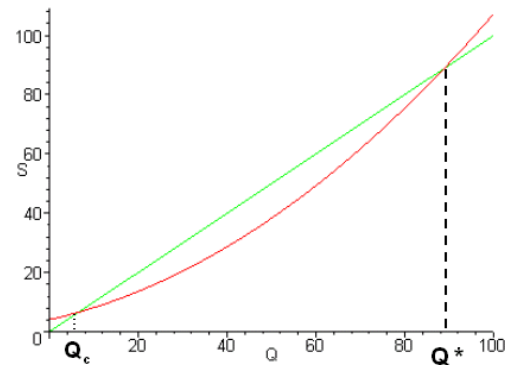
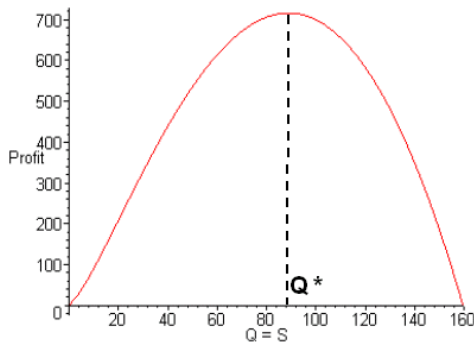
(1) Quantities measured in millions of units. Number of potential customers is 174.5 million (there willingness to pay in the fulfilled-expectations equilibrium will be zero, i.e., $p(174.5, 174.5) = 0$)

WHAT IS THE CRITICAL MASS? Let's compute it! (Cont'd)



$$P(Q, S) = 4 - Q/4 + 3\sqrt{S}$$

$$\Pi(Q) = Q \cdot P(Q, Q) - 2Q$$



BUSINESS IMPLICATIONS FOR SELLERS OF NETWORK GOODS

- **Penetration pricing** (initially possibly < 0) to reduce adoption costs for the consumer. Pulling one consumer over is likely to induce further consumers ("herding") to adopt, also due to the effect of network externalities
- **Growth** is a strategic imperative
 - Production-side economies can help: e.g., lower marginal cost lowers optimal (monopoly) price, which in turn lowers critical mass
 - Demand-side economies are most important for achieving market dominance
- **Strategic pre-announcements** to reduce uncertainty. Market uncertainty can prevent the consumers from exploiting beneficial network externalities since consumers fear being stranded with a new technology⁽¹⁾
- **Tradeoff among current and future benefits** through lock-in. Difficult tradeoff and frequent cause of business failure:
 - Myopia (too high prices) vs. overestimating future benefits (too low prices)

(1) Cf. Farrel, J., and Saloner, G., Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation, AER 76(5):940—955, 1986.

AGENDA

Some Special Utility Functions

Wealth Effects

Price Effects

Demand Aggregation

Standard Welfare Measures

Welfare Changes

Key Concepts to Remember

MEASURING WELFARE CHANGES

Consider a simple example of **valuing a nonmarket good** (e.g., a national park).

- Assume that there are N standard market goods and one nonmarket good.
- A consumer has preferences represented by a smooth increasing utility function $u(x, q)$, where x denotes the consumption in the market goods and q the consumption of the nonmarket good
- The consumer's income (wealth) is $y > 0$

Given any q , the consumer's indirect utility function is

$$v(p, q, y) = \max_{x \in \{\hat{x} \in \mathbb{R}_+^N : p \cdot \hat{x} \leq y\}} u(x, q)$$

where p is the price vector for the market goods.

Question. How much is an exogenous change of q from q^0 to $q^1 > q^0$ worth to the consumer?

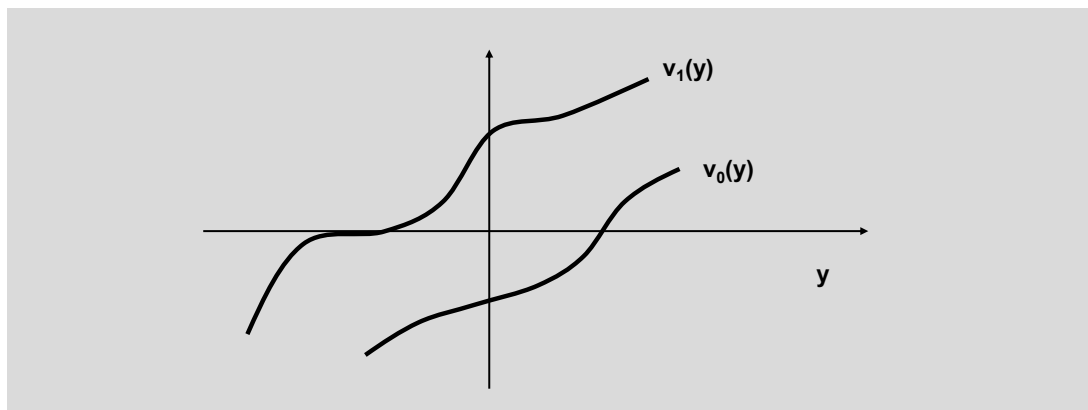
MEASURING WELFARE CHANGES (Cont'd)

One can interpret q^0 and q^1 as two “states” of the economy, and the consumer has some value for the change of the state (assume that $q^1 > q^0$, without loss of generality).

Let

$$v_i(y) = v(p, q^i, y)$$

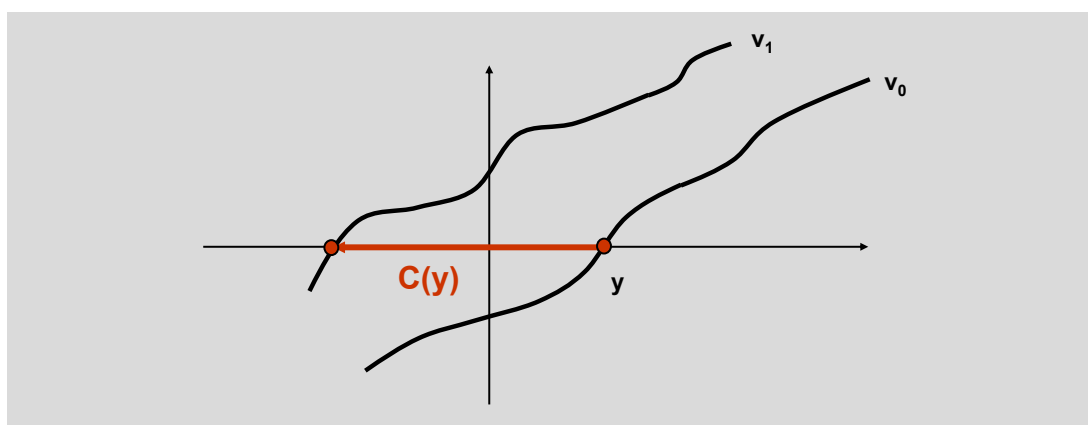
denote the consumer's indirect utility as a function of his income y for i in $\{0,1\}$.⁽¹⁾



From standard demand theory we know that $v_i(y)$ is strictly increasing in y .

(1) We suppress the dependence on the constant price vector for simplicity. More generally, $v_i(y)$ can denote the consumer's (indirect) utility function in state i of the economy as a function of his or her wealth. Thus, the analysis here can also be applied when the state of the economy is defined by different commodity price vectors (corresponding to the treatment in standard economics textbooks such as MWG).

MEASURING WELFARE CHANGES (Cont'd)



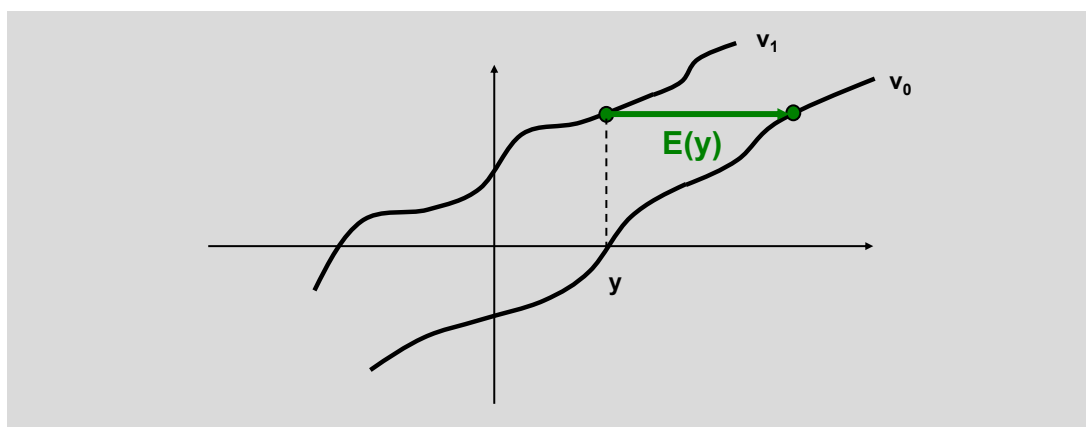
Question I. How much would a consumer of income y be **willing to pay** for the transition from q^0 to q^1 ?

Answer I. A income change of $(-C(y))$ would “compensate” the consumer for having q^1 instead of q^0 . [The word *compensate* means here to bring the consumer back to the original utility level before the change.]

$$v_1(y - C(y)) = v_0(y)$$

$C(y)$ is the Compensating Variation

MEASURING WELFARE CHANGES (Cont'd)



Question II. How much would a consumer of income y be **willing to accept** for the transition from q^1 to q^0 ?

Answer II. A income change of $+E(y)$ would make the consumer feel “equivalent” between having q^0 (at income $y+E(y)$) and having q^1 (at income y).

$$v_1(y) = v_0(y + E(y))$$

$E(y)$ is the Equivalent Variation

COMPENSATING AND EQUIVALENT VARIATIONS

Definition. Let $v_i(y)$ be a consumer’s increasing (indirect) utility function for an economy in state $i \in \{0,1\}$ as a function of income y .

(i) The **compensating variation $C(y)$** is defined as the consumer’s maximum **willingness to pay** to transition from state 0 to state 1, i.e.,

$$C(y) = \sup \{c \in \mathcal{R} : v_0(y) \leq v_1(y - c)\}$$

(ii) The **equivalent variation $E(y)$** is defined as the consumer’s minimum **willingness to accept** to transition from state 1 to state 0, i.e.,

$$E(y) = \inf \{e \in \mathcal{R} : v_1(y) \leq v_0(y + e)\}$$

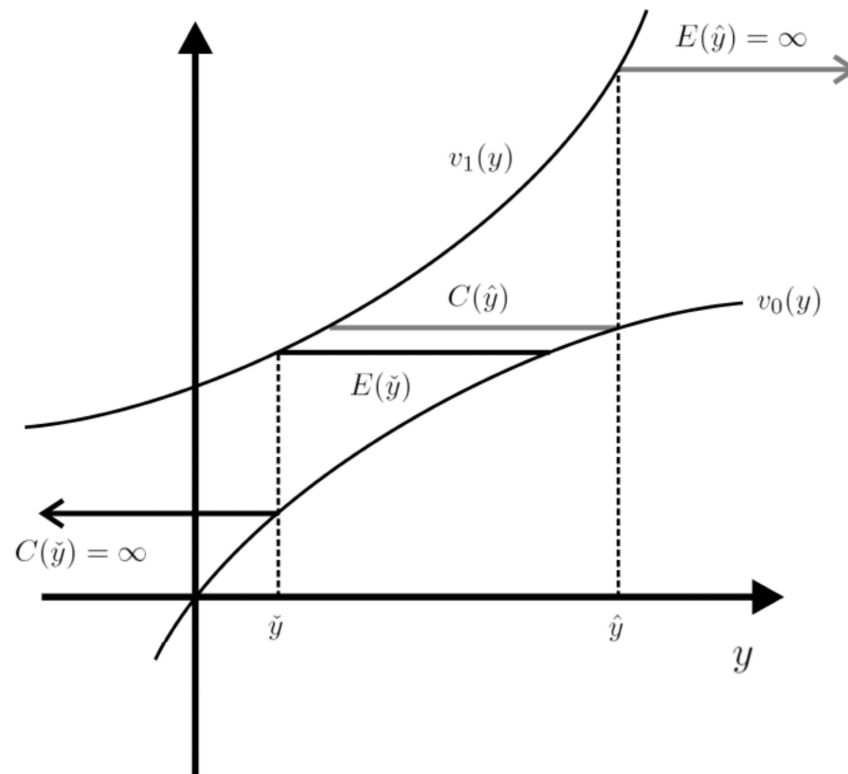
Remark. If $C(y)$ and $E(y)$ are bounded, we have that

$$v_0(y) = v_1(y - C(y)) \quad \text{and} \quad v_0(y + E(y)) = v_1(y),$$

corresponding to the standard definition of these two welfare measures.

COMPENSATING AND EQUIVALENT VARIATIONS

They can be very different!



WHAT IS THE RELATION BETWEEN C(y) AND E(y)?

The answer is simple. Since both v_0 and v_1 are invertible functions, we obtain from the definition of C and E that

$$C(y) = y - v_1^{-1}(v_0(y)) \quad \text{and} \quad E(y) = v_0^{-1}(v_1(y)) - y$$

This immediately implies that C and E are independent of the particular utility representation of the consumer's preferences (why?).

As a result, we could choose the utility representation such that $v_0(y) = y$, so that $v_1(y) = E(y) + y$ (from the definition of E(y)). Thus, from the definition of C(y) we know that we simply need to form the inverse of v_1 to find C(y), so that

$$C(y) = y - w_{01}(y) = E(w_{01}(y)),$$

where the compensated income $w_{01}(y)$ is such that $y = w_{01}(y) + E(w_{01}(y))$.

Similarly, one can show that

$$E(y) = w_{10}(y) - y = C(w_{10}(y)),$$

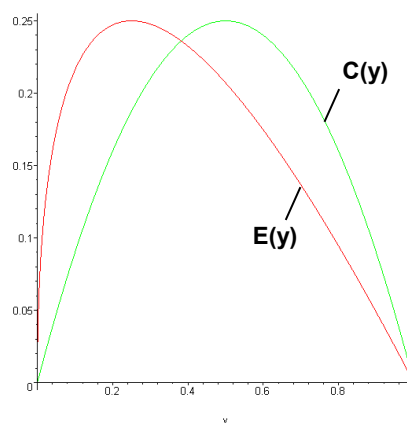
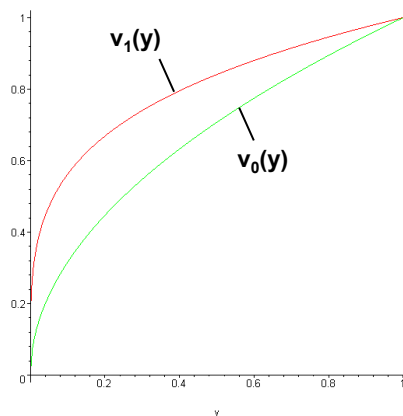
where the compensated income $w_{10}(y)$ is such that $y = w_{10}(y) - C(w_{10}(y))$

EXAMPLE: COMPUTATION OF C(y) AND E(y)

Consider a consumer with indirect utility functions $v_0(y) = y^\alpha$ and $v_1(y) = y^\beta$, where $\alpha = 1/2$ and $\beta = 1/4$, and the income y lies in $[0,1]$.⁽¹⁾

$$C(y) = y - v_1^{-1}(v_0(y)) = y - (\sqrt{y})^4 = (1-y)y$$

$$E(y) = v_0^{-1}(v_1(y)) - y = (y^{1/4})^2 - y = \sqrt{y} - y$$

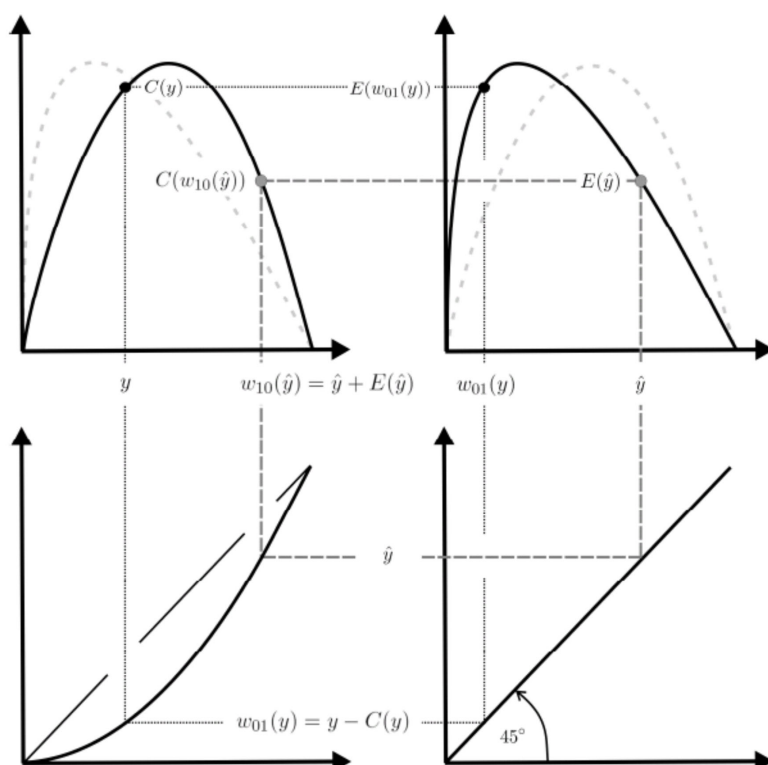


(1) These indirect utility functions can be obtained after solving the utility maximization problem for appropriate Cobb-Douglas utilities.

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EXAMPLE (Cont'd)



$$C(y) = E(w_{01}(y))$$

$$E(\hat{y}) = C(w_{10}(\hat{y}))$$

$$w_{10}(\hat{y}) = w_{01}^{-1}(\hat{y}) = \hat{y} + E(\hat{y})$$

$$w_{01}(y) = y - C(y)$$

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EXAMPLE: TRANSFER OF A NONMARKET GOOD

Assume that there are **two consumers**, the first has welfare measures $C(y), E(y)$, while the second has the welfare measures $\hat{C}(y), \hat{E}(y)$. For simplicity, we assume that both start with the same income level y . **The first consumer holds one unit of a nonmarket good, while the second consumer possesses none.**

Questions. (i) At what transfers t will there be a transaction of the nonmarket good?

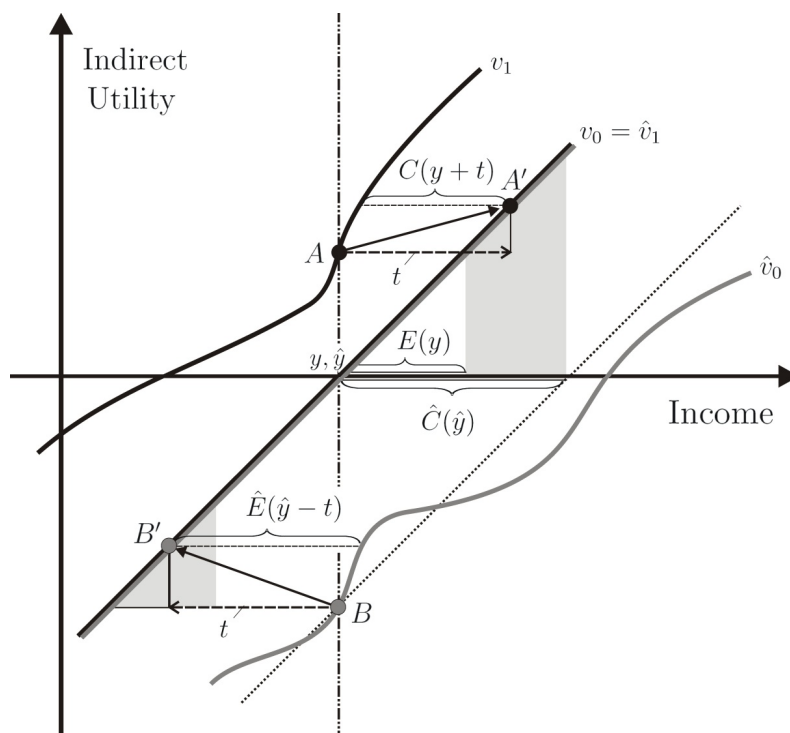
(ii) Is it possible that after the first transfer takes place, another such transfer occurs moving the good back to the first consumer?

Answers.

(i) A necessary and sufficient condition for a transfer is that $\hat{C}(y) \geq E(y)$

(ii) A necessary and sufficient condition for a second transfer (after the good had been exchanged under (i) at price t) is that $C(y+t) \geq \hat{E}(y-t)$. This can never happen if the first transaction realized gains from trade!

TRANSFER OF A NONMARKET GOOD (Cont'd)



The first transfer leads to a Pareto improvement! A second transfer cannot generate another (strict) Pareto improvement.

AGENDA

Some Special Utility Functions

Wealth Effects

Price Effects

Demand Aggregation

Standard Welfare Measures

Welfare Changes

Key Concepts to Remember

HOW TO COMPUTE $E(y)$ AND $C(y)$ FOR PRICE CHANGES?

Price change from p to $\hat{p}$

$$\begin{aligned} C(y) &= e(p, v(p, y)) - e(\hat{p}, v(p, y)) \\ &= y - e(\hat{p}, v(p, y)) \end{aligned}$$

Compensating Variation

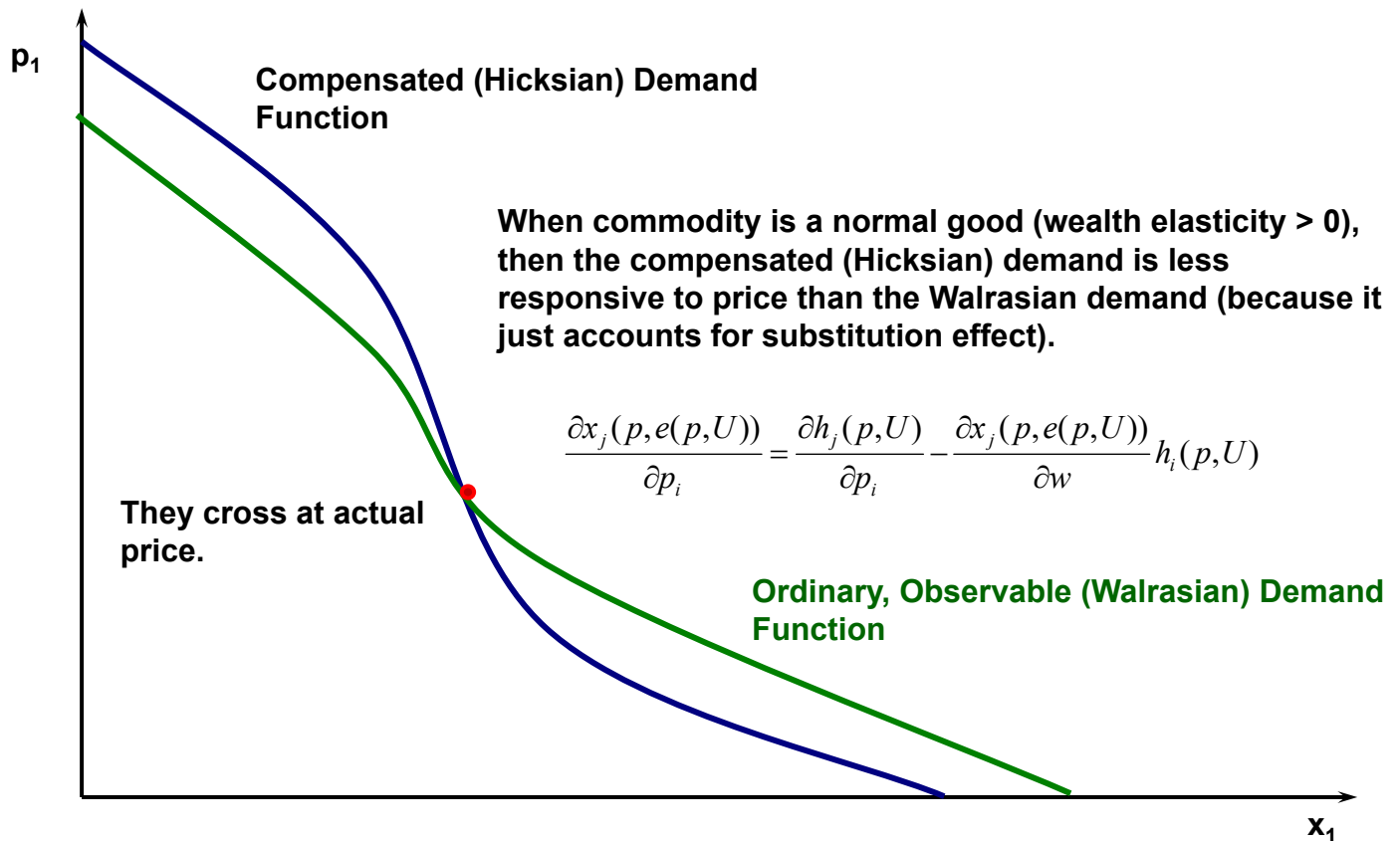
Expenditure at initial price minus expenditure at final price, evaluated at initial utility level

$$\begin{aligned} E(y) &= e(p, v(\hat{p}, y)) - e(\hat{p}, v(\hat{p}, y)) \\ &= e(p, v(\hat{p}, y)) - y \end{aligned}$$

Equivalent Variation

Expenditure at initial price minus expenditure at final price, evaluated at final utility level

WALRASIAN DEMAND VS. COMPENSATED (HICKSIAN) DEMAND



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COMPENSATED DEMAND FUNCTION

Slope of the actual demand function, or Walrasian demand function is $\partial x_i / \partial p_i$

Can use Slutsky equation to construct the “compensated demand function”, also called the “Hicksian demand function”

Construct compensated demand function around some specific combination of p_i and the resulting x_i but with the slope S_{ii}

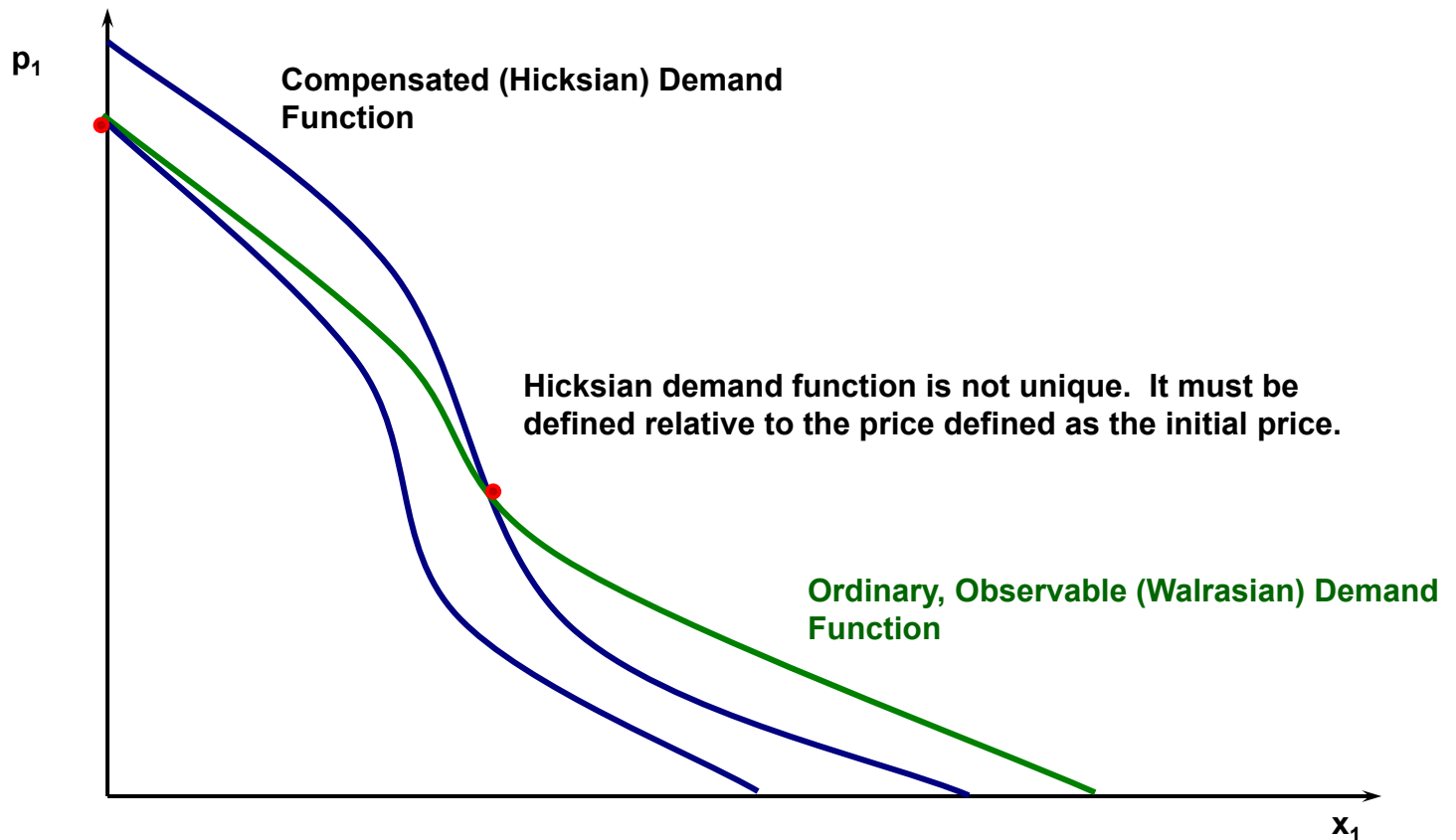
$$S_{ii} = \partial x_i / \partial p_i + \partial x_i / \partial w x_i$$

This is the slope of the artificial demand function, constructed as if at the same time the price is increasing, consumer is given exactly enough additional wealth to keep utility constant.

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COMPENSATED (HICKSIAN) DEMAND FUNCTIONS FROM TWO DIFFERENT INITIAL PRICES



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SIGNIFICANCE

Hicksian demand curve is used to create two conceptually correct measurements of welfare impacts of a price change (called “compensating variation” and “equivalent variation”)

- **Compensating Variation:** Negative of dollar amount to compensate consumer for facing price change, so that utility remains unchanged.
- **Equivalent Variation:** Dollar amount consumer would accept in place of a price change, so utility change would be the same as it would be with the price change.

Compensating Variation and Equivalent variation are both positive for price decrease and negative for a price increase.

- A less conceptually correct measure, the change in **consumer's surplus**, will be approximately equal to compensating variation and to equivalent variation. Consumer's surplus is somewhat easier to calculate

CALCULATION OF WELFARE MEASURES FOR PRICE CHANGE

Compensating Variation: Negative of dollar amount to compensate consumer for facing price change, so that utility remains unchanged.

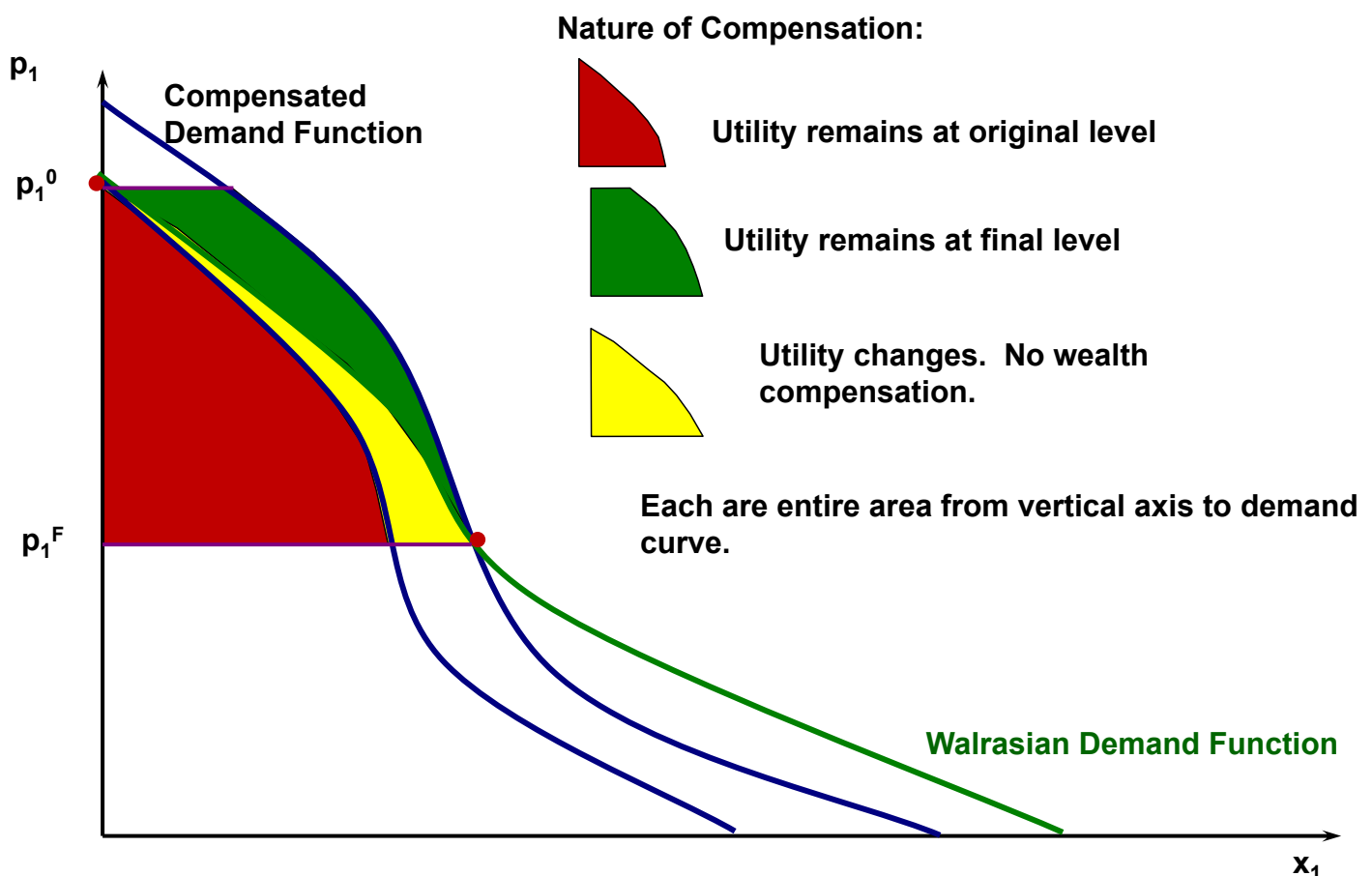
- Integrate along **Hicksian** demand curve, crossing through the **original** price and quantity

Equivalent Variation: Dollar amount consumer would accept in place of a price change, so utility change would be the same as it would be with the price change.

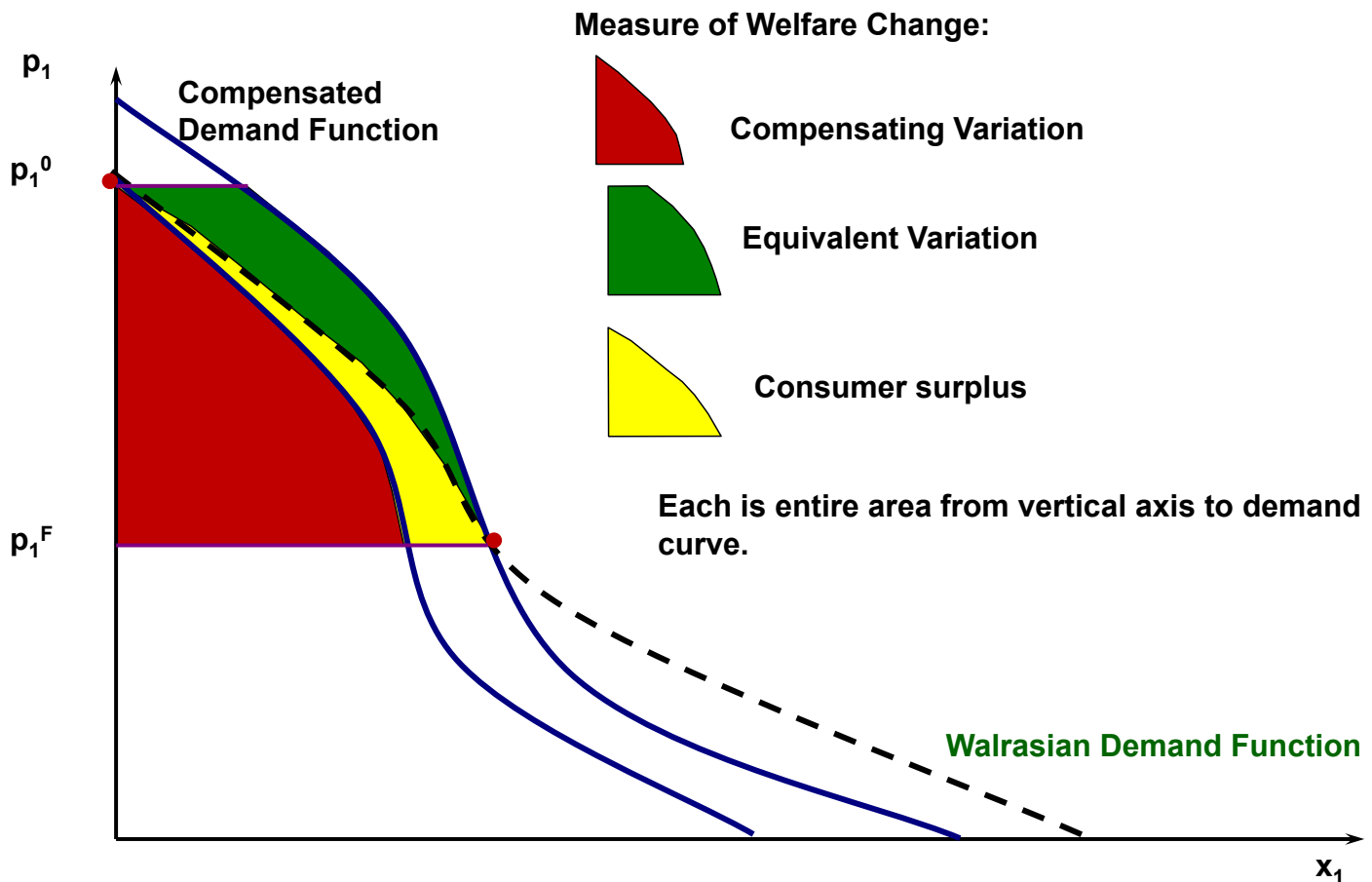
- Integrate along **Hicksian** demand curve, crossing through the **final** price and quantity

Consumer Surplus: Integrate along **ordinary (Walrasian)** demand curve.

WELFARE IMPACTS OF PRICE REDUCTION



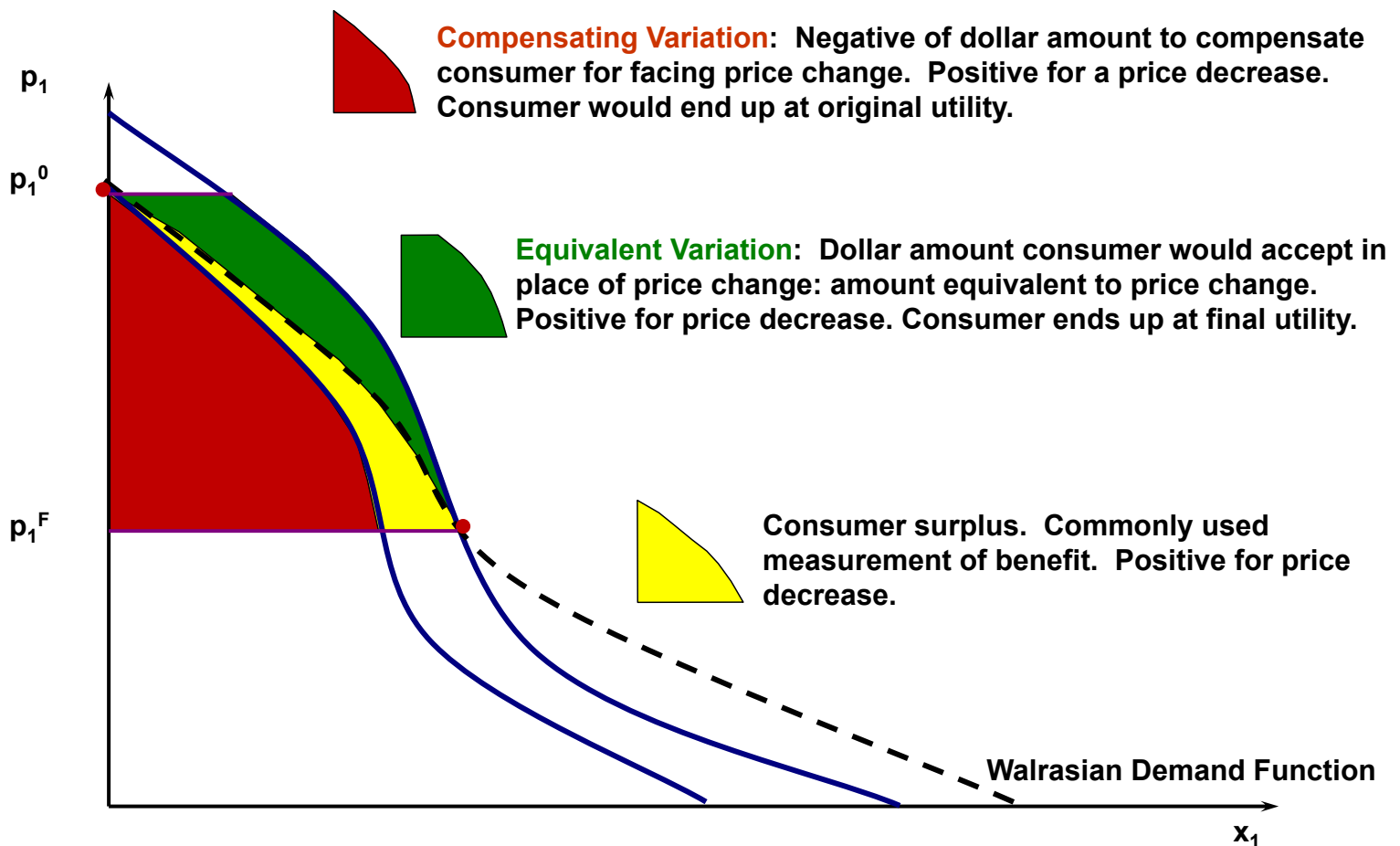
WELFARE IMPACTS OF PRICE REDUCTION (Cont'd)



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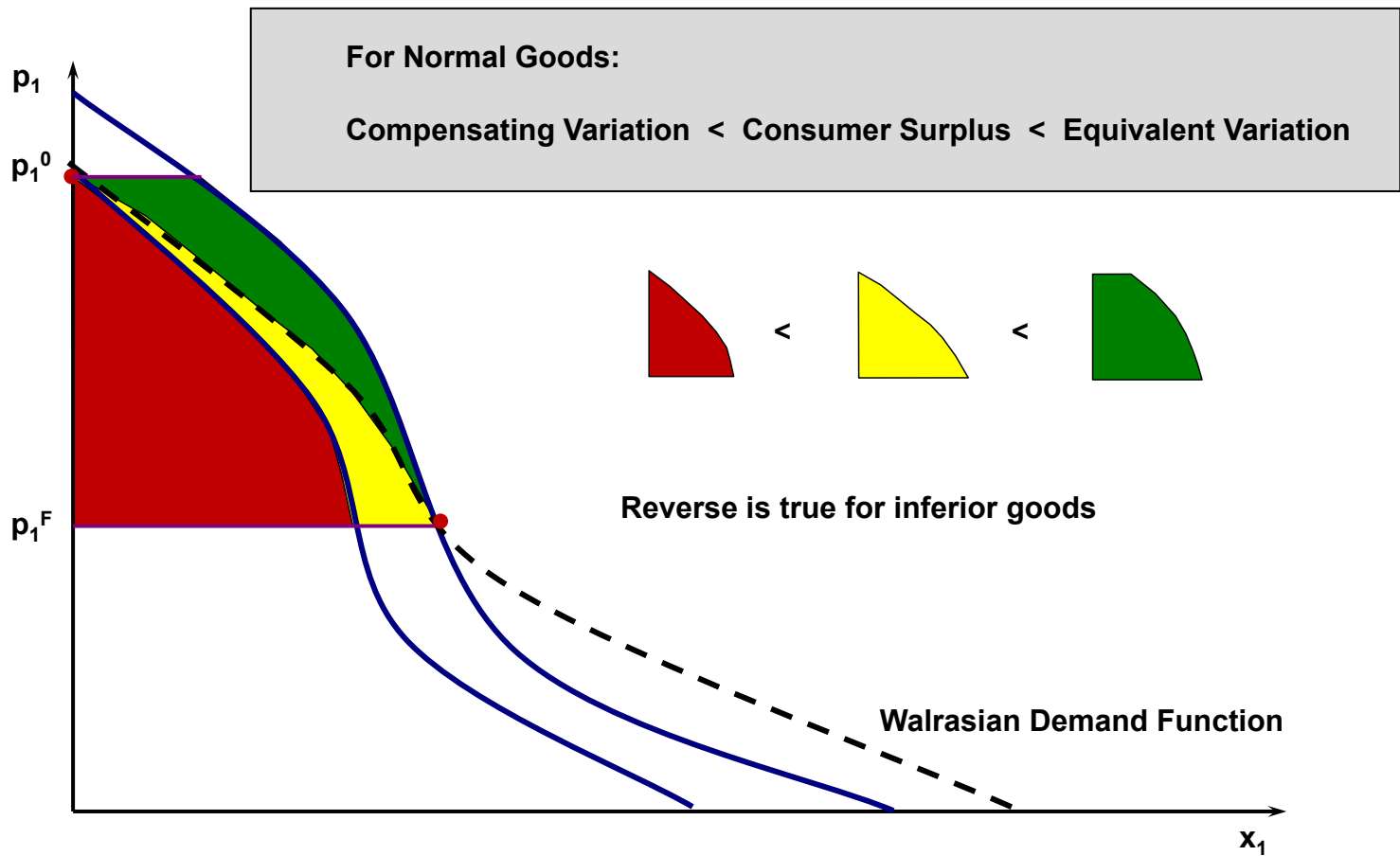
WELFARE IMPACTS OF PRICE REDUCTION (Cont'd)



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WELFARE IMPACTS OF PRICE REDUCTION (Cont'd)



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EXAMPLE: CONSTANT ELASTICITY OF DEMAND Compute Hicksian Compensated Demand Function

$$x(p, w) = \begin{bmatrix} \alpha(w / p_1) \\ (1 - \alpha)(w / p_2) \end{bmatrix}$$

Walrasian Demand Function
corresponding to a Cobb-Douglas
utility function $u(x) = Kx_1^\alpha x_2^{1-\alpha}$
in a two-good economy,
where $\alpha \in (0, 1)$, $K > 0$.

$$U = K(x_1(p, w))^\alpha (x_2(p, w))^{1-\alpha} = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{p_1^\alpha p_2^{1-\alpha}} Kw \Rightarrow e(p, U) = \frac{p_1^\alpha p_2^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{U}{K}$$

**Hicksian Compensated
Demand Function**

$$h_1(p, U) = x_1(p, e(p, U)) = \left(\frac{\alpha p_2}{(1-\alpha) p_1} \right)^{1-\alpha} \left(\frac{U}{K} \right)$$

$$h_2(p, U) = x_2(p, e(p, U)) = \left(\frac{(1-\alpha) p_1}{\alpha p_2} \right)^\alpha \left(\frac{U}{K} \right)$$

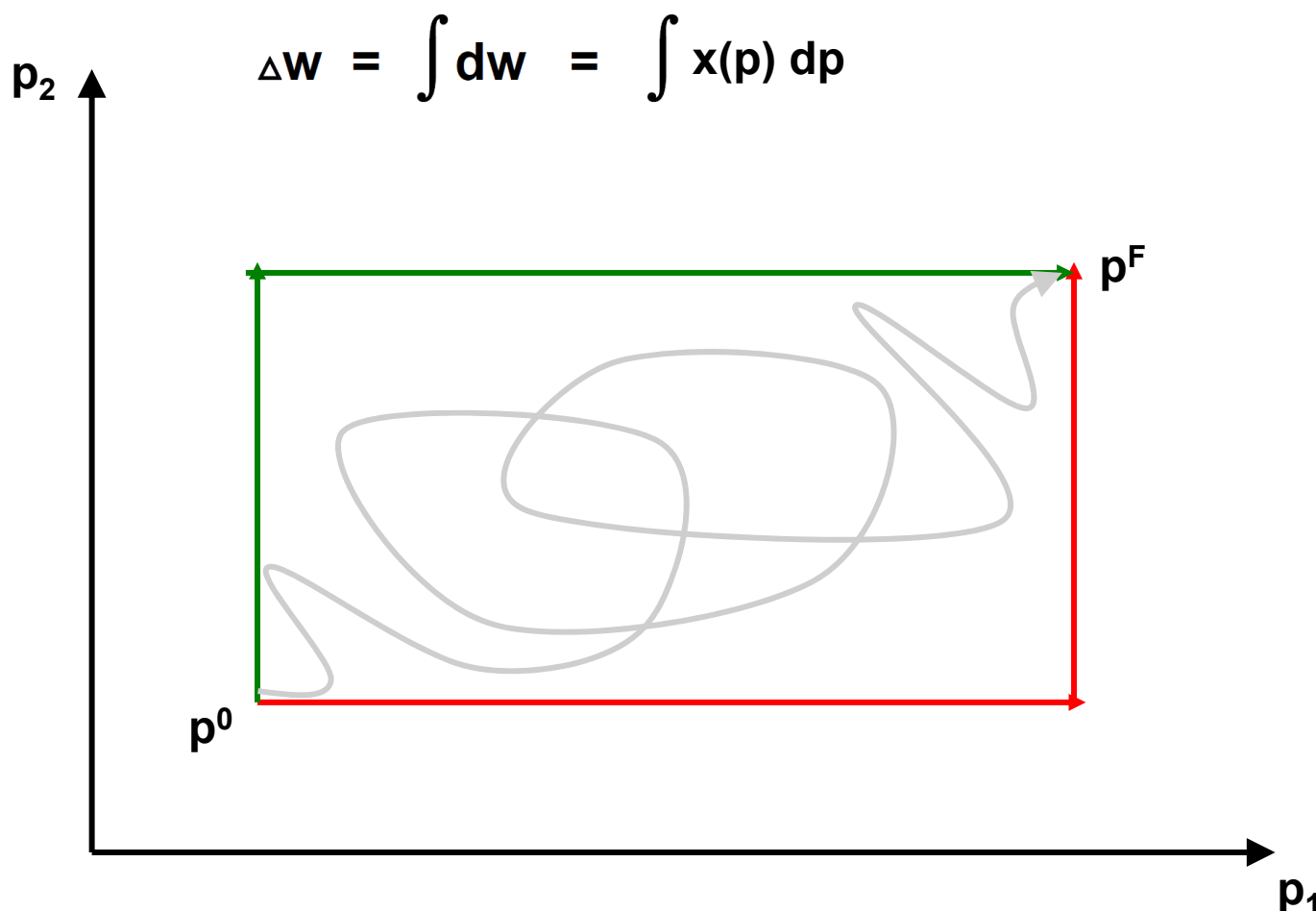
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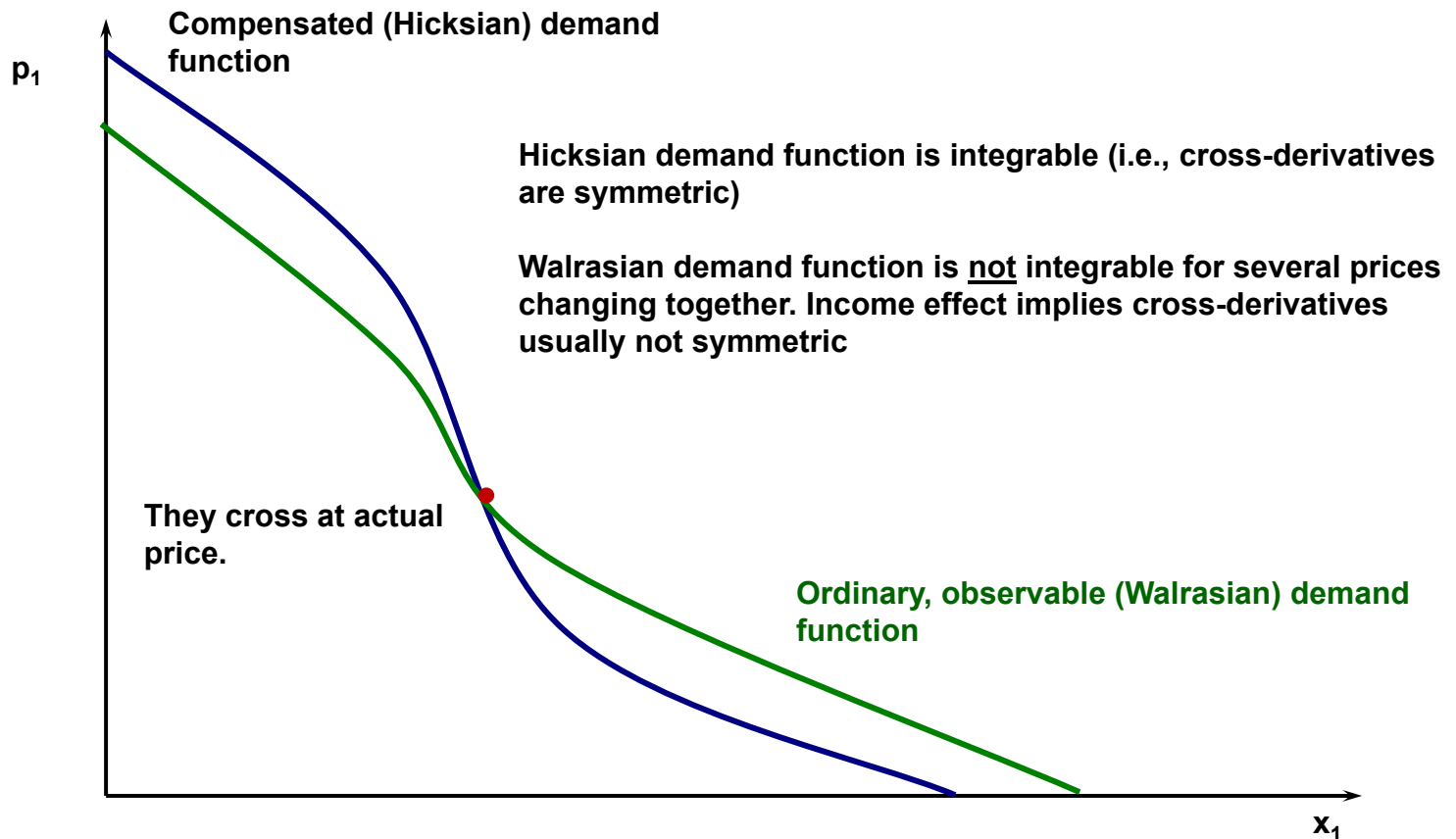
INTEGRABILITY

- To calculate welfare effects we integrate $x(p)$
- Integration is along *some* path of p from p^0 to p^F
- For the welfare measure to be unique, the integral must not depend on which particular path of p (from p^0 to p^F) was chosen for the integration
- Integrability conditions provide us with criterion for whether the measure is unique, that is, whether the integral is path-dependent
- Note that if only one price changes, all continuous functions are integrable
- For multiple prices changing, the demand function is integrable if and only if all cross-derivatives are symmetric, so

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i} \quad (\text{for all } i, j)$$



WALRASIAN DEMAND VS. COMPENSATED (HICKSIAN) DEMAND



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SUMMARY OF WELFARE MEASURES

Equivalent variation and compensating variation

- conceptually precise measurements
- based on integrable demand function

Consumer surplus

- not based on conceptually precise concept (e.g., not based on integrable demand functions)
- very easy to measure
- measure most often seen in calculations

All three measures tend to be similar UNLESS income effect is large

Welfare measures for state changes in the economy are routinely used in benefit-cost calculations

AGENDA

Some Special Utility Functions

Wealth Effects

Price Effects

Demand Aggregation

Standard Welfare Measures

Welfare Changes

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Special Utility Functions (Cobb-Douglas, CES, Leontief)**
- **Envelope Theorem**
- **Monotone Comparative Statics**
- **Expenditure Minimization Problem**
- **Hicksian Demand**
- **Law of Compensated Demand**
- **Slutsky Compensation (Wealth Compensation)**
- **Indirect Utility (and Gorman Form)**
- **Roy's Identity**
- **Slutsky Equation**
- **Income Effect & Substitution Effect of a Price Change**
- **Aggregate Demand**
- **Bandwagon Effect & Network Externalities & Fulfilled-Expectations Demand & Critical Mass**
- **Compensating Variation / Equivalent Variation / Consumer Surplus**

MGT 621 – MICROECONOMICS

Thomas A. Weber

3. Choice Under Uncertainty

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

Elements of Probability

Choice Under Uncertainty

Expected Utility Theory

Risk Aversion and Decision Biases

Key Concepts to Remember

CHOICE UNDER UNCERTAINTY

So far in this course we have assumed that a consumer (decision maker) knows perfectly the consequences of choice.

However, in most practical economic decision situations there is uncertainty.



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UNCERTAINTY IN CHOICE Some Examples

- How much will my education help me in the job market?
- Will I be given valuable assignments if I accept this job offer?
- Is the used car I am buying a lemon? Or will it be dependable?
- If I put effort into developing a proposal, will it be accepted?
- Will an R&D program be successful?
- How capable is the person I am considering hiring?
- Will my competitors introduce superior new products?
- Will potential customers purchase the product I offer?
- Will I enjoy the movie?
- What will the weather be like in the city I plan to visit?
- Will my home catch on fire in the next year and be destroyed?
- Will the price of the stock I purchase go up or down?
- Will prices for a commodity go up or down? Sign fixed-price contract?
- If I take a litigation to trial rather than settling, will I win?
- ...

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RISK VS. UNCERTAINTY

Distinction between risk and uncertainty due to Frank Knight (1921):

“The practical difference between the two categories, risk and uncertainty, is that *in the former the distribution of the outcome in a group of instances is known (...), while in the case of uncertainty this is not true*, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique” (p. 233)

“We can also employ the terms “**objective**” and “**subjective**” probability to designate the risk and uncertainty respectively, as these expressions are already in general use with a signification akin to that proposed” (ibid.)

- **Uncertainty**: unknowable (e.g., the success probability of your new startup company, or the likelihood of an unforeseen contingency in your project)
- **Risk**: knowable (e.g., the outcome of a die roll)

In this course, *no explicit distinction between risk and uncertainty*, since in order to formally analyze optimal choice, **need to introduce a probability space in either case**.

SUBJECTIVE PROBABILITY Some Examples

- All would not necessarily agree on the likelihood of events
 - Probability relevant for your decision is determined by all your knowledge
 - Most economic situations can best be described by subjective probabilities
- Will the used car be a lemon? Or will it be dependable?
 - Seller knows, i.e., for seller probability of being lemon is 0 or 1
 - Buyer does not know, i.e., buyer must assign a probability (= “belief”)
- What will the weather be like in the city I plan to visit?
 - Probability assessment may change after you read weather forecast
- How capable is the person I am considering for a job?
 - Potential employee has more information about work habits

SUBJECTIVE PROBABILITY (Cont'd)

New information (e.g., observing other agents' actions) can change beliefs

- What does this imply? (value of information, ... disinformation)

Can different people have very different probability assessments given the same choice, the same events, and the same information?

- Agreeing to disagree ...

Most of the time we assume that probabilities are subjective

DISCRETE RANDOM VARIABLES

Definition: A **discrete random variable** (or **lottery**) $X = [p_1, x_1; p_2, x_2; \dots; p_n, x_n]$ is a variable x that can take on one of the values

- $x_1, x_2, x_3, \dots, x_n$

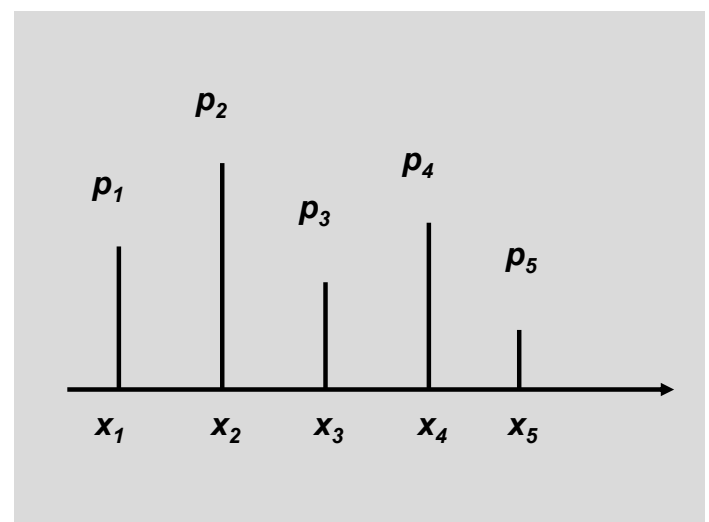
with the respective probabilities

- $p_1, p_2, p_3, \dots, p_n$

where each $p_i \geq 0$ and

$$\sum_{i=1}^n p_i = 1 \quad (\text{normalization})$$

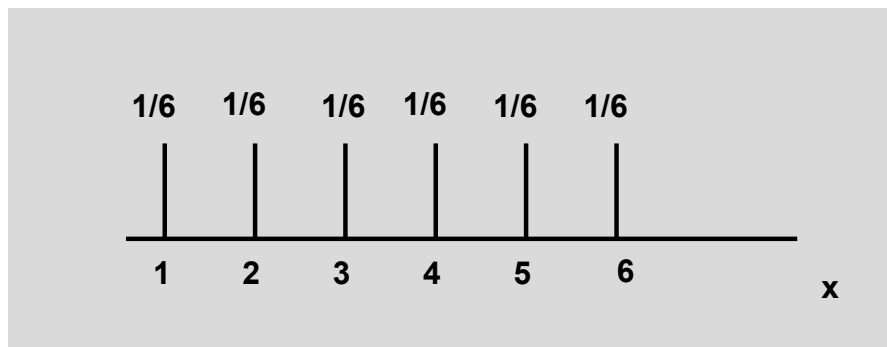
Using the **probability mass function** $p(\cdot)$ we can write the probabilities p_i as a function of x_i : $p_i = p(x_i)$.



EXAMPLE: ROLL OF A DIE

	x_i	p_i
$x =$ {	1	1/6
	2	1/6
	3	1/6
	4	1/6
	5	1/6
	6	1/6

w/ probability



EXPECTATION

The **expectation** of a (discrete) random variable X is defined as:

$$\bar{X} = E[X] = \sum_{i=1}^n p_i x_i$$

Key Property:

Linearity

If X and Y are random variables, and a and b are constants.
 $E[aX+bY] = a E[X] + b E[Y]$

Other Properties:

Sign Preservation

If X can take on only positive values, then $E[X] > 0$.

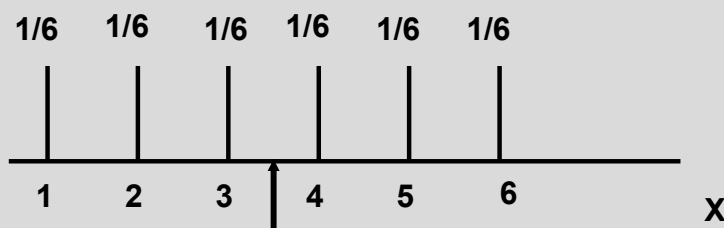
Certain Value

If X is perfectly known (and equal to x), then $E[X] = x$.

EXAMPLE (Cont'd)

X = {	1	w/ probability	1/6
	2		1/6
	3		1/6
	4		1/6
	5		1/6
	6		1/6

$$\begin{aligned}
 E[X] &= \sum_{i=1}^6 p_i x_i \\
 &= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \dots + \frac{1}{6} \cdot 6 \\
 &= \sum_{i=1}^6 \frac{1}{6} \cdot i = 3.5
 \end{aligned}$$



The expectation is where this picture would balance on your finger

VARIANCE AND STANDARD DEVIATION

The **variance** is a measure of the spread of a random variable around its mean.

$$\begin{aligned}
 V[X] &= E[(X - \bar{X})^2] = E[(X^2 - 2X\bar{X} + \bar{X}^2)] \\
 &= E[X^2] - 2\bar{X}E[X] + \bar{X}^2 = E[X^2] - \bar{X}^2
 \end{aligned}$$

The **standard deviation** is the square root of the **variance**.

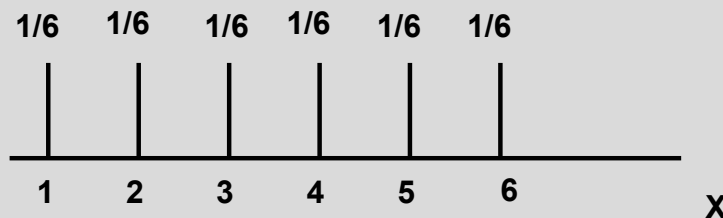
$$\sigma_X = \sqrt{V[X]}$$

It has the same units as X.

EXAMPLE (Cont'd)

$$V[X] = E[(X - \bar{X})^2]$$

$$\begin{aligned}
 X = \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \right. \quad \text{w/ probability} \quad & \begin{array}{l} 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{array} \\
 & = \frac{1}{6}(1-3.5)^2 + \frac{1}{6}(2-3.5)^2 + \dots \\
 & \quad + \frac{1}{6}(6-3.5)^2 \\
 & = 2.92
 \end{aligned}$$



CONDITIONAL PROBABILITY

Definition: The conditional probability $P(A|B)$ of event A conditional on event B having realized is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Question. Given a six-sided die what is the probability of rolling a 3 conditional on rolling a 1, 2 or 3?

$$\begin{aligned}
 P(\{3\} | \{1,2,3\}) &= \frac{P(\{3\} \cap \{1,2,3\})}{P(\{1,2,3\})} = \frac{P(\{3\})}{P(\{1,2,3\})} \\
 &= \frac{1/6}{1/2} = 1/3
 \end{aligned}$$

INDEPENDENCE

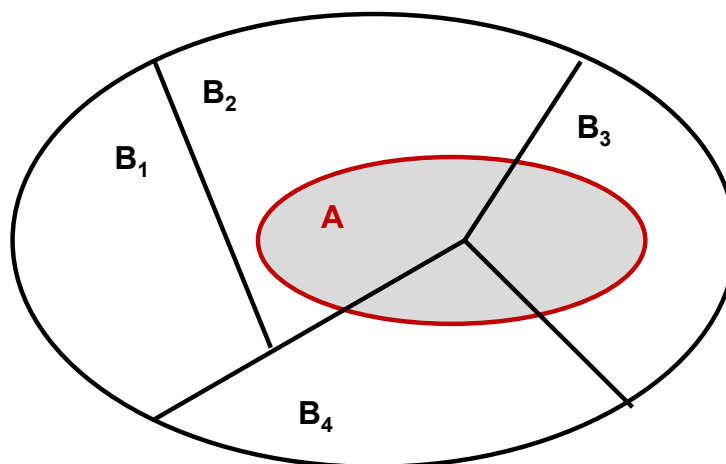
Definition: A and B are **independent events**, if

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

$$P(A \cap B) = P(A|B)P(B)$$

THE LAW OF TOTAL PROBABILITY

$$P(A) = \sum_{n=1}^N P(A \cap B_n) = \sum_{n=1}^N P(A|B_n)P(B_n)$$



AGENDA

Elements of Probability

Choice Under Uncertainty

Expected Utility Theory

Risk Aversion and Decision Biases

Key Concepts to Remember

LOTTERIES ARE (DISCRETE) RANDOM VARIABLES

Let X be a **random variable** with possible outcomes in the set $X = \{x_1, x_2, \dots, x_n\}$ (the “outcome space”). Each outcome x_i occurs with probability p_i .

The random variable X is sometimes also called a **lottery** and denoted

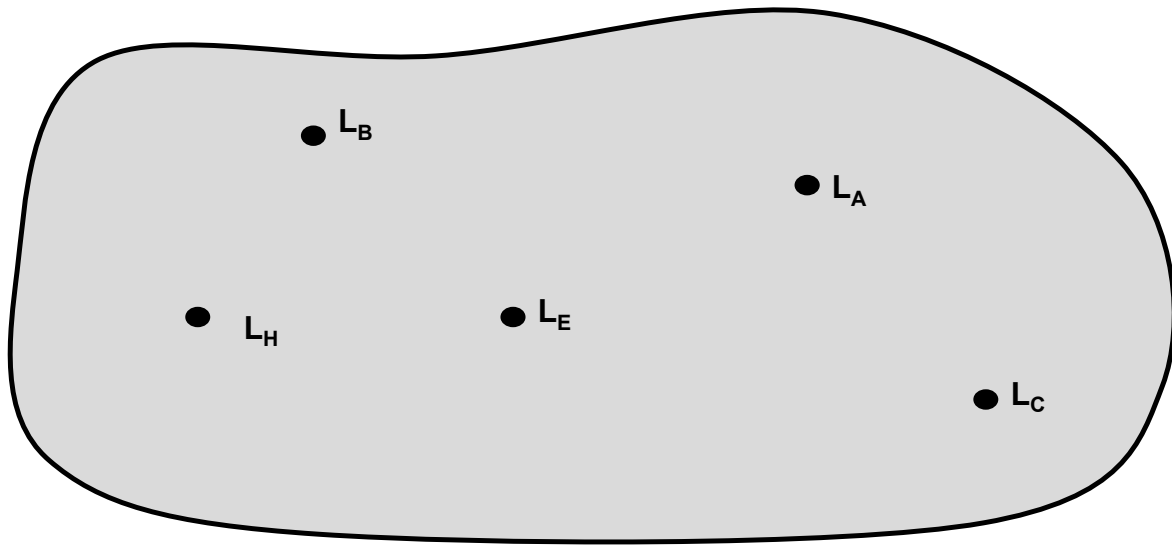
$$X = [p_1, x_1; p_2, x_2; \dots; p_n, x_n]$$

If all outcomes x_i are real and measured in dollars (or any other currency), then X is commonly referred to as a “**money lottery**.”

The set of all lotteries with outcomes in X is the “**lottery space**” $L(X)$.

Example: A coin-flip lottery X (with an unbiased coin) pays \$1 if heads and zero if tails. Then $X = [0.5, \$1; 0.5, \$0]$.

THEORY OF CHOICE UNDER UNCERTAINTY



The choice set contains all simple lotteries over the various outcomes.

PREFERENCES OVER LOTTERIES

For a decision maker (DM), *choosing between actions corresponds generally to choosing between lotteries* (given a decision d in the set of possible decisions D , the outcome x_i in X occurs with probability $P(x_i|d)$).

Example: wearing a helmet on a motorcycle changes the probability of injury. **In a decision tree, each decision node represents a lottery.**

Hence, the DM needs to be able to order lotteries according to his preference implying a **complete preference (pre-)ordering**⁽¹⁾ over elements in the lottery space $L(X)$.

More specifically, if A and B are elements of $L(X)$, then

- $A > B$ means DM prefers A to B
- $A \geq B$ means DM does not prefer B to A or DM “weakly” prefers A to B
- $A \sim B$ means DM is indifferent between A and B , i.e., DM will take either one and would play a 50-50 lottery to choose between them.

(1) Formally, a pre-ordering R is a *binary relation* (i.e., it takes two inputs) that is *reflexive* (i.e., xRx) and *transitive* (i.e., xRy and yRz implies xRz). The pre-ordering is *complete* if for any elements x and y in X it is either xRy or yRx (or both).

PREFERENCES OVER LOTTERIES: EXAMPLE

Question. Jane likes to play ping pong and she wonders about how to respond to an opponent's serve.

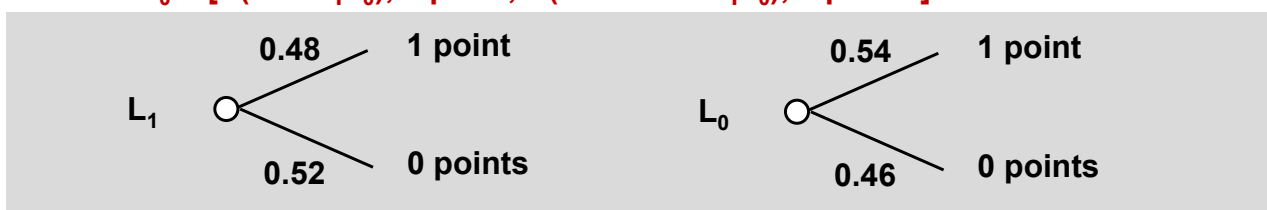
- If she hits a top spin (decision d_1), the ball is going to be on the table with probability 0.6 and given that it is, she is going to score with probability 0.8.
- If she does not play a top spin (decision d_0), the ball is going to land on the table with probability 0.9, but she is only going to score with probability 0.6.

What decision should she take?

Solution:

Jane needs to choose between the following two lotteries:

- $L_1 = [P(\text{score}|d_1), 1 \text{ point}; P(\text{don't score}|d_1), 0 \text{ points}]$
- $L_0 = [P(\text{score}|d_0), 1 \text{ point}; P(\text{don't score}|d_0), 0 \text{ points}]$



Thus, she should prefer L_0 (i.e., $L_0 \geq L_1$) which implies “**don't play top spin**” as her decision.

PREFERENCES OVER OUTCOMES Utility Representation (Reminder)

Preferences over lotteries imply preferences over particular (certain) outcomes in X . Indeed, for any x and y in X one could just consider the lotteries X and Y that produce the outcomes x and y with probability one respectively.

Thus, $x \geq y$ if the outcome x is (weakly) preferred to the outcome y .

Definition: A **real-valued function u with domain X represents the DM's preferences over outcomes in X** , if for any x, y in X :

$$x \geq y \text{ if and only if } u(x) \geq u(y).$$

The function u is called the DM's **utility function**.

For some preferences no utility function representation exists (e.g., lexicographic preferences). We typically take a utility function as an input for a decision model.⁽¹⁾ A utility function always exists for finite sets of outcomes.

A utility representation of a DM's preferences is generally not unique: given any **utility function u and a strictly increasing function ϕ** (from real numbers to real numbers), **the function $v = \phi(u)$ is an equivalent utility representation.**

(1) For more details on utility theory see Kreps, D.M. (1988) *Notes on the Theory of Choice*, Westview Press, Boulder, CO, or Fishburn, P.C. (1970) *Utility Theory for Decision Making*, Wiley, New York, NY.

UTILITY REPRESENTATION

Mark has the utility function $u(x) = x^{1/2}$ for any nonnegative amount of money x (in dollars).

Thus, he prefers \$100 to \$36, since $u(100) = 10 > u(36) = 6$.

Similarly, for $\phi(y) = y^2$, we have that $\phi(u(100)) = 100 > \phi(u(36)) = 36$, and Mark would have the same preferences for outcomes (amounts of money) for any other ϕ , as long as ϕ is strictly increasing.

- $x \geq y$ if and only if $u(x) \geq u(y)$ if and only if $\phi(u(x)) \geq \phi(u(y))$

Thus, in the absence of uncertainty Mark can just maximize x instead of $u(x)$.

AGENDA

Elements of Probability

Choice Under Uncertainty

Expected Utility Theory

Risk Aversion

Key Concepts to Remember

EXPECTED UTILITY MAXIMIZATION

Given a utility representation u of the DM's preferences over outcomes, we would like to infer his preferences over lotteries of outcomes (random variables), which corresponds to his preferences over actual decisions (e.g., at which speed to drive a car).

Under certain axioms (= assumptions on the DM's preferences over lotteries, typically one uses the Von Neumann-Morgenstern axioms), the DM's **expected utility of a particular decision d in D** which induces a lottery $X(d)$ with probability distribution $P(\cdot|d)$ is

$$EU(X(d)) = \sum_{x \in X} P(x|d)u(x)$$

Thus, under uncertainty the **DM maximizes expected utility**, i.e., he solves

$$d^* \in \arg \max_{d \in D} EU(X(d))$$

EXPECTED UTILITY MAXIMIZATION: EXAMPLE

Question. Joe needs to decide how fast to drive on highway E25 from Lausanne to Geneva. Any minute saved he values at \$1. At 120 km/h it takes about 40 min, and at 140 km/h about 32 min. However, if he drives 140 km/h there is a chance p that he gets pulled over and has to pay a ticket worth \$280 plus a delay of 20 min (over the 120 km/h time).

Let d_0 : drive 120 km/h, d_1 : drive 140 km/h. He thus needs to choose between the lotteries $X(d_0) = [1, \$0]$ and $X(d_1) = [p, -\$300; 1-p, \$8]$.



If Joe's utility function for money is $u(x) = -\exp(-x/1000)$, at what detection probability p would he be indifferent between d_0 and d_1 ?

Answer: $EU(X(d_0)) = u(\$0) = EU(X(d_1)) = pu(-\$300) + (1-p)u(\$8)$

$$\Rightarrow p = \frac{u(\$8) - u(\$0)}{u(\$8) - u(-\$300)} = 2.2\% \quad (\text{i.e., for } p > 2.2\%, \text{ Joe drives 65 mph!})$$

VON NEUMANN-MORGENSTERN AXIOMS IMPLY EXPECTED UTILITY REPRESENTATION

Fundamental Justification for Expected Utility Maximization

Let $X = \{x_1, x_2, \dots, x_n\}$ be a set of outcomes and $L(X)$ the corresponding lottery space. Consider arbitrary elements A, B, C of $L(X)$ and x, y, z in X .⁽¹⁾

1. **Completeness:** $A \geq B$ or $B \geq A$ (can compare any two lotteries)
2. **Reflexivity:** $A \geq A$
3. **Transitivity:** $A \geq B$ and $B \geq C$ implies $A \geq C$ (o/w construct a “money pump”)
4. **Continuity:** If $x > y > z$, then there exists p in $(0,1)$ such that one can achieve indifference between the lottery $A = [1, y]$ and the lottery $B(p) = [p, x; 1-p, z]$, i.e., there is a p such that $A \sim B(p)$

- Example: What happens if z is “death”?

5. **Independence (of Irrelevant Alternatives):** If $x > y$, then for any z :
 $[p, x; (1-p), z] > [p, y; (1-p), z]$

(1) Note that if x is an outcome in X , then $[1, x]$ is a lottery in $L(X)$. Thus, any outcome can be viewed as a (degenerate) lottery.

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KEY IMPLICATION OF THE VNM AXIOMS

Theorem (Von Neumann-Morgenstern, 1944): If a DM's preferences “ $\geq$ ” on $L(X)$ satisfy the Von Neumann-Morgenstern axioms, then there exists an expected utility function $EU(\cdot)$ for that DM which represents his preferences in the sense that for any two lotteries A, B in $L(X)$:

$$A \geq B \text{ if and only if } EU(A) \geq EU(B)$$

Proof: See notes on “Risk and Uncertainty” posted on the course website.

In more detail, suppose:

$A = [p_1, x_1; p_2, x_2; \dots; p_n, x_n]$, where $p_1 + \dots + p_n = 1$

$B = [q_1, x_1; q_2, x_2; \dots; q_n, x_n]$, where $q_1 + \dots + q_n = 1$

If DM's preferences satisfy the VNM Axioms, then the DM prefers A to B ($A \geq B$) if and only if

$$EU(A) = \sum_{i=1}^n p_i u(x_i) \geq \sum_{i=1}^n q_i u(x_i) = EU(B)$$

Note: Two expected utility functions $EU(x)$ and $EV(x)$ represent the same preferences over lotteries if and only if $EU(x) = \alpha EV(x) + \beta$, where $\alpha > 0$ and β is any real number. In the same vein, a “positive affine transformation” of the DM's utility function u (to $v = \alpha u + \beta$, for $\alpha > 0$) does not change the DM's preferences over lotteries.

The original reference for the Von Neumann-Morgenstern result is their immensely readable text Von Neumann, J., Morgenstern, O. (1944) *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

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KEY IMPLICATION OF THE VNM AXIOMS (Cont'd)

Example: Consider the utility functions $u(x) = 1 - 2e^{-\rho x}$ and $v(x) = 3 - 5e^{-\rho x}$, where $\rho > 0$ is some constant.

Since $u(x) = .4v(x) - .2 = \alpha v(x) + \beta$ (with $\alpha > 0$), one can check that for any two lotteries A and B in $L(X)$:

$$EU(A) \geq EU(B) \quad \text{if and only if} \quad EV(A) \geq EV(B)$$

More generally, **invariance with respect to positive affine transformations implies that we can fix any two values of a DM's utility function** without disturbing his preference ordering over lotteries.

ALLAIS PARADOX

Imagine the following two decision situations—each involving a pair of gambles.

SITUATION I

	<u>P(Winning)</u>	<u>AMOUNT TO WIN</u>
Lottery A	100%	\$1,000,000
Lottery B	10%	\$5,000,000
	89%	\$1,000,000
	1%	-0-

SITUATION II

	<u>P(Winning)</u>	<u>AMOUNT TO WIN</u>
Lottery C	11%	\$1,000,000
	89%	-0-
Lottery D	10%	\$5,000,000
	90%	-0-

ALLAIS PARADOX

Implies Critique of 'Independence of Irrelevant Alternatives'

Lottery	Ticket Numbers		
	1	2-11	12-100
A	\$1 million	\$1 million	\$1 million
B	\$0 million	\$5 million	\$1 million
C	\$1 million	\$1 million	\$0 million
D	\$0 million	\$5 million	\$0 million

NOTE: Ticket numbers 12-100 are the same for A and B so they are irrelevant for choices between them; the same for C and D

If you eliminate these ticket numbers then
A and C are identical and so are B and D

AGENDA

Elements of Probability

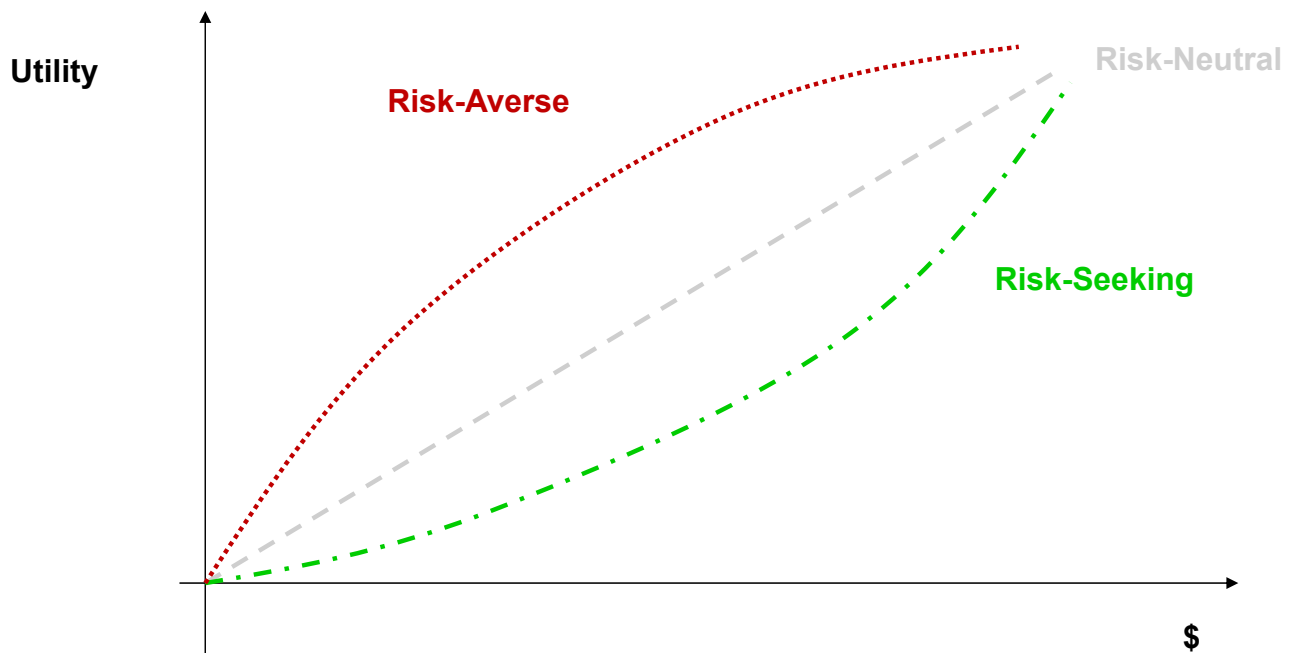
Choice Under Uncertainty

Expected Utility Theory

Risk Aversion and Decision Biases

Key Concepts to Remember

UTILITY FUNCTIONS: SOME COMMON SHAPES



RISK AVERSION: EXAMPLES

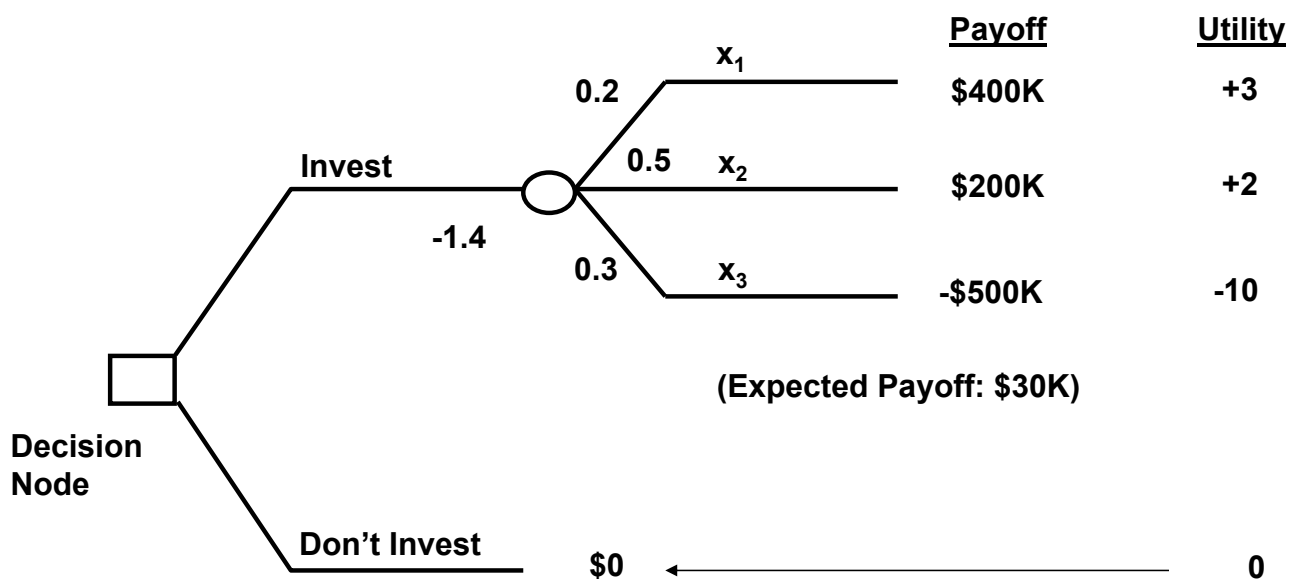
People will buy insurance, even though the expected value of the payment from insurance is smaller than its price. That is the result of risk aversion.

People purchase a portfolio of stocks and bonds, rather than only one. Such diversification reduces risk and is consistent with risk aversion.

People will incur costs to purchase hedges, assets that reduce the risk of the overall portfolio.

Typically the larger the monetary lottery, the greater the degree of risk aversion people exhibit.

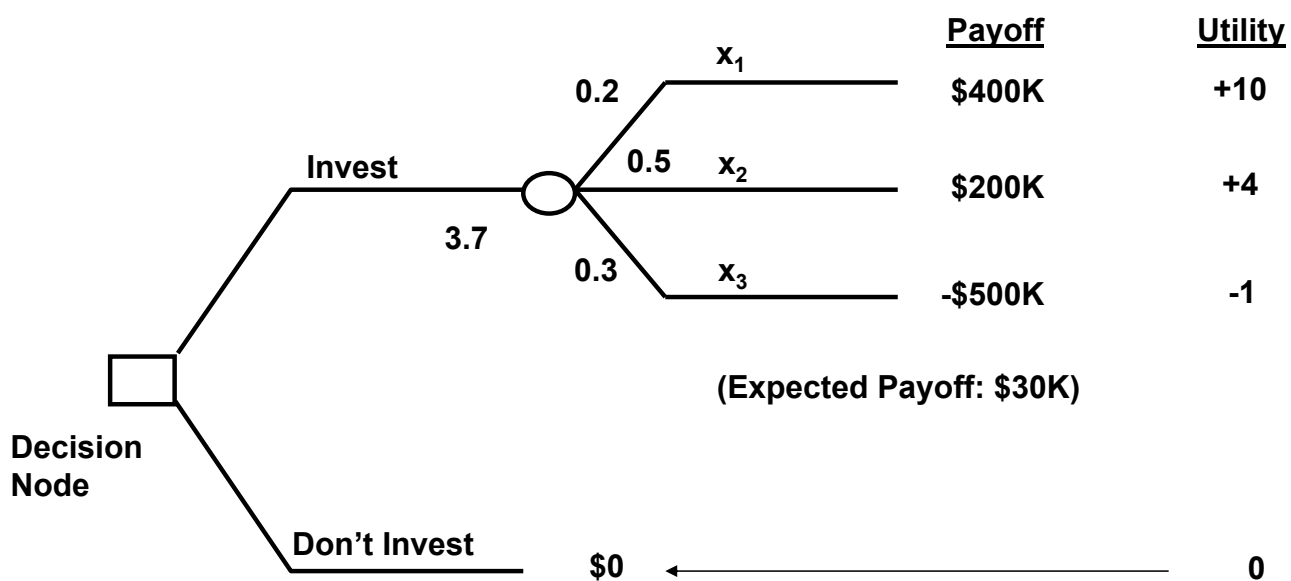
A SIMPLE DECISION: RISK-AVERSE DM



Decision Criterion: Maximize Expected Utility

$$EU = (0.2)(3) + (0.5)(2) + (0.3)(-10) = -1.4 < 0 \rightarrow \text{Don't Invest!}$$

A SIMPLE DECISION TREE: RISK-SEEKING DM



Decision Criterion: Maximize Expected Utility

$$EU = (0.2)(10) + (0.5)(4) + (0.3)(-1) = 3.7. \rightarrow \text{Invest!}$$

ABSOLUTE AND RELATIVE RISK AVERSION

DESCRIBE A DM'S RISK ATTITUDE

The level of risk aversion may be measured by the (Arrow-Pratt) **absolute-risk-aversion** coefficient,

$$R(x) = -\frac{u''(x)}{u'(x)}$$

or the **relative-risk-aversion coefficient**,

$$r(x) = xR(x)$$

If $R(x) > 0$, the DM is **risk-averse**. Similarly, if $R(x) < 0$, the DM is **risk-seeking**, while $R(x)=0$ for a **risk-neutral** DM.

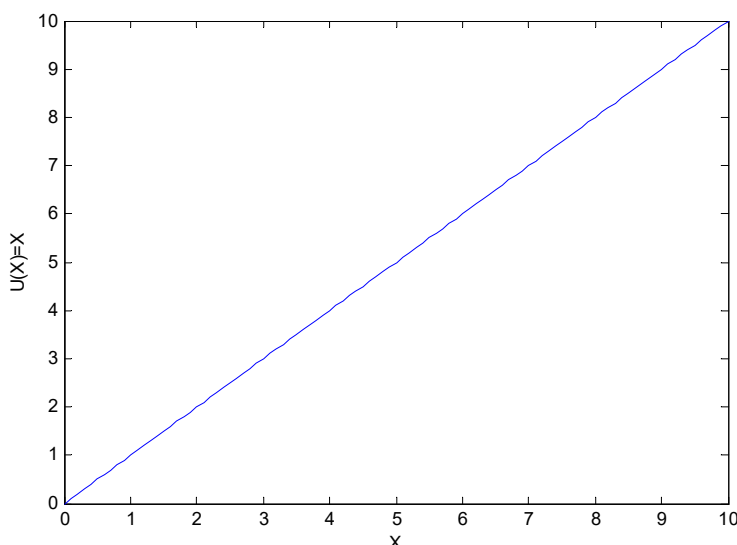
Both absolute and relative risk aversion are local properties:
they can vary for different outcomes.

RISK NEUTRALITY: LINEAR UTILITY

$$u(x) = \alpha x \quad u'(x) = \alpha \quad u''(x) = 0$$

$$(\alpha > 0)$$

$$R(x) = -\frac{u''(x)}{u'(x)} = 0$$

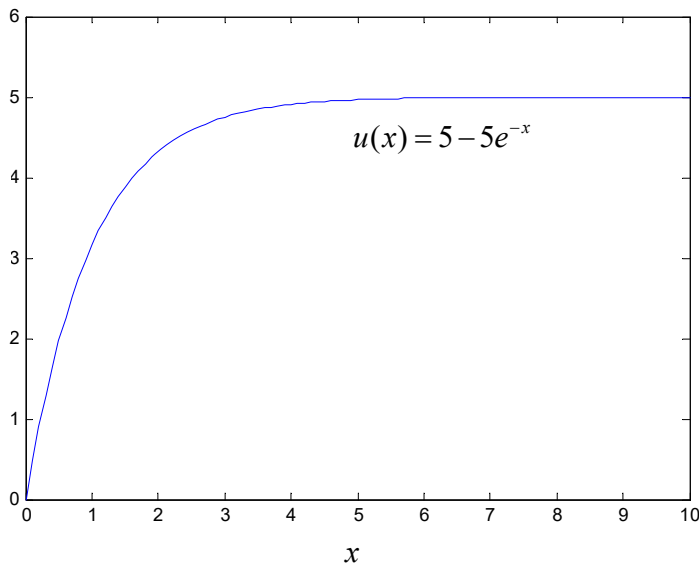


CONSTANT ABSOLUTE RISK AVERSION

Exponential utility: $u(x) = \alpha - \beta e^{-\rho x}$

$$u'(x) = \rho \beta e^{-\rho x}$$

$$u''(x) = -\rho^2 \beta e^{-\rho x}$$



$$R(x) = -\frac{u''(x)}{u'(x)} = \rho$$

Exponential utility functions exhibit **constant absolute risk aversion (CARA)**.⁽¹⁾

(1) CARA utility functions are often used in financial modeling, since it allows obtaining conclusions free from wealth effects (adding a constant w to an individual's wealth just amounts to a positive linear transformation and thus leads to the same decisions, since the expected utility representation of the individual's preferences does not change).

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CERTAINTY EQUIVALENT

The certainty equivalent of a lottery is a single certain outcome for which the DM is indifferent between receiving the outcome for sure and participating in the lottery.

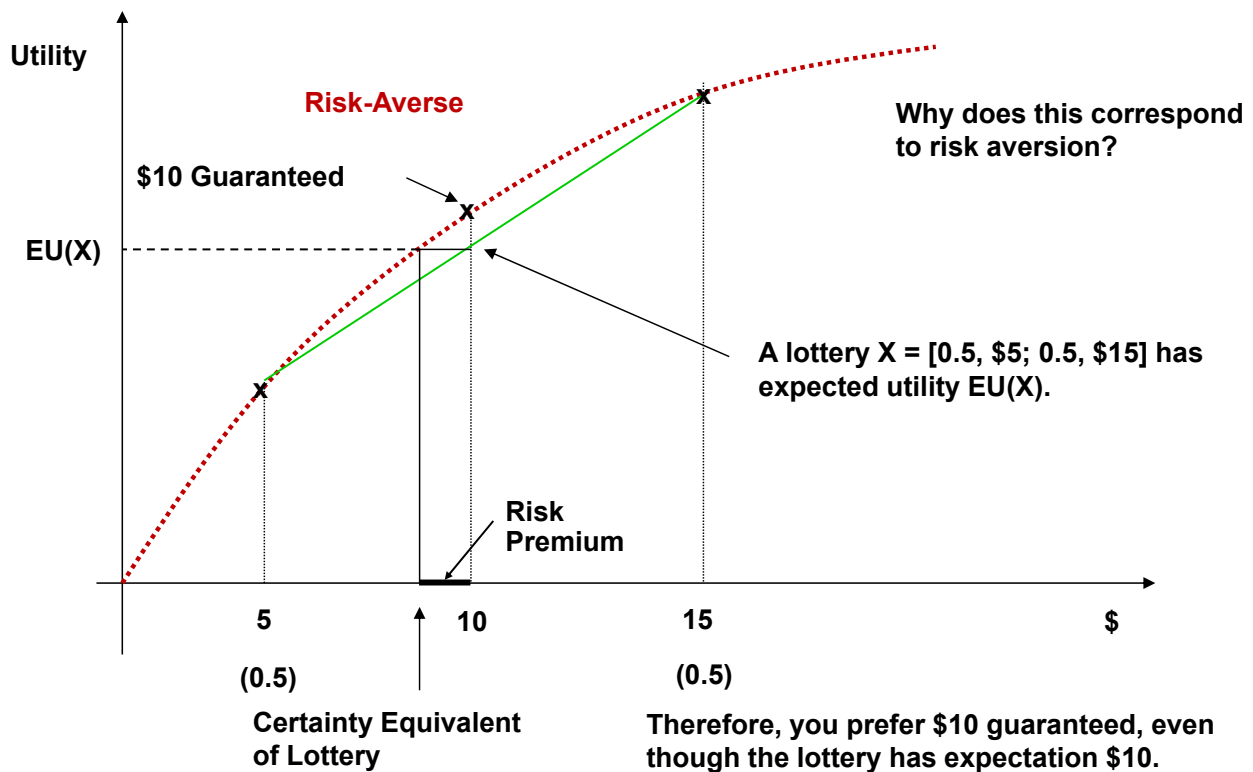
It represents the “selling price” of the lottery.

Denote the **certainty equivalent of a lottery X** by $CE(X)$

Then: $u(CE(X)) = EU(X)$ (DM is indifferent)

$$\Rightarrow CE(X) = u^{-1}(EU(X))$$

CERTAINTY EQUIVALENT: EXAMPLE



CERTAINTY EQUIVALENT: ANOTHER EXAMPLE

Consider the lottery $X = [.25, \$100; .5, \$49; .25, \$0]$ and the utility function $u(x) = \sqrt{x}$

	<u>Payoff (x)</u>	<u>Utility u(x)</u>
0.25	\$100	10
0.50	\$49	7
0.25	\$0	0

Expected utility: $EU(X) = (0.25)(10) + (0.5)(7) + (0.25)(0) = 6$

$$EU(X) = u(CE) = \sqrt{CE} \implies \sqrt{CE} = 6 \implies CE = 36$$

EXAMPLE: CONSTRUCTING A UTILITY FUNCTION FOR MONEY

Arbitrarily assign utilities to two real-valued outcomes, x_1 and x_2 (say, measured in dollars). For example, $x_1 = -\$128$ and $x_2 = \$128$, and

$$u(-\$128) = -100 \text{ and } u(\$128) = 100.$$

Use continuity axiom to specify other utilities.

Certainty Equivalence Method: Fix p and two outcomes x_1 and x_2 . Then find an outcome y which makes you indifferent between having y for certain or taking the lottery $[p, x_1; 1-p, x_2]$.⁽¹⁾

The value y is commonly referred to as the **certainty equivalent (CE)** of the lottery $[p, x_1; 1-p, x_2]$: $y = CE$,

$$u(CE) = p u(x_1) + (1-p) u(x_2).$$

(1) In a set of discrete outcomes such an element y might not be available. Then one needs to adjust the probability p accordingly, which by the continuity axiom can always achieve indifference.

A MARKET FOR COIN FLIPS

Consider the following game. You flip a fair coin.

- If the first flip is heads (H) you win \$2 and you flip the coin again. If it is tails (T), then you win \$0 and the game is over.
- If the second flip is H you win \$4 and you flip the coin again. If it is T, then you keep the \$2 you won on the first flip and the game is over.
- If the n -th flip is H you win $\$2^n$ and you flip the coin again. If it is T, then you keep the $\$2^{n-1}$ you won on the $(n-1)$ st flip and the game is over.

The following table summarizes the outcome (we restrict the length to $n \leq 7$ to avoid bankruptcy of players).

Number of Heads in a Row (n)	Total Winnings
1	\$2
2	\$4
3	\$8
4	\$16
5	\$32
6	\$64
7	\$128

A MARKET FOR COIN FLIPS (Cont'd)

Please answer the following question (depending on your role):

- **Bankers:** if you are a banker (i.e., act as a bank in this game), how much would you need to be paid for sure to run the game? The person with the lowest amount will serve as the banker and play the game for real.
- **Players:** if you are a player (i.e., you get to potentially win in this game), how much would you be willing to pay to participate in the game? The person with the highest amount will play the game for real.

MARKET FOR COIN FLIPS: ACTUARIAL VALUE

Let us compute the actuarial value of the coin-flip game X:

$$\begin{aligned} E[X] &= \left(\frac{1}{2}\right) \$0 + \left(\frac{1}{4}\right) \$2 + \left(\frac{1}{8}\right) \$4 + \dots + \left(\frac{1}{2^{n-1}}\right) \$2^{n-1} + \left(\frac{1}{2^n}\right) \$2^n \\ &= (n-1) (\$0.5) + \$1 = \$4. \end{aligned}$$

↑
n = 7

The certainty equivalent $CE(X)$ of the coin-flip lottery X for a player,⁽¹⁾ given a utility function u , satisfies therefore:

$$u(CE(X)) = \left(\frac{1}{2}\right) u(\$0) + \left(\frac{1}{4}\right) u(\$2) + \left(\frac{1}{8}\right) u(\$4) + \dots + \left(\frac{1}{2^{n-1}}\right) u(\$2^{n-1}) + \left(\frac{1}{2^n}\right) u(\$2^n) = EU(X),$$

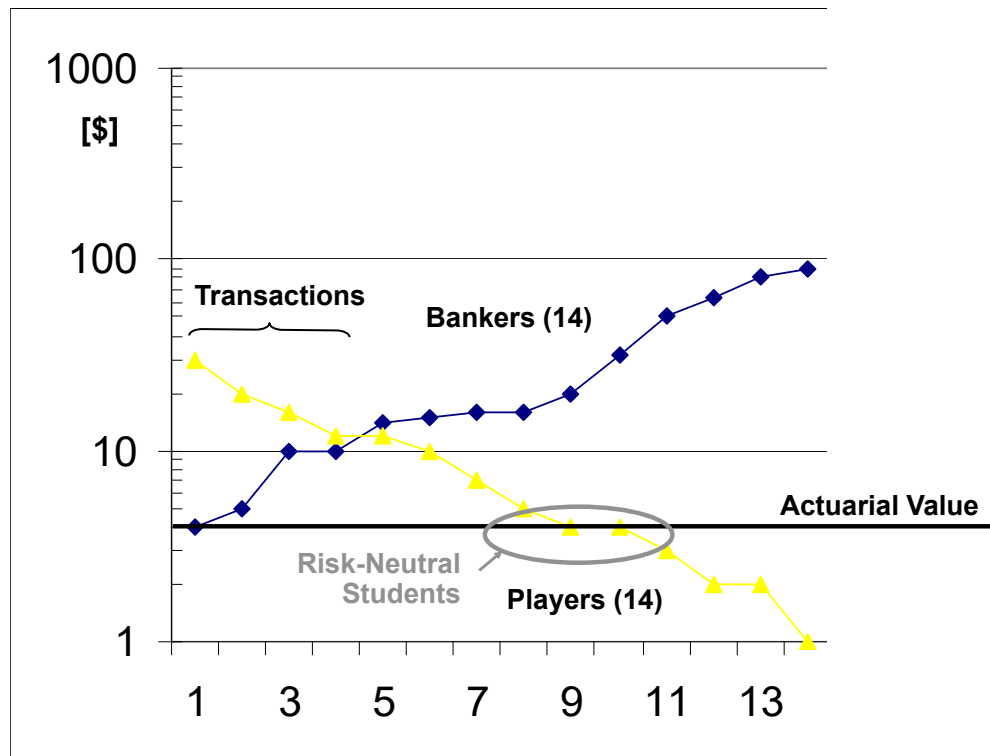
so that

$$CE = u^{-1}(EU(X))$$

You can read the utilities off your utility function constructed a couple of slides ago (for $n = 7$)

(1) For a *banker* one can compute the CE in a similar way, just by taking the outcomes in terms of losses instead of gains.

MARKET FOR COIN FLIPS: CLASS RESULTS



The plotted values correspond to the certainty equivalents of bankers and players respectively. Why are they not the same?

INDIVIDUALS ARE RISK-AVERSE IN GAINS AND RISK-SEEKING IN LOSSES (SENSITIVITY TO REFERENCE POINT) We already did this experiment!

1. You have been given \$200 and have a choice between the following two options

- A: Win \$150 with certainty
- B: Win \$300 with probability .5
Win \$0 with probability .5

- Do you prefer A or B ?

2. You have been given \$500 and have a choice between the following two options

- C: Lose \$150 with certainty
- D: Lose \$300 with probability .5
Lose \$0 with probability .5

- Do you prefer C or D?

SENSITIVITY TO REFERENCE POINT: CLASS RESULTS

		Loss Averse	
		Gamble C	Gamble D
Risk Averse	Gamble A	35	28
	Gamble B	7	8

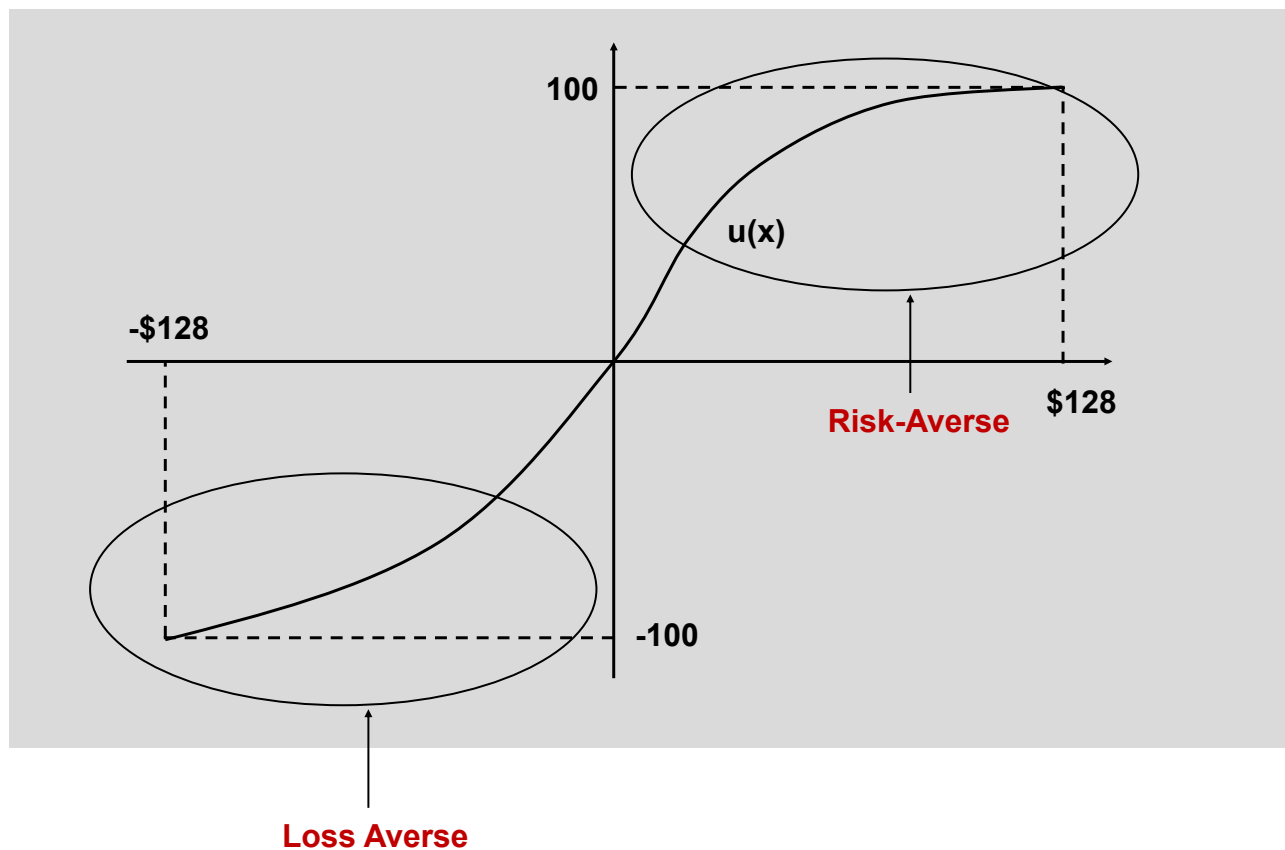
Rational choices (satisfying VNM axioms)

According to utility theory $A = C$ and $B = D$; so if A is preferred to B then C should be preferred to D and vice versa.

The “modal choices” are (i.e., “most people prefer”) A and D to avoid losses.

Remark: This sensitivity to the framing of the lottery choice in terms of either gains or losses is sometimes also referred to as *reflection effect*: preferences tend reverse if the lottery is “reflected” from the domain of gains (with respect to the *status quo*) to the domain of losses. For details, see Kahneman, D., Tversky, A. (1979) “Prospect Theory: An Analysis of Decision Under Risk,” *Econometrica*, Vol. 47, No. 2, pp. 263—291.

“REAL” UTILITY FUNCTIONS OFTEN LOOK LIKE THIS



COMPARISON OF RISK AVERSION

Theorem (Pratt, 1964). Assume that agents U and V have the same initial wealth w and suppose that their utility functions u and v are twice differentiable. Then the following statements are equivalent:

- (i) Agent U is more risk averse than agent V .
- (ii) There is a strictly increasing concave function φ such that $u = \varphi \circ v$.
- (iii) Agent U 's absolute risk aversion is larger than agent V 's absolute risk aversion, i.e., $\rho_A(u; w) \geq \rho_A(v; w)$ for all w .
- (iv) The risk premium that agent U is willing to pay exceeds the risk premium that agent V is willing to pay, i.e., $\pi(u; w) \geq \pi(v; w)$ for all w .

Proof: See notes on “Risk and Uncertainty” posted on the course website.

WHEN IS ONE LOTTERY PREFERRED TO ANOTHER FOR A CLASS OF UTILITY FUNCTIONS?

$$Eu(\tilde{x}) \leq Eu(\tilde{y}) \quad \forall u \in \mathcal{U}. \quad (**)$$

Definition. If $(**)$ holds, the risk $\tilde{y}$ is said to stochastically dominate $\tilde{x}$ with respect to $\mathcal{U}$, denoted by $\tilde{x} \preceq_{\mathcal{U}} \tilde{y}$. Necessary and sufficient conditions on $\tilde{x}$ and $\tilde{y}$ for $(**)$ to hold are called a stochastic dominance order (representation) relative to $\mathcal{U}$.

$\uparrow$
 Class of Utility Functions (= Class of Agents)

Answer: Construct a stochastic dominance order

- **First-order stochastic dominance:** all agents with **increasing utility**
- **Second-order stochastic dominance:** all agents with **increasing concave utility**

STOCHASTIC DOMINANCE: DEFINITIONS

Let $X = [p_1, x_1; p_2, x_2; \dots; p_n, x_n]$ and $Y = [q_1, y_1; q_2, y_2; \dots; q_m, y_m]$ be two given **discrete random variables**, each with a finite number of realizations.

Without any loss of generality we can assume that $m = n$ and that $x_i = y_i$ for all i in $\{1, \dots, n\}$, and that $x_1 < x_2 < \dots < x_n$. This situation can always be achieved by extending the discrete variables X and Y to all events in the union of $\{x_1, \dots, x_n\}$ and $\{y_1, \dots, y_m\}$ assigning zero probabilities if necessary and subsequent relabeling.

Definition. Let $m = n$ and $x_i = y_i$ for all i in $\{1, \dots, n\}$. **Y first-order stochastically dominates X if**

$$\sum_{i=1}^k q_i \leq \sum_{i=1}^k p_i$$

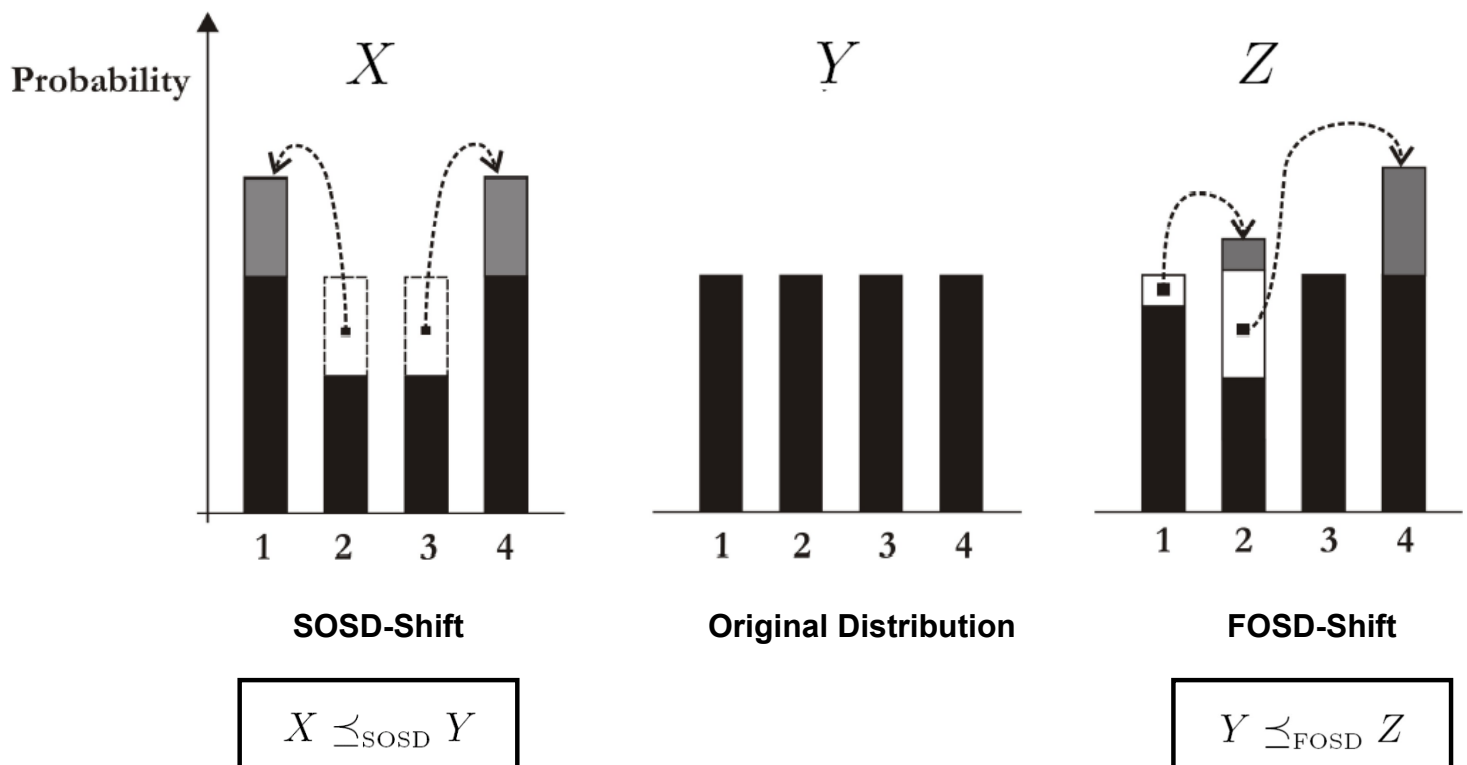
for all k in $\{1, \dots, n\}$.

Definition. Let $m = n$ and $x_i = y_i$ for all i in $\{1, \dots, n\}$. **Y second-order stochastically dominates X if**

$$\sum_{j=1}^k \sum_{i=1}^j q_i \leq \sum_{j=1}^k \sum_{i=1}^j p_i$$

for all k in $\{1, \dots, n\}$.

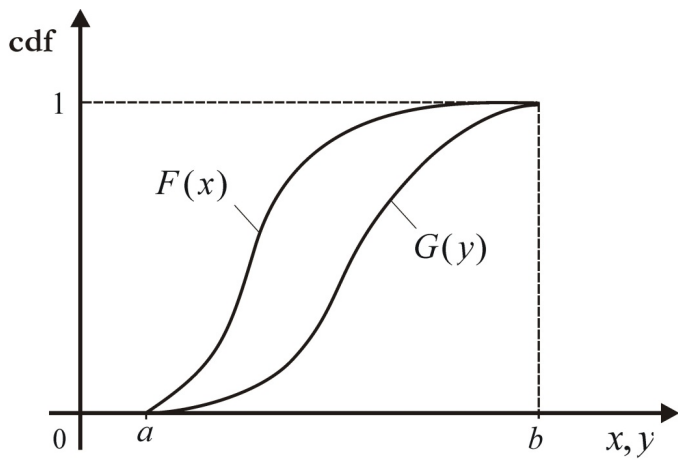
FIRST- AND SECOND-ORDER STOCHASTIC DOMINANCE



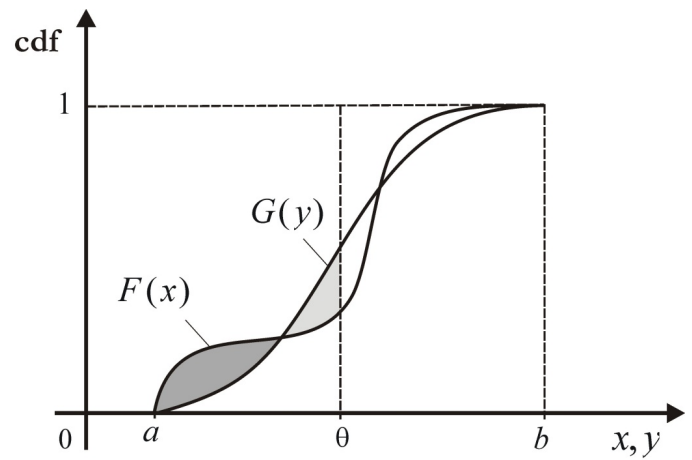
FIRST- AND SECOND-ORDER STOCHASTIC DOMINANCE

General Case

Let F, G be two cumulative distribution functions (measures) for random variables X and Y , respectively, distributed on the set $[a, b]$. When does Y FOSD/SOSD-dominate X ?



FOSD



SOSD

$$X \preceq_{\text{FOSD}} Y \Leftrightarrow G(z) \leq F(z) \quad \forall z \in [a, b]$$

$$X \preceq_{\text{SOSD}} Y \Leftrightarrow \int_a^z G(y) dy \leq \int_a^z F(x) dx \quad \forall z \in [a, b]$$

AGENDA

Elements of Probability

Choice Under Uncertainty

Expected Utility Theory

Risk Aversion and Decision Biases

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Risk and Uncertainty**
- **Objective and Subjective Probability**
- **Discrete Random Variable**
- **Lottery**
- **Von Neumann-Morgenstern Axioms → Expected Utility Representation**
- **Allais Paradox**
- **Expected Utility Maximization**
- **Risk Aversion (Absolute & Relative)**
- **Sensitivity to Reference Point (Reflection Effect)**
- **Stochastic Dominance (First-Order & Second-Order)**

MGT 621 – MICROECONOMICS

Thomas A. Weber

4. Theory of the Firm

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

Introduction

Production Sets

Profit Maximization: Some Intuition

The Firm's Cost Function

Profit Maximization

Key Concepts to Remember

THEORY OF THE FIRM

Production Side of the Economy

Including ...

- **Corporations** (... General Motors, Microsoft, Virgin Atlantic)
- **Public utilities** (... Pacific Gas and Electric Co., Metropolitan Water District)
- **Partnerships** (... law firm, McKinsey, start-up firm)
- **Small businesses** (... retail store, individual consultant, restaurant, internet radio station)
- **Home production** (... home improvement, prepare meals)
- **Educational institutions** (... UC Berkeley, Stanford U)
- **Non-profit organizations** (... community hospital, YMCA)

THE FIRM AS AN OPTIMIZER

Possible Objectives

- **Maximize (expected) profit**
- **Maximize (expected) utility (which takes profits as one argument)**
- **Minimize cost (e.g., given fixed outputs)**

Feasible Actions

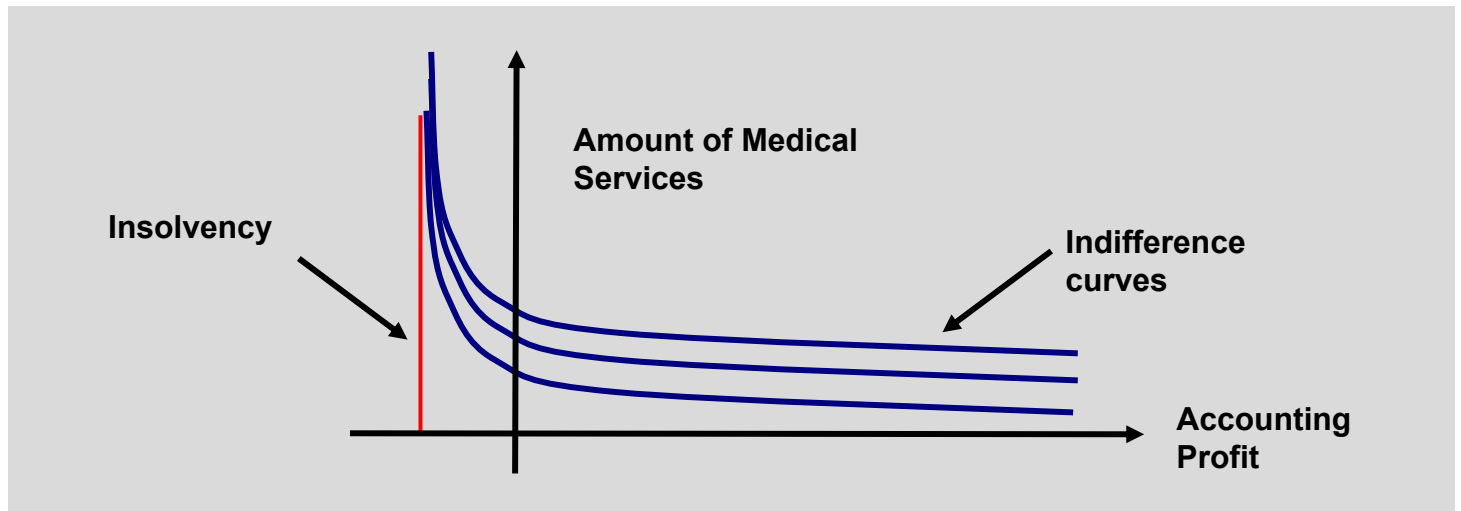
- **Production possibilities**
- **Legal constraints**
- **Competitive necessities**

Firm chooses a most preferred action from the set of all feasible actions.

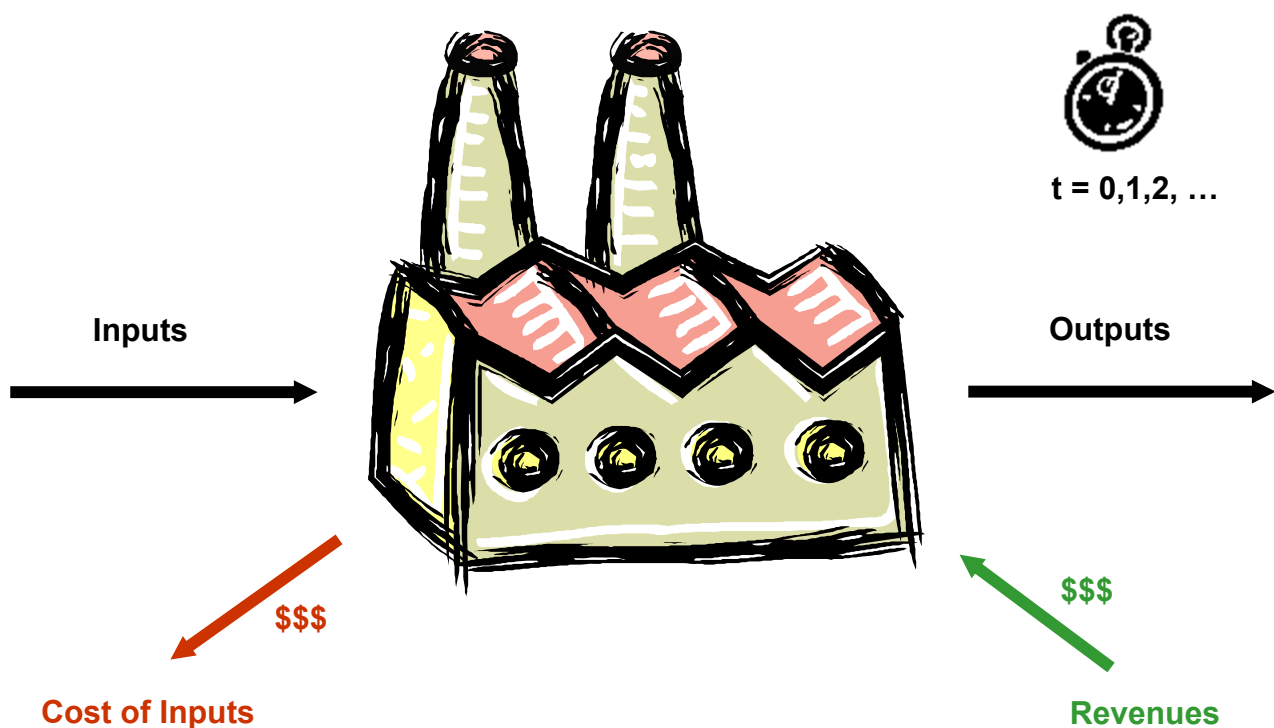
A FIRM'S POSSIBLE OBJECTIVES: EXAMPLE

Maximization of (Expected) "Utility" (Objective Function)

- Could include accounting profits as only one variable among several others (such as a measure of output)
 - Example: a community hospital may put a high value on accounting profits if it is losing money (negative profit), and a high value on the provision of medical services otherwise

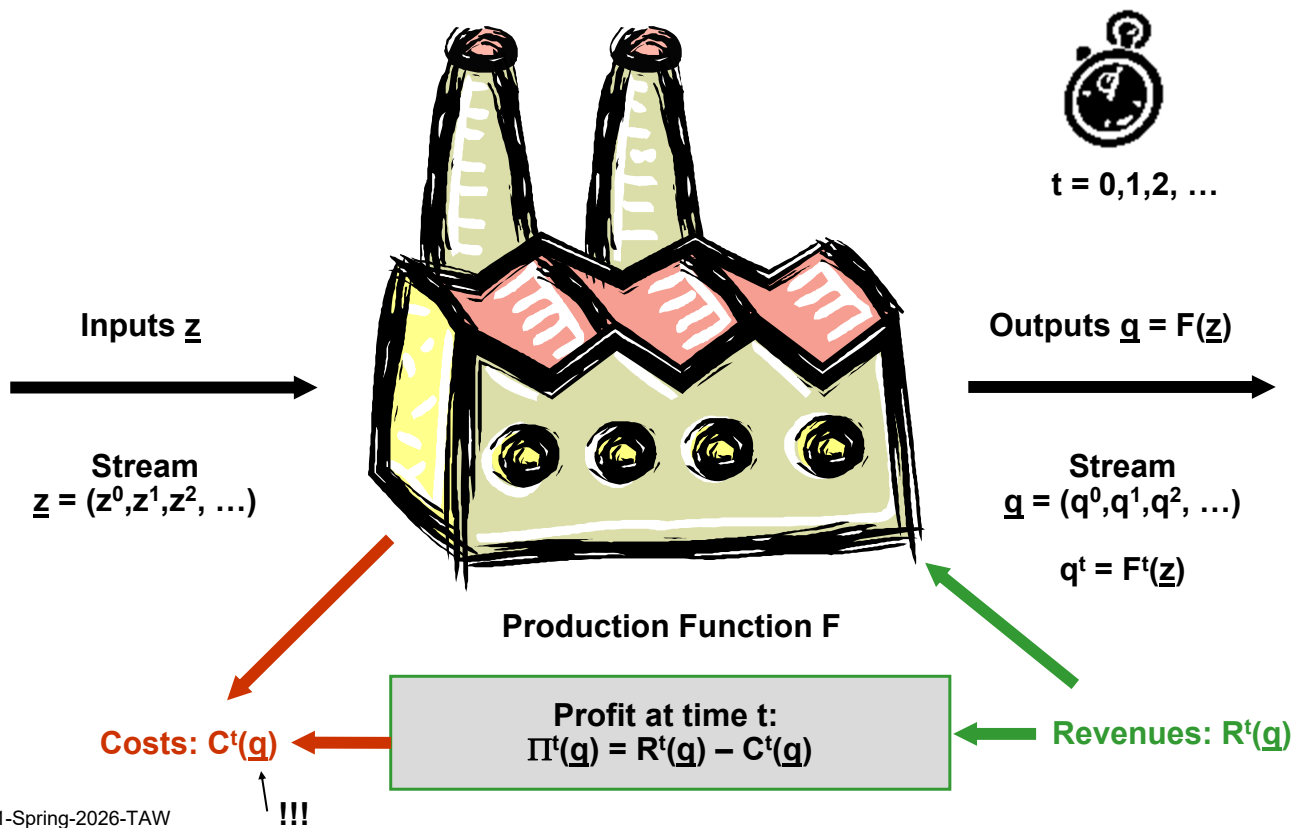


THE FIRM AS A BLACK BOX Compare Inflows and Outflows



THE FIRM AS A BLACK BOX (Cont'd)

Converts Inputs to Outputs



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PRODUCTION FUNCTION

The production function at time t might depend on current and/or (anticipated) future inputs,

$$q^t = F^t(\underline{z}) = F^t(z^0, z^1, z^2, \dots)$$

Question: Why?

Answer: There are many real-world phenomena which may produce dependencies such as

- Learning-Curve Effects⁽¹⁾
- Demand Effects (e.g., Saturation)

In some practical applications current output depends only on current input, i.e.,

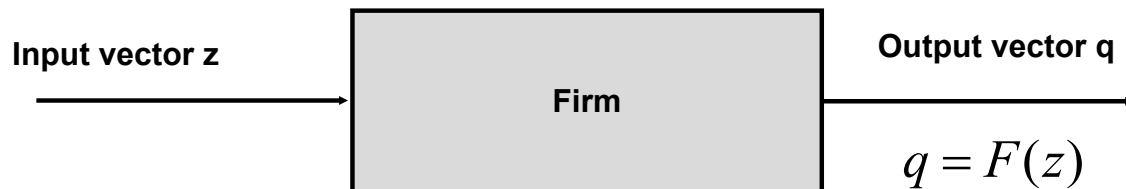
$$q^t = F^t(\underline{z}) = F^t(z^t)$$

(1) The commander of Wright-Patterson Air Force Base in Ohio observed in 1925 that the required direct labor hours for the assembly of a plane decreased in the number of planes built. First academic observations include Hirschman, W.B. (1964) "Profit from the Learning Curve," Harvard Business Review, Jan/Feb, and Arrow, K.J. (1962) "Economic Welfare and the Allocation of Resources to Invention," in: Nelson, R. (ed.) *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton University Press, Princeton, NJ.

A FIRM CONVERTS INPUTS TO OUTPUTS

Simplification: Static Conversion → Omit Time Index

Production function F summarizes conversion



Sometimes it is difficult to decide what exactly is input and what is output, so that it is useful to consider the “general” output (or “production possibility”)

$$y = (-z, q)$$

(Net) inputs have negative components and (net) outputs are represented by nonnegative components of y .

GENERAL (STATIC) MODEL OF THE FIRM

Firm chooses vector $z \geq 0$ of inputs to produce vector $q \geq 0$ of outputs

=

Firm chooses production possibility vector $y = (-z, q)$.

Feasible Set

- Production Set (or Production Possibility Set) Y

Objective Function

- For simplicity, first assume that profit maximization is the objective, given a price $p(y)$ for outputs (it turns out that cost minimization for inputs is a necessary condition)

→ Profit-Maximization Problem

- Choose y to maximize the product $\Pi(y) = p(y) y$ subject to y being feasible, i.e., solve

$$\max_{y \in Y} \Pi(y)$$

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Profit Maximization

Key Concepts to Remember

PRODUCTION SET Y

The firm's production set Y contains all input-output bundles y it might choose.

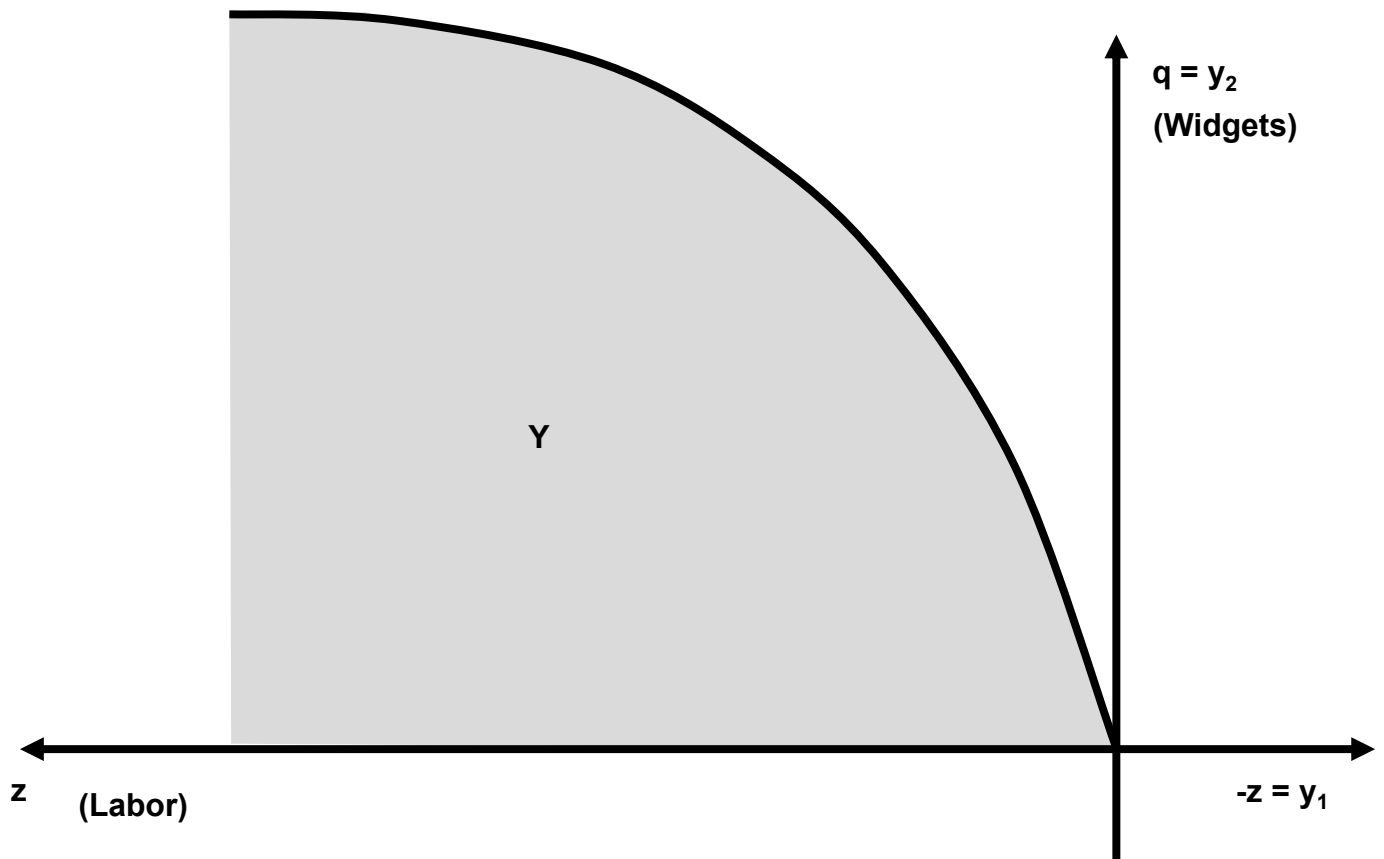
Possible inputs: labor, land, buildings, raw materials, capital equipment, services, intermediate goods (produced by other firms), etc.

Possible outputs: finished goods, services, knowledge, intermediate goods (to be used by other firms), etc.

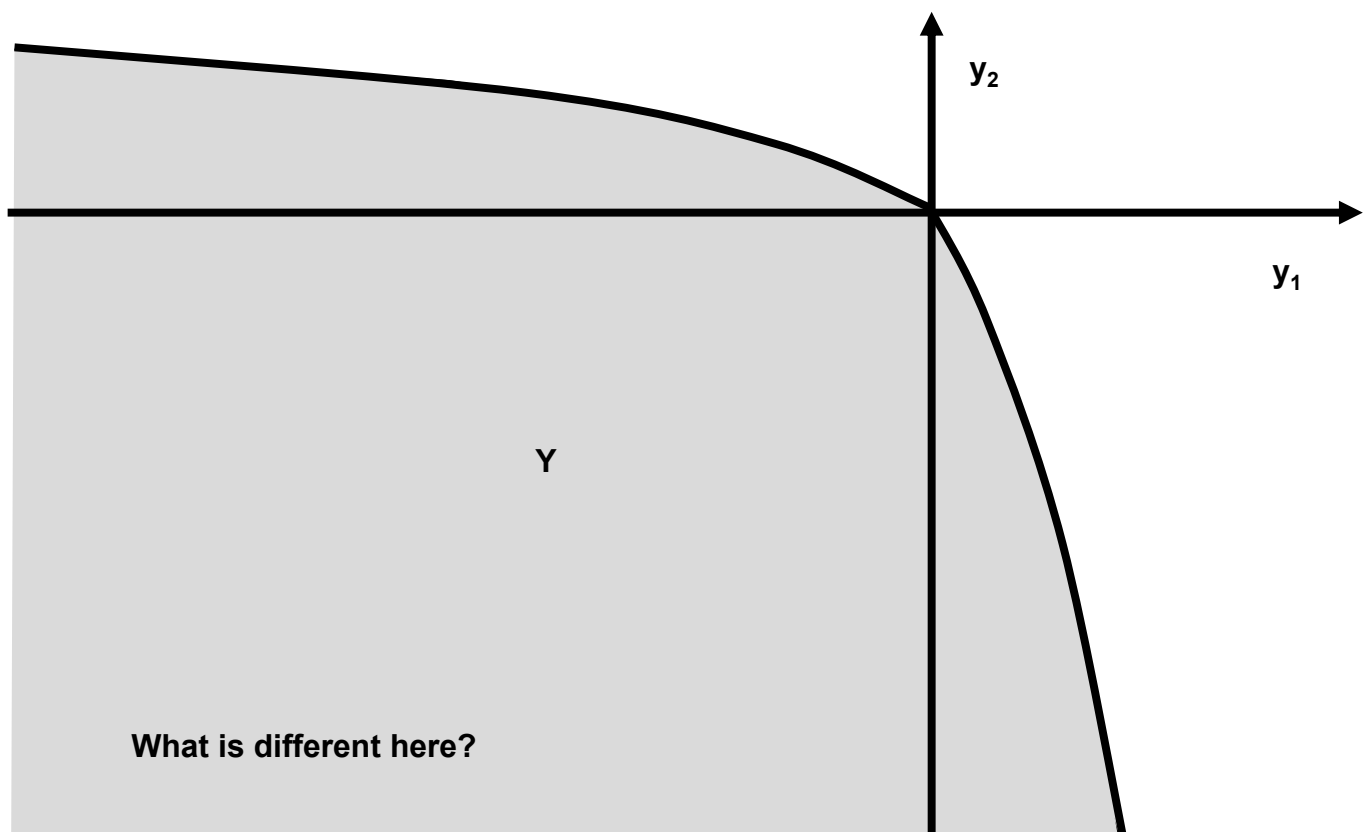
Shape of Y implied by

- technological possibilities
- legal constraints

PRODUCTION SET: EXAMPLE



PRODUCTION SET: ANOTHER EXAMPLE



PROPERTIES OF PRODUCTION SETS

We now discuss standard assumptions on a firm's production set $Y \subset \mathfrak{R}_+^L$.

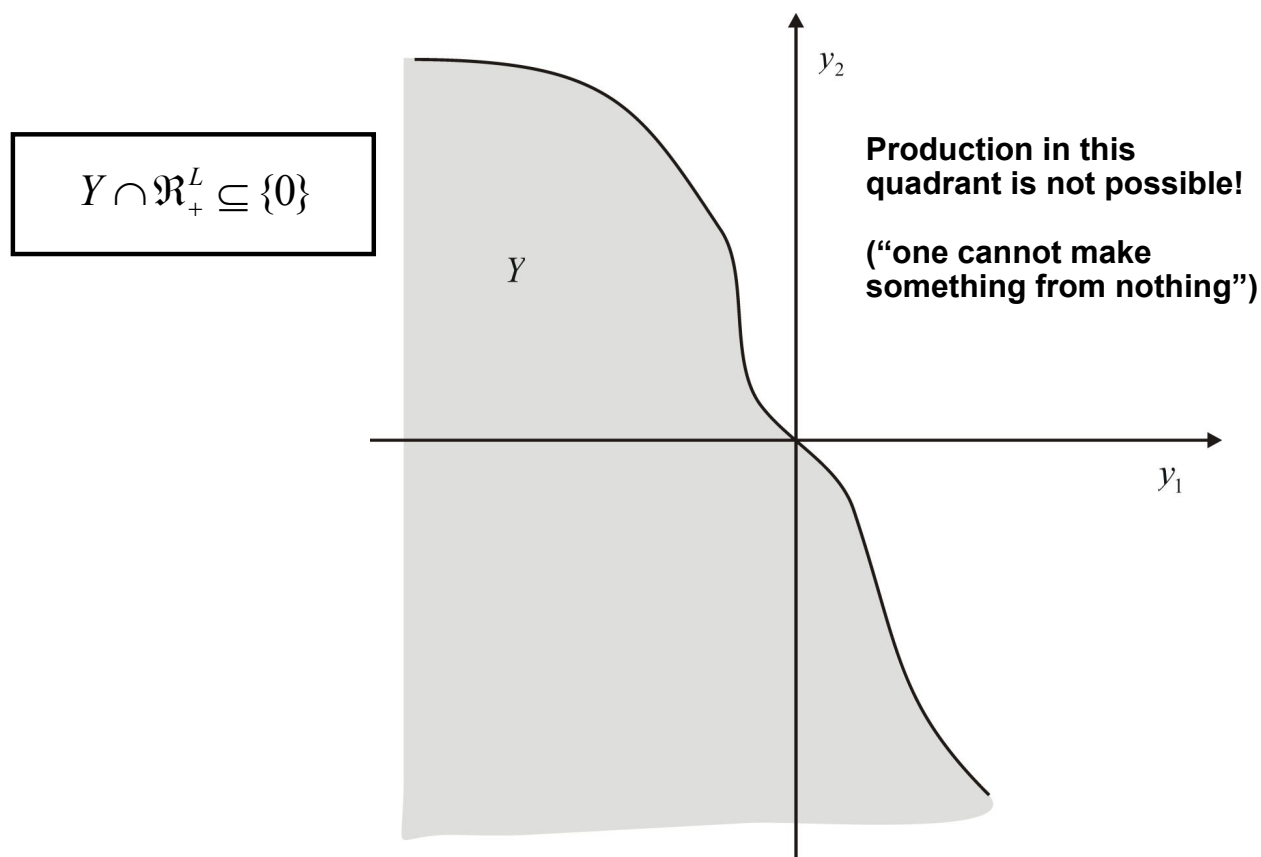
A1 (Non-Emptiness). The set Y contains at least one element.

A2 (Closedness). It is not possible to construct a sequence of elements of Y with limit outside Y .

A3 (No Free Lunch). $Y \cap \mathfrak{R}_+^L \subseteq \{0\}$: it is not possible to have a positive net output $y > 0$ (i.e., one cannot produce at least one positive net output and not use any positive net inputs).

A1 – A3 are fundamental requirements which we assume are always satisfied.

NO FREE LUNCH



PROPERTIES OF PRODUCTION SETS (Cont'd)

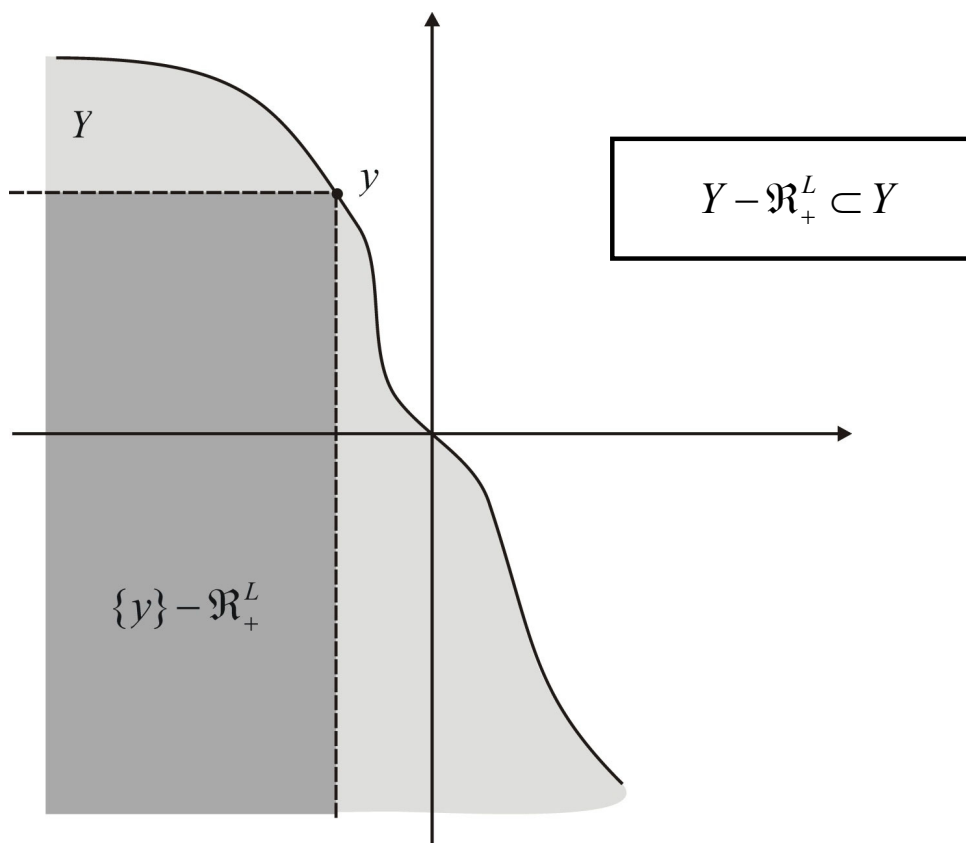
A4 (Possibility of Inaction). $0 \in Y$, i.e., it is possible to do nothing.

A5 (Free Disposal). $Y - \mathfrak{R}_+^L \subset Y$, i.e., it is always possible to use more net inputs for the same net output.

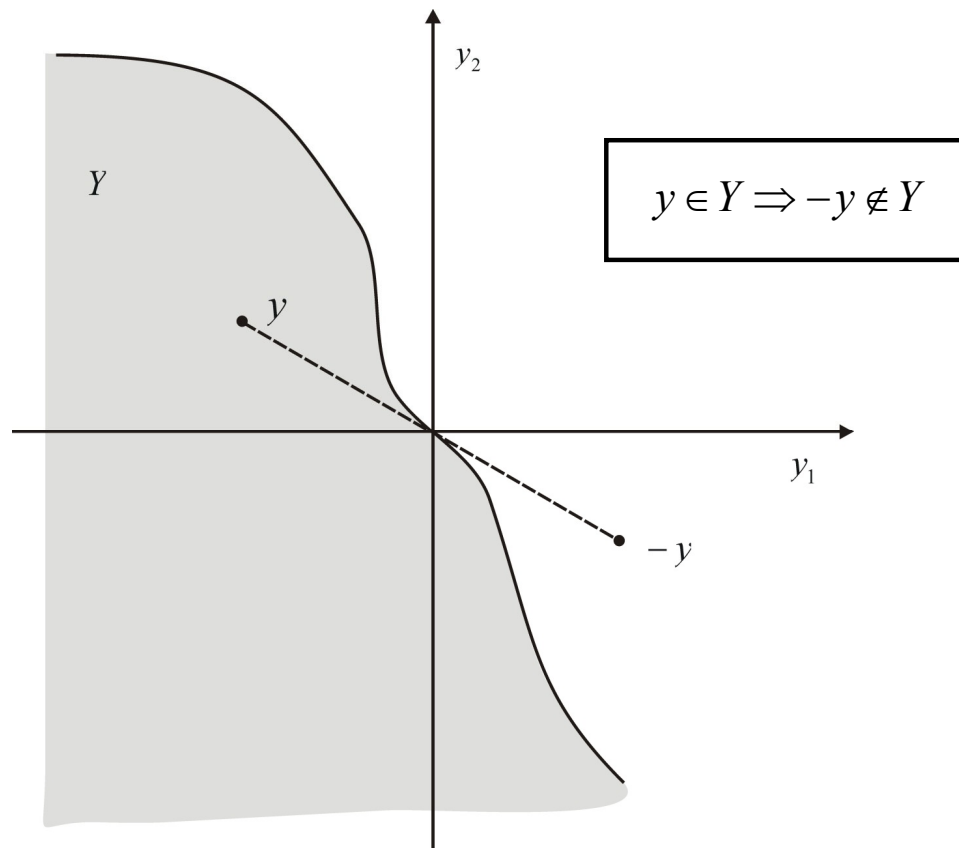
A6 (Irreversibility). $y \in Y \Rightarrow -y \notin Y$: it is not possible to obtain the inputs back once the outputs have been created.

A4 – A6 are basic properties of production sets that may sometimes be violated.

FREE DISPOSAL



IRREVERSIBILITY



PROPERTIES OF PRODUCTION SETS (Cont'd)

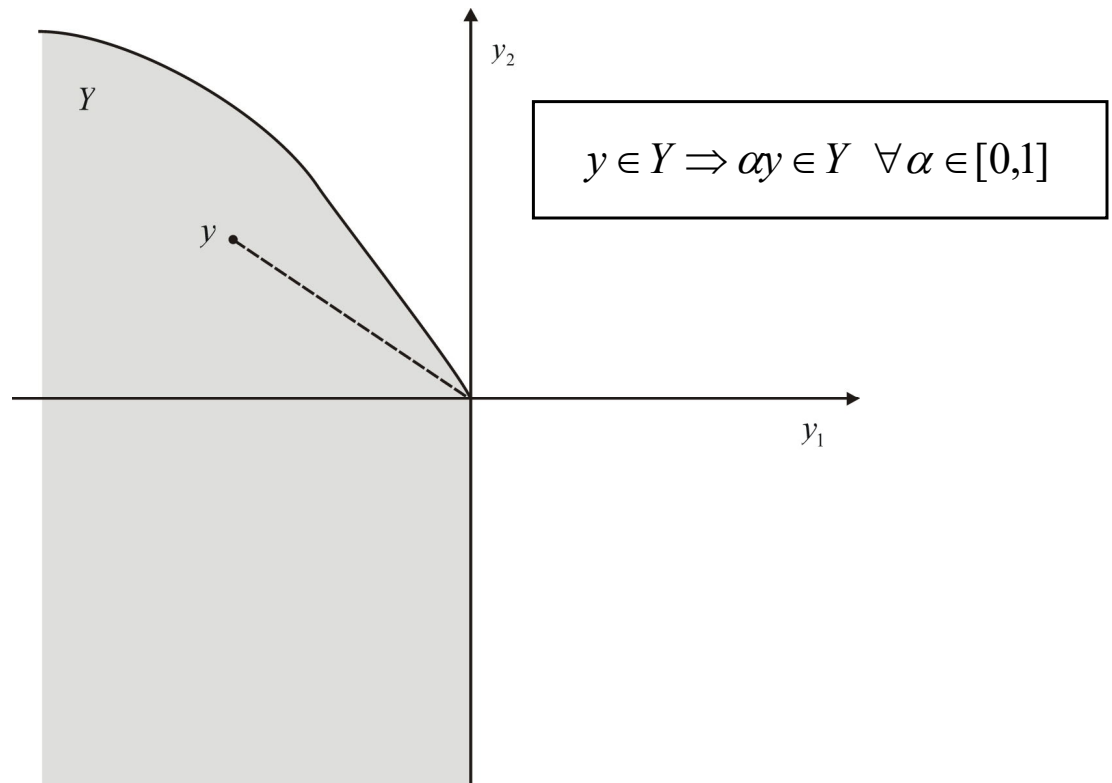
A7 (Nonincreasing Returns to Scale). $y \in Y \Rightarrow \alpha y \in Y \quad \forall \alpha \in [0,1]$, i.e., it is possible to scale down any feasible production vector. Mathematically, this means that Y is “star-shaped” with respect to the origin.

A8 (Nondecreasing Returns to Scale). $y \in Y \Rightarrow \alpha y \in Y \quad \forall \alpha \geq 1$, i.e., it is always possible to scale up any feasible production vector.

A9 (Constant Returns to Scale). $y \in Y \Rightarrow \alpha y \in Y \quad \forall \alpha \geq 0$, i.e., any feasible production vector is completely scalable. Mathematically, this means that Y is a cone.

A7 – A9 are useful properties of production sets that may or may not be satisfied.

NONINCREASING RETURNS TO SCALE

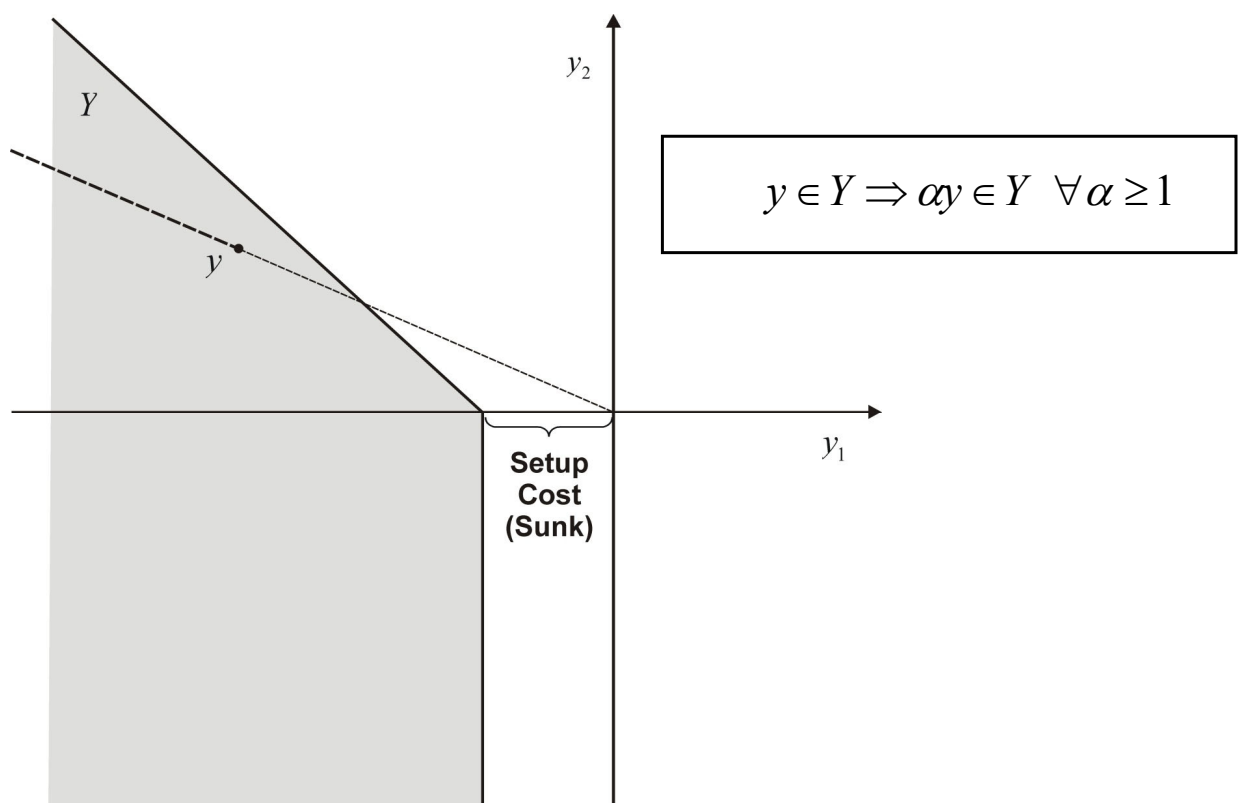


Remark: The set Y exhibits *decreasing* returns to scale if $y \in Y \Rightarrow \alpha y \in \text{int } Y \quad \forall \alpha \in (0,1)$

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NONDECREASING RETURNS TO SCALE

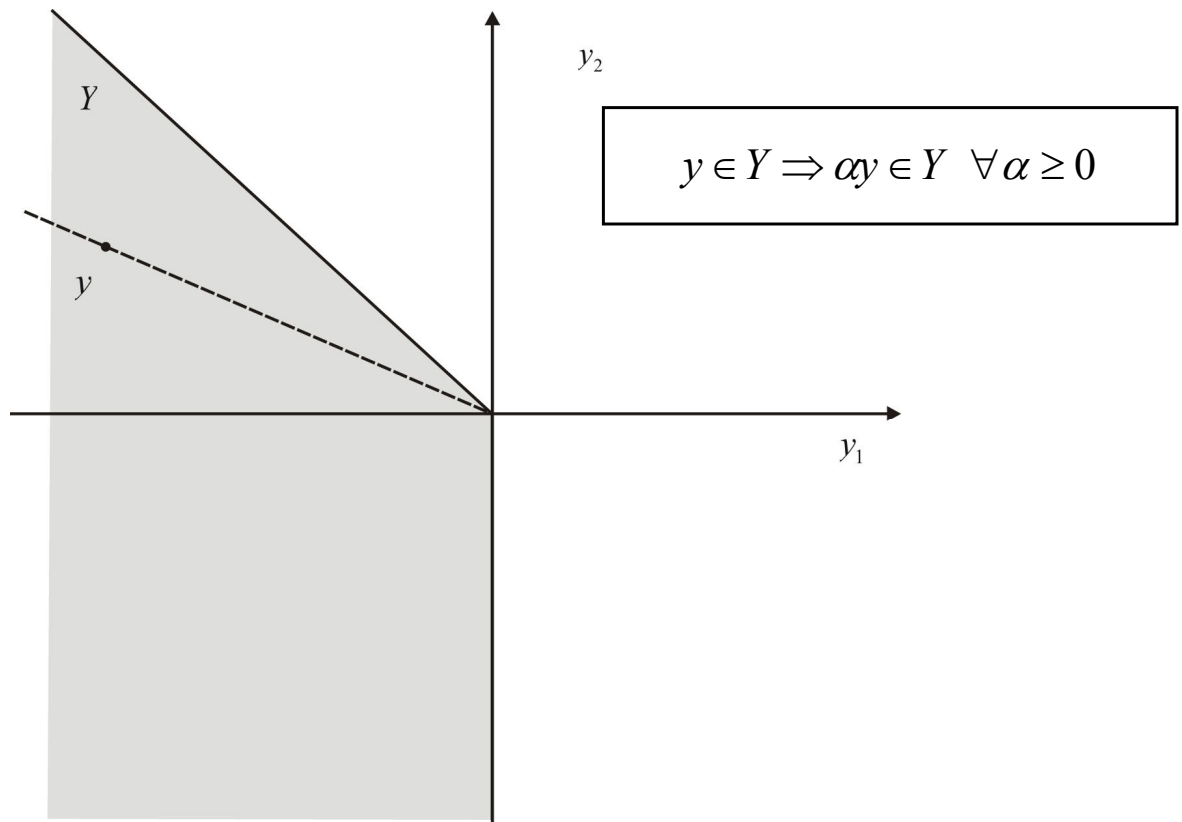


Remark: The set Y exhibits *increasing* returns to scale if $y \in Y \Rightarrow \alpha y \in \text{int } Y \quad \forall \alpha > 1$.

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CONSTANT RETURNS TO SCALE



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PROPERTIES OF PRODUCTION SETS (Cont'd)

A10 (Additivity/Free Entry). $y, \hat{y} \in Y \Rightarrow y + \hat{y} \in Y$, i.e., it is possible to combine any feasible production vectors. For an economy this means that the aggregate production possibilities are obtained by summing up the firms' individual production possibilities, provided each firm is free to contribute or not (= free entry).

A11 (Convexity). $y, \hat{y} \in Y \Rightarrow \alpha y + (1 - \alpha)\hat{y} \in Y \quad \forall \alpha \in (0, 1)$, i.e., any convex combination of feasible production vectors is feasible.

A12 (Convex Cone Property). $y, \hat{y} \in Y \Rightarrow \alpha y + \beta \hat{y} \in Y \quad \forall \alpha, \beta \geq 0$, i.e., any feasible production vectors can be combined and scaled. The convex cone property is equivalent to the combination A9 and A11.

A10 – A12 are structural properties of production sets that affect the optimization methods when looking for optimal production vectors.

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Profit Maximization

Key Concepts to Remember

GENERAL (STATIC) MODEL OF THE FIRM

Firm chooses vector $z \geq 0$ of inputs
to produce vector $q \geq 0$ of outputs

=

Firm chooses production possibility
vector $y = (-z, q)$.

Feasible Set

- Production Set (or Production Possibility Set) Y

Objective Function

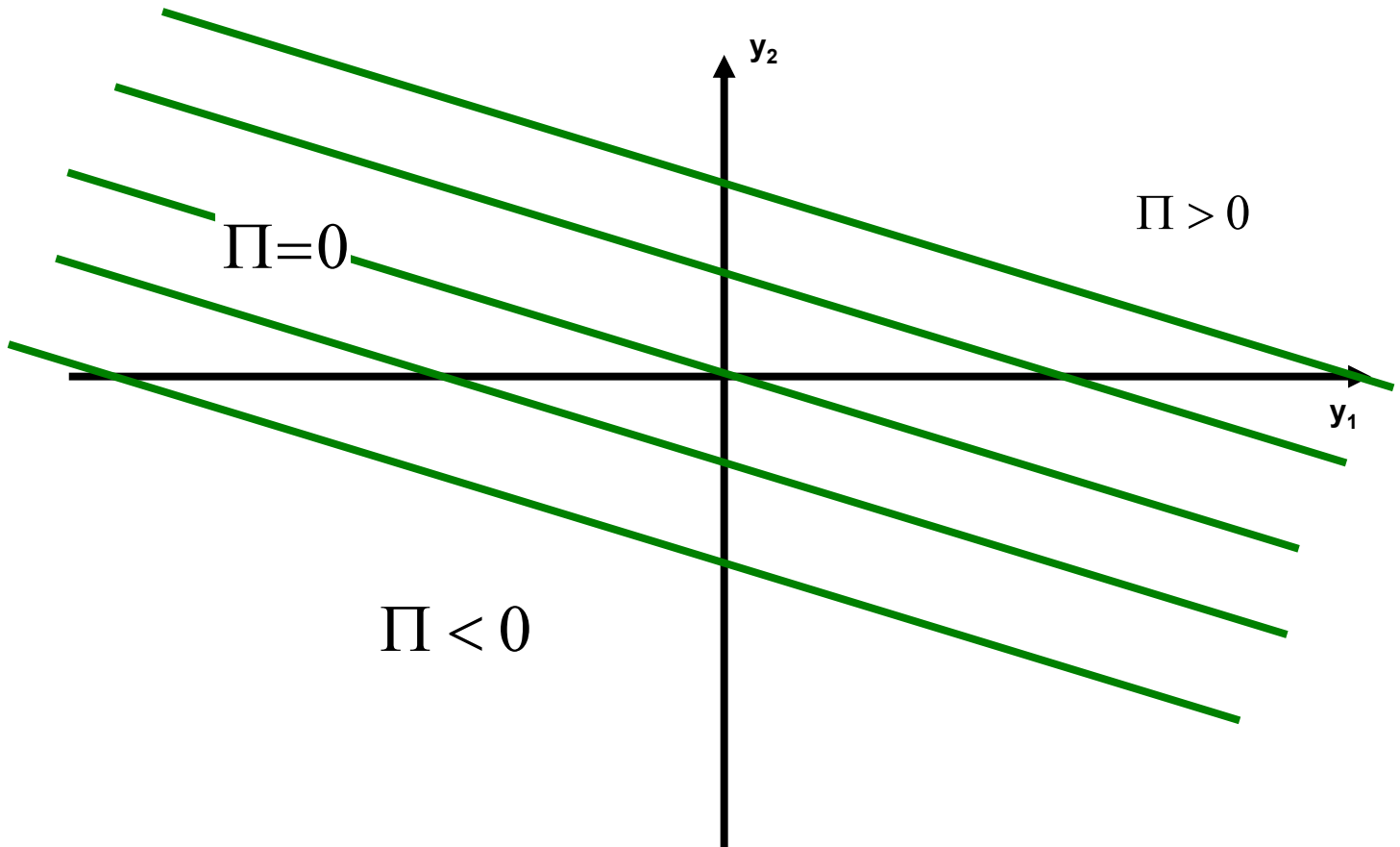
- For simplicity, first assume that profit maximization is the objective, given a price $p(y)$ for outputs
(it turns out that cost minimization for inputs is a necessary condition)

→ Profit-Maximization Problem

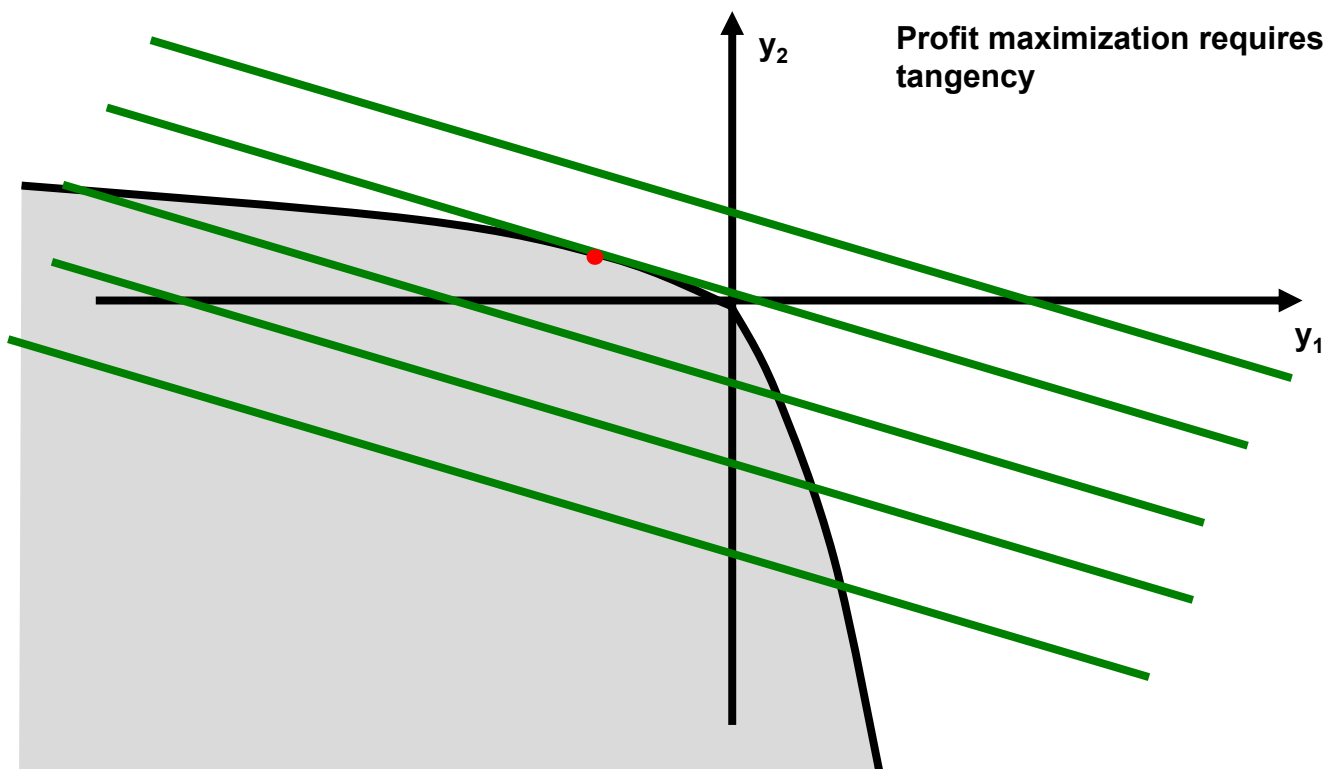
- Choose y to maximize the product $\Pi(y) = p(y)y$ subject to y being feasible, i.e., solve

$$\max_{y \in Y} \Pi(y)$$

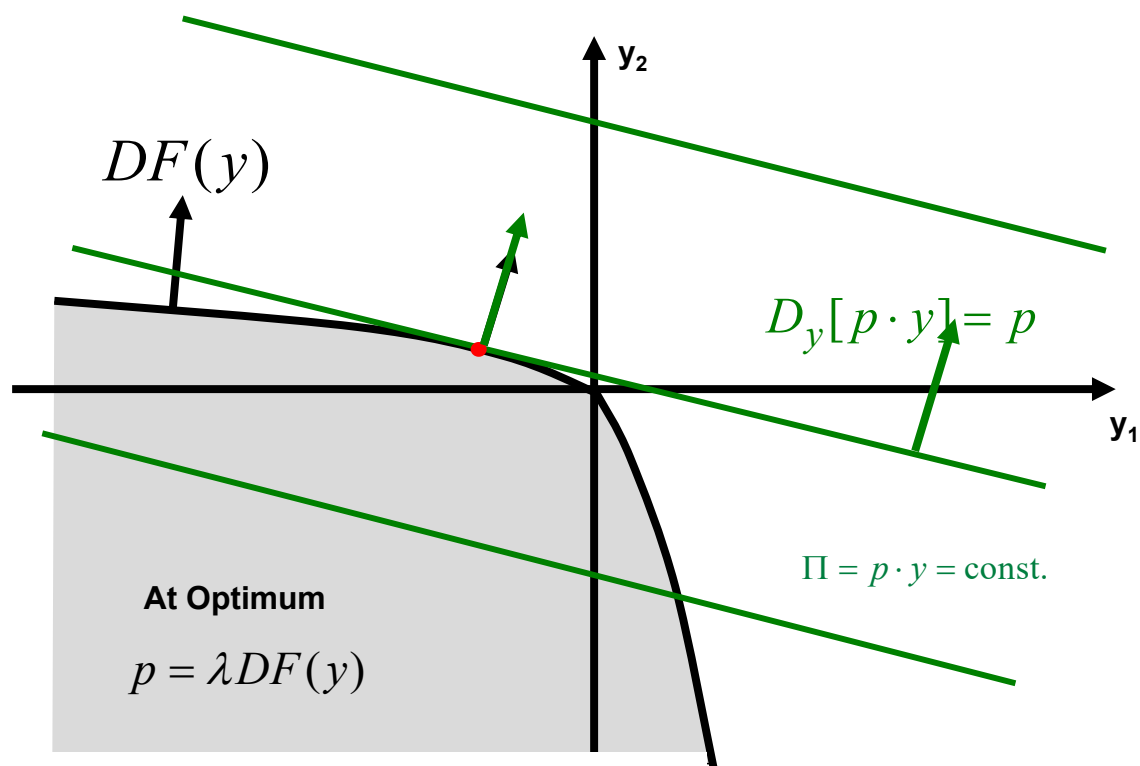
**ASSUMPTION THAT $p(y)=p$ (I.E., LINEAR PRICING) ...
... YIELDS CONSTANT ISO-PROFIT CURVES**



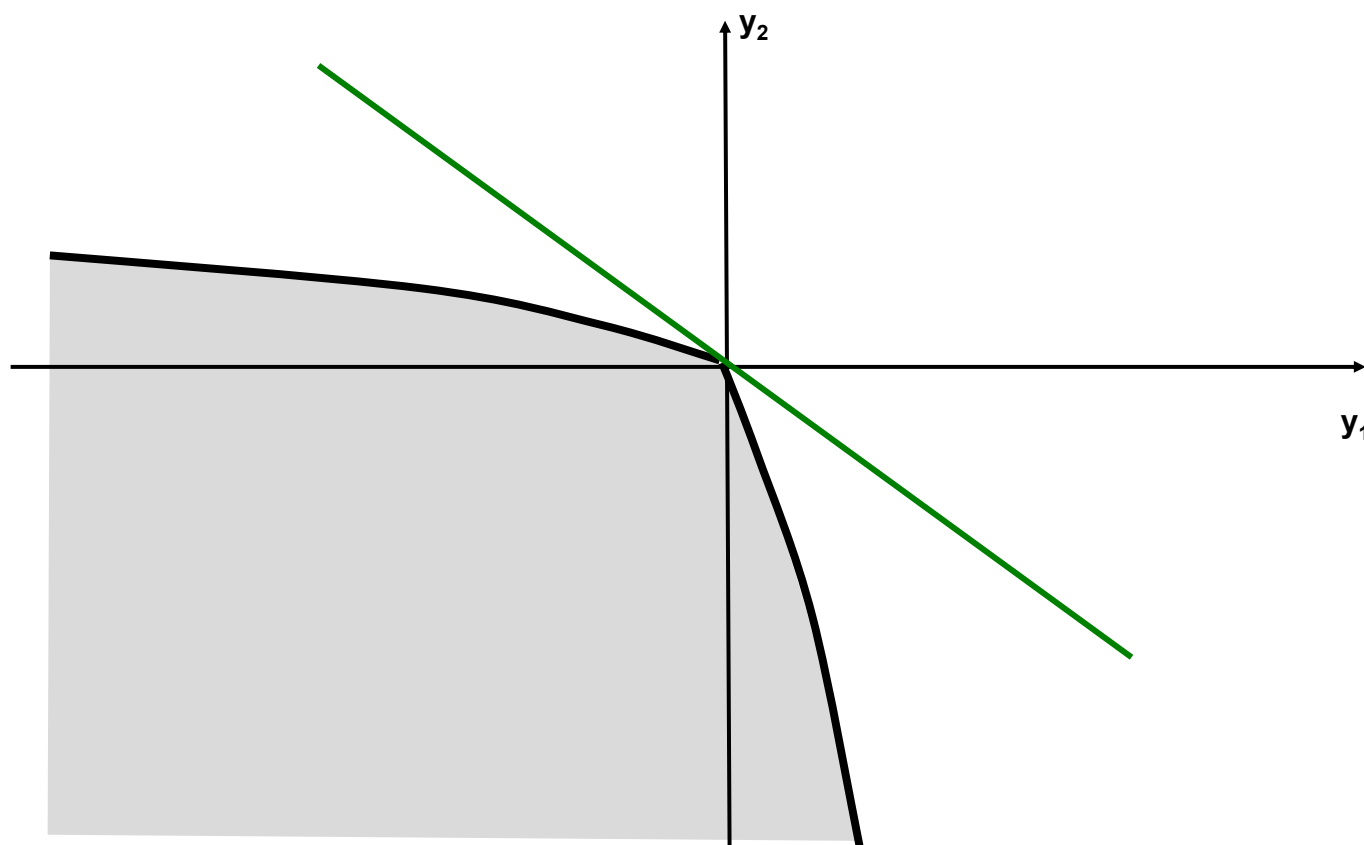
PROFIT MAXIMIZATION OVER PRODUCTION SET



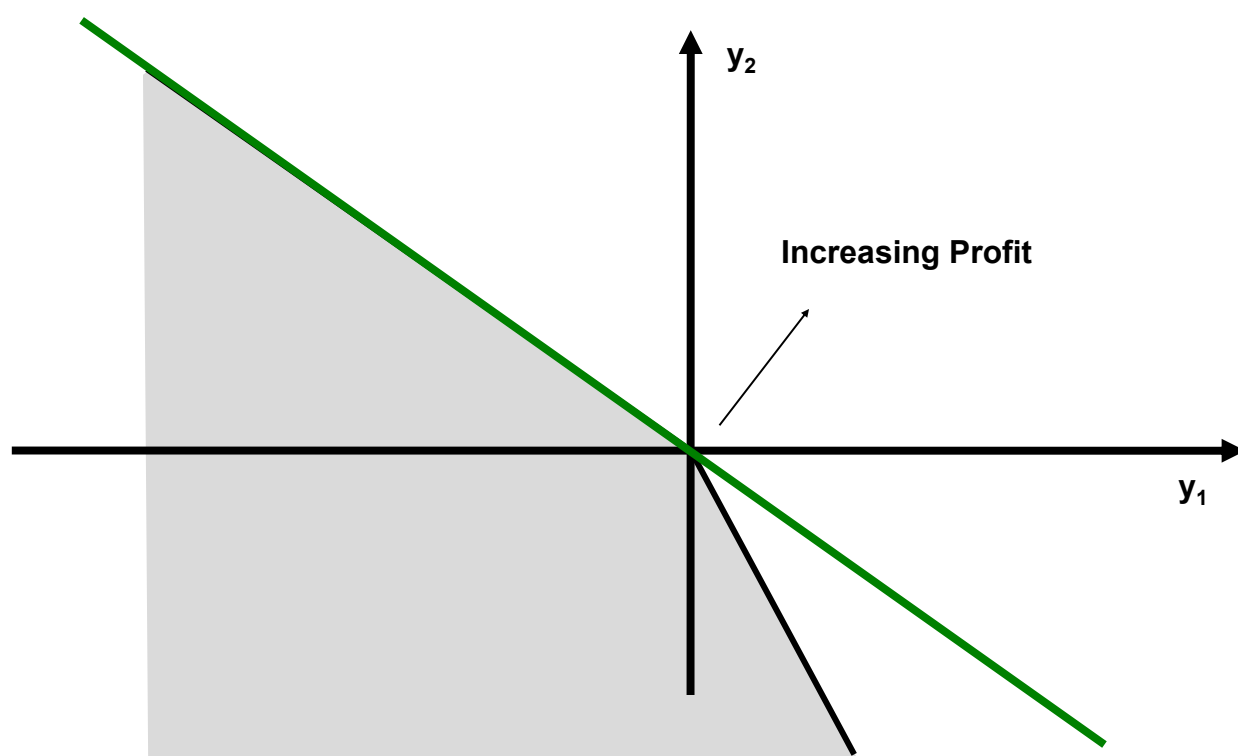
PROFIT MAXIMIZATION OVER PRODUCTION SET (Cont'd)



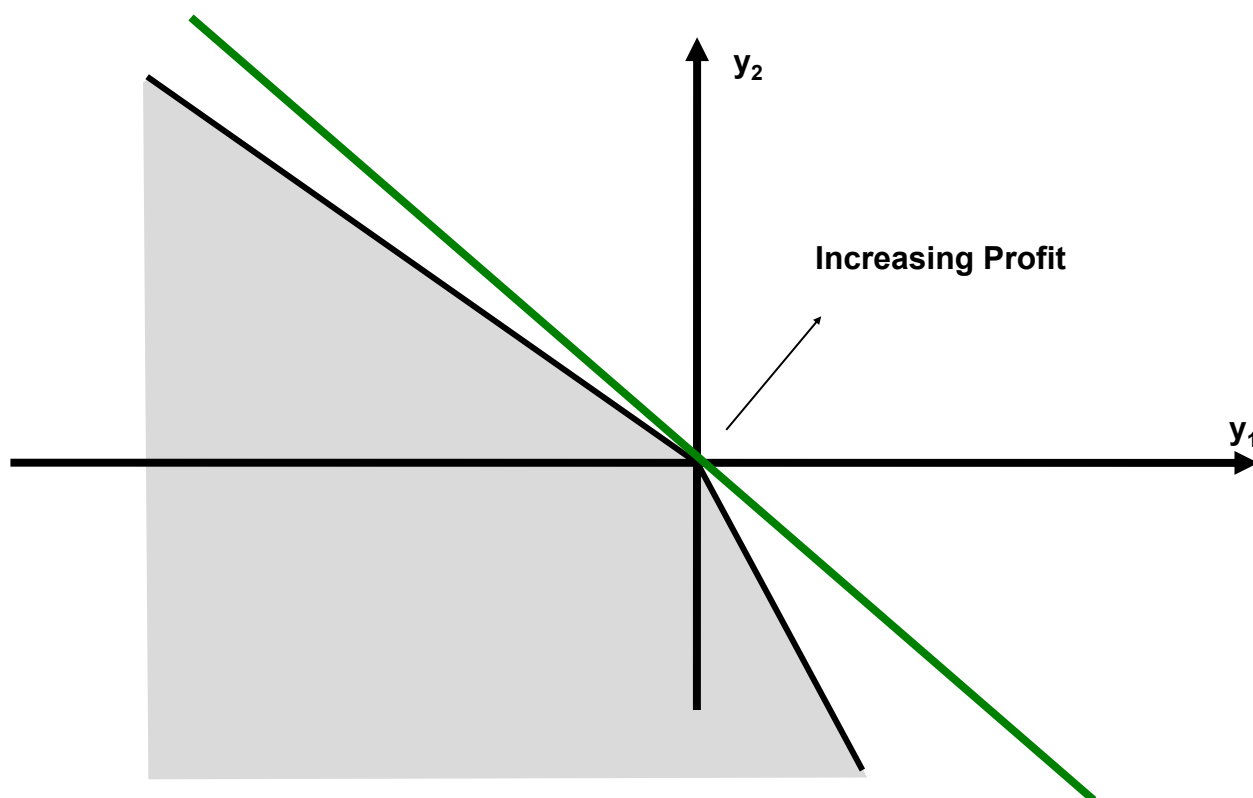
WHAT HAPPENS WHEN THERE ARE KINKS IN PRODUCTION SET?



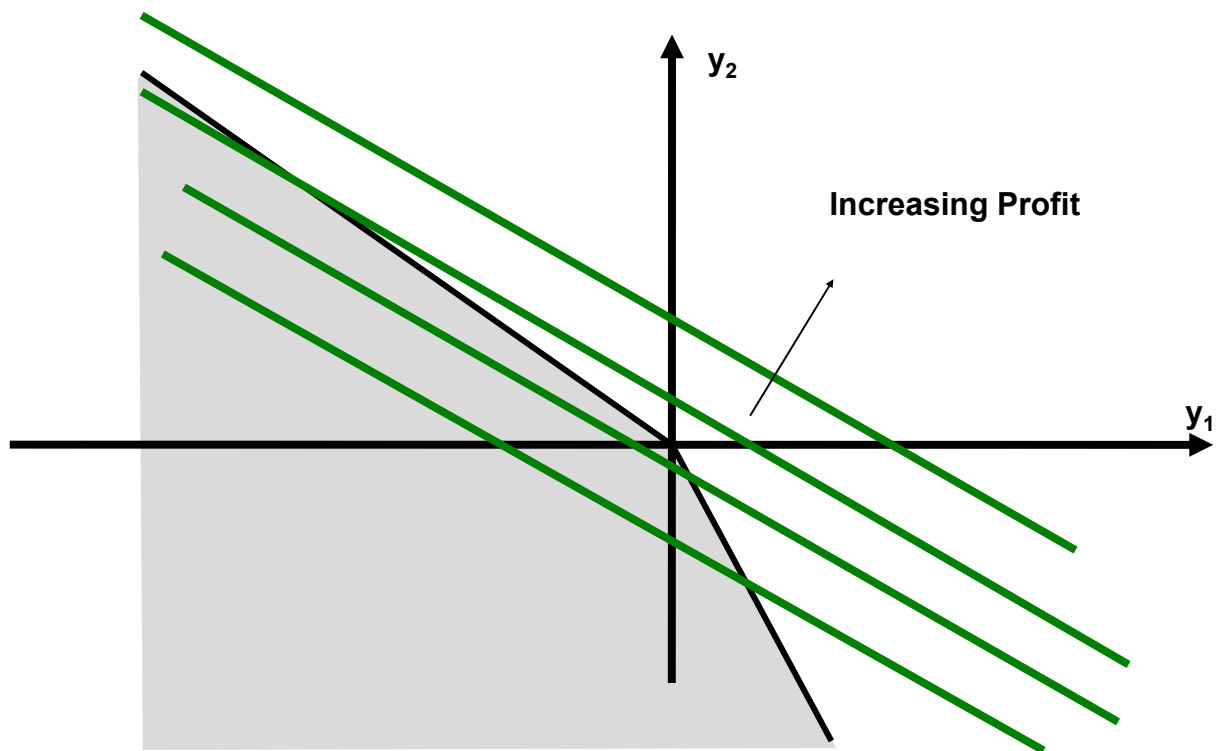
PROFIT-MAXIMIZING FIRM WITH CONSTANT RETURNS TO SCALE



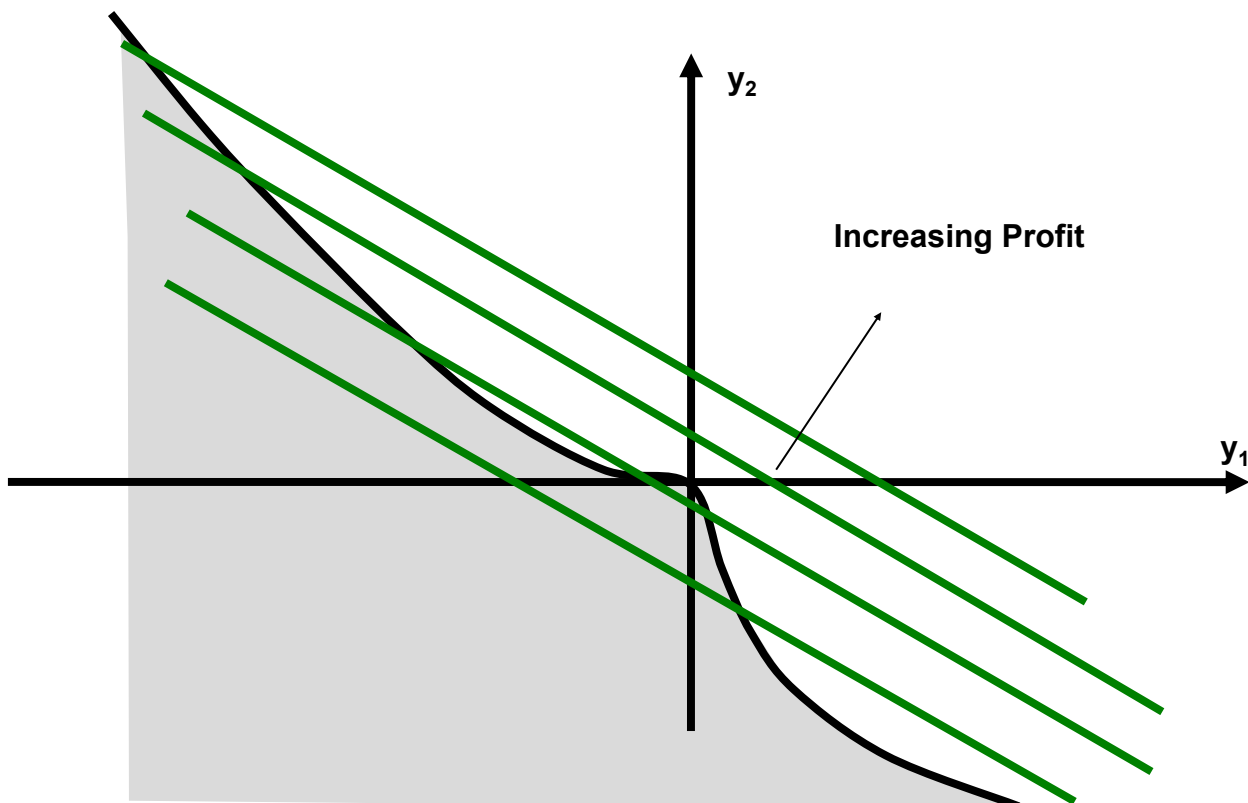
PROFIT MAXIMIZATION WITH CONSTANT RETURNS TO SCALE



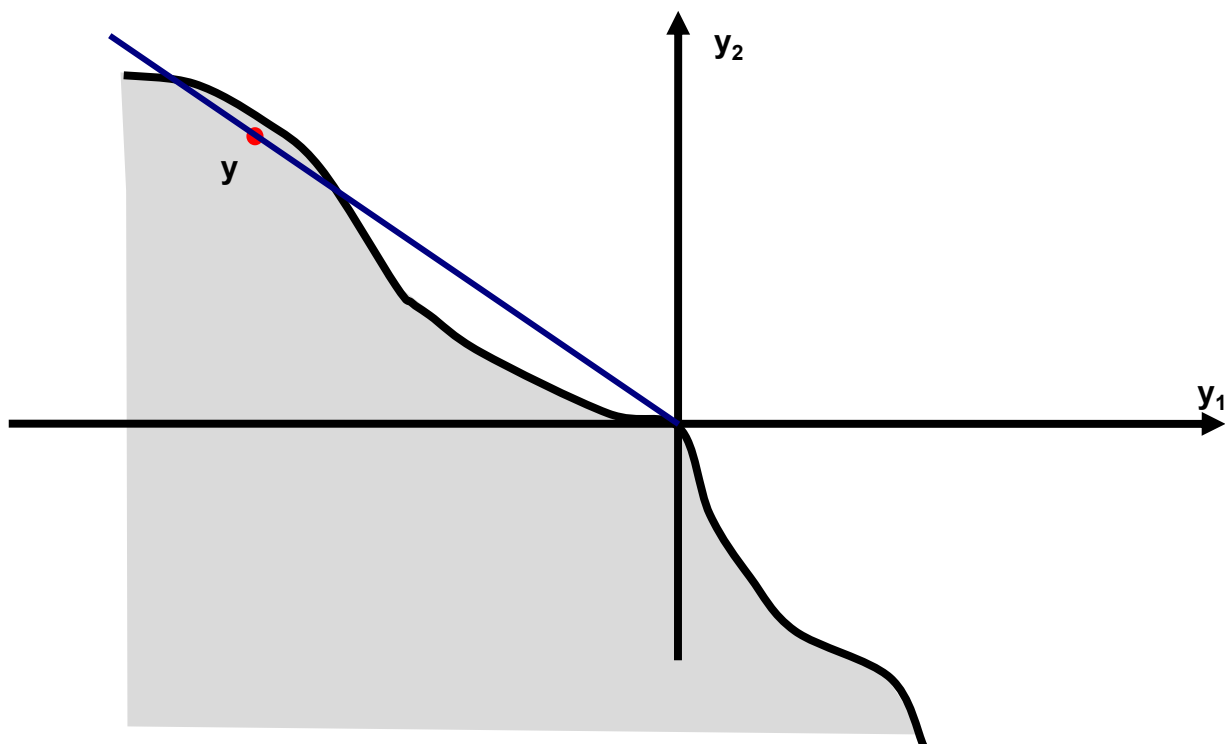
PROFIT MAXIMIZATION WITH CONSTANT RETURNS TO SCALE?



PROFIT-MAXIMIZATION WITH INCREASING RETURNS TO SCALE?



NEITHER INCREASING NOR DECREASING RETURNS TO SCALE



AGENDA

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Key Concepts to Remember

PRODUCTION FUNCTION

Assume that a firm can clearly distinguish between inputs and outputs, i.e., any y in the production set Y can be written in the form $y = (-z, q)$ where $z \geq 0$ is the vector of inputs and $q \geq 0$ is the vector of outputs.(1)

Then it is possible to represent Y in the form

$$Y = \{(-z, q) \in \mathbb{R}^L : q \leq F(z)\}$$

where $F(z)$ is referred to called the firm's **production function**.

(1) This holds when the firm's production is "irreversible," i.e., Y satisfies Assumption A6.

COST FUNCTION

Question. Given an increasing production function $F(z)$, determine the firm's cost function $C(q)$, i.e., the firm's minimum cost to produce a (feasible) output vector $q \geq 0$.

Answer: Given a feasible vector q of outputs, the firm solves the *expenditure minimization problem* (or '*cost minimization problem*' in this context)

$$\min_{y=(-z,q) \in Y} \{w(z) \cdot z\} = \min_{z:F(z) \geq q} \{w(z) \cdot z\}$$

where $w(z)$ is the vector of (positive) input prices. The firm's **cost function** $C(q)$ is its minimal expenditure,

$$C(q) = \min_{z:F(z) \geq q} \{w(z) \cdot z\} = \min \{w(z) \cdot z : F(z) \geq q\}$$

COST FUNCTION: EXAMPLE

Problem Set. Find the cost function $C(q)$ implied by the production set

$$Y = \left\{ (-z_1, -z_2, q) : (z_1, z_2, q) \in \mathbb{R}_+^3, z_1^\alpha z_2^\beta \geq q \right\}$$

where α and β are positive constants with $\alpha + \beta < 1$.

ECONOMIES/DISECONOMIES OF SCALE

A **cost function** $C(q)$ with a **scalar output** $q > 0$ exhibits **economies of scale** if the **average cost** decreases in $q > 0$, i.e.,

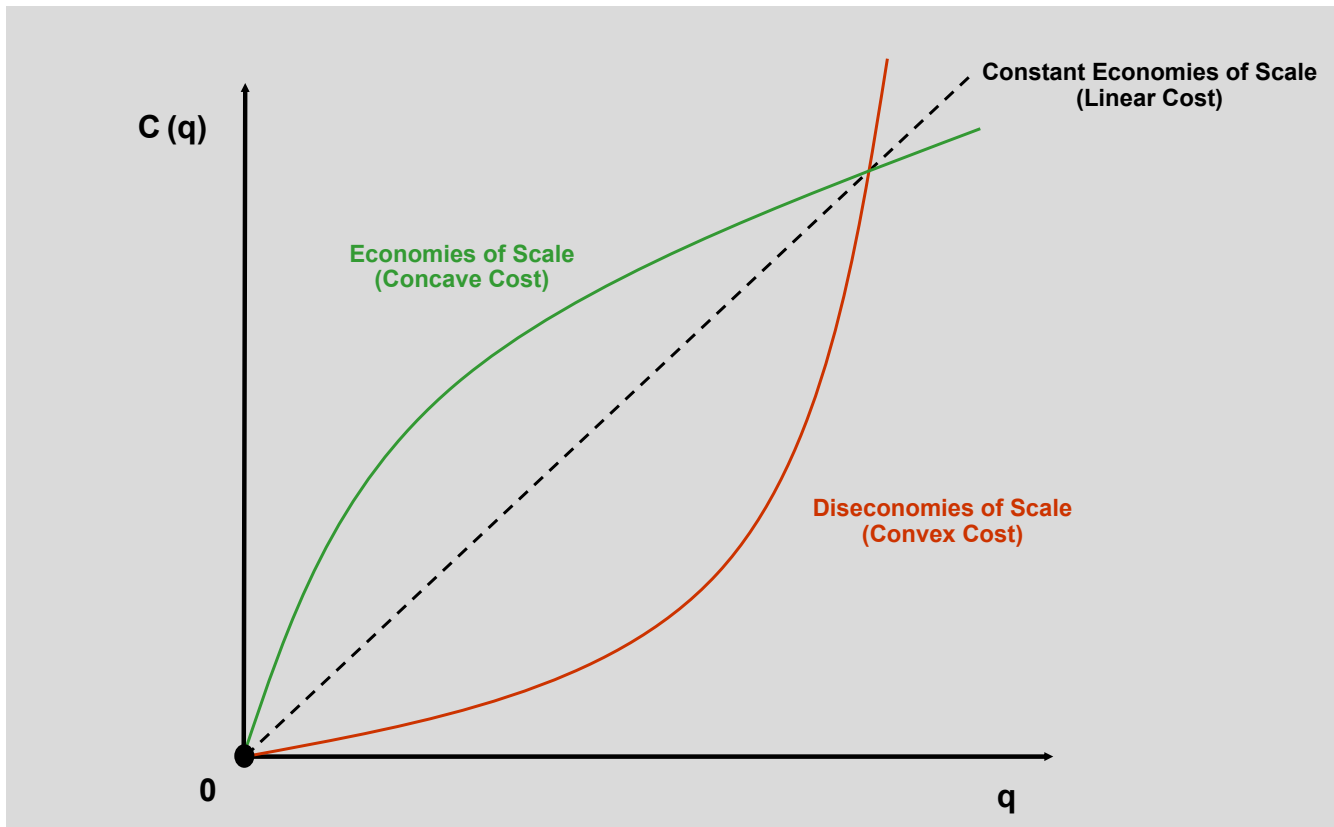
$$AC(q) = \frac{C(q)}{q} \quad \text{goes down, as } q \text{ goes up.}$$

If average costs increase in q , then $C(\cdot)$ exhibits **diseconomies of scale**; if average costs stay constant, then $C(\cdot)$ exhibits **constant economies of scale**.

[Remark. **Marginal cost** is the cost “at the margin,” corresponding to the slope of $C(q)$ at q , i.e., $MC(q) = C'(q)$.]

Similarly, a **production function** $F(z)$ with a scalar input z exhibits **economies of scale** (**diseconomies of scale/constant economies of scale**) if the conversion rate $F(z)/z$ increases (decreases/stays constant) in $z > 0$.

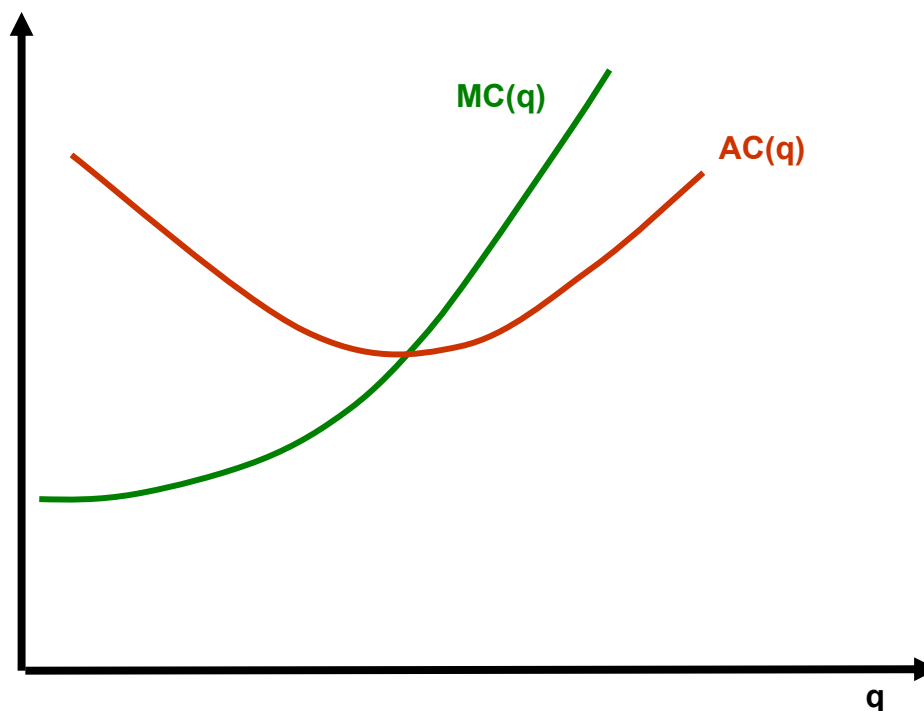
ECONOMIES/DISECONOMIES OF SCALE



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AVERAGE COST AND MARGINAL COST



$MC(q) = AC(q)$ at the minimizer of the average cost (= "minimum efficient scale")
 You should be able to prove this relation yourself

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PROFIT

Difference between value to firm of outputs and cost to the firm of inputs.

For firm that sells outputs:

- Profit $\Pi(q)$ is difference between total dollar revenues $R(q)$ received by the firm and total dollar costs $C(q)$ the firm incurs, as a function of its output q

$$\Pi(q) = R(q) - C(q)$$

Revenues include

- Money for selling the outputs, service fees, royalties, license fees, etc.

Costs include

- Expenditures for purchased goods, wages paid for labor, taxes
- Normal rate of return on invested capital
- Value of firm-owned resources allocated to alternative uses
- Value of time of the firm owner, allocated to firm activities

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Key Concepts to Remember

PROFIT MAXIMIZATION

$$\Pi(q) = R(q) - C(q)$$

First-order necessary optimality conditions for local **unconstrained** (or “interior”) maximum.

$$\frac{\partial \Pi}{\partial q_i} = \frac{\partial R(q)}{\partial q_i} - \frac{\partial C(q)}{\partial q_i} = 0 \Leftrightarrow \textcolor{red}{MR_i(q) = \frac{\partial R(q)}{\partial q_i} = \frac{\partial C(q)}{\partial q_i} = MC_i(q)}$$

Second-order necessary optimality condition for local **unconstrained** maximum:

$$\left[\frac{\partial^2 \Pi}{\partial q_i \partial q_i} \right]_{i,j=1}^n \leq 0$$

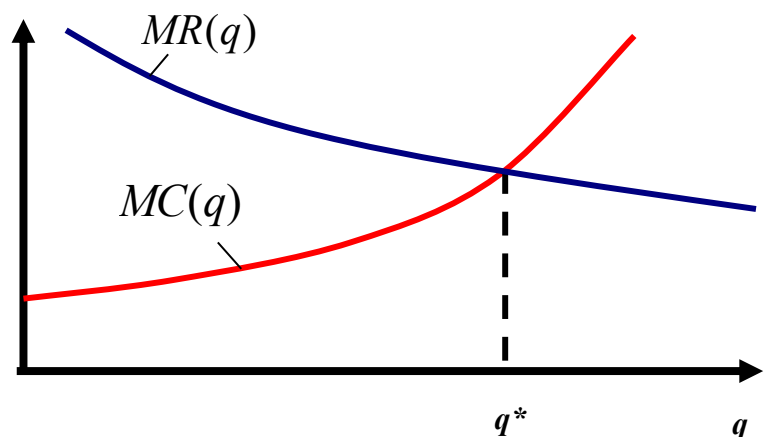
That is, the Hessian $D^2\Pi(q)$ must be a negative semi-definite matrix at the local maximizer.

PROFIT MAXIMIZATION (Cont'd)

Necessary optimality conditions in the one-dimensional case

$$\frac{\partial R(q)}{\partial q} = \frac{\partial C(q)}{\partial q}$$

$$R''(q) - C''(q) \leq 0$$



PROFIT MAXIMIZATION (Cont'd)

Necessary optimality conditions for profit-maximization (one-dimensional case)

$$MR(q) = MC(q)$$

$$R''(q) - C''(q) \leq 0$$

Different Perspectives

- **External Analyst**

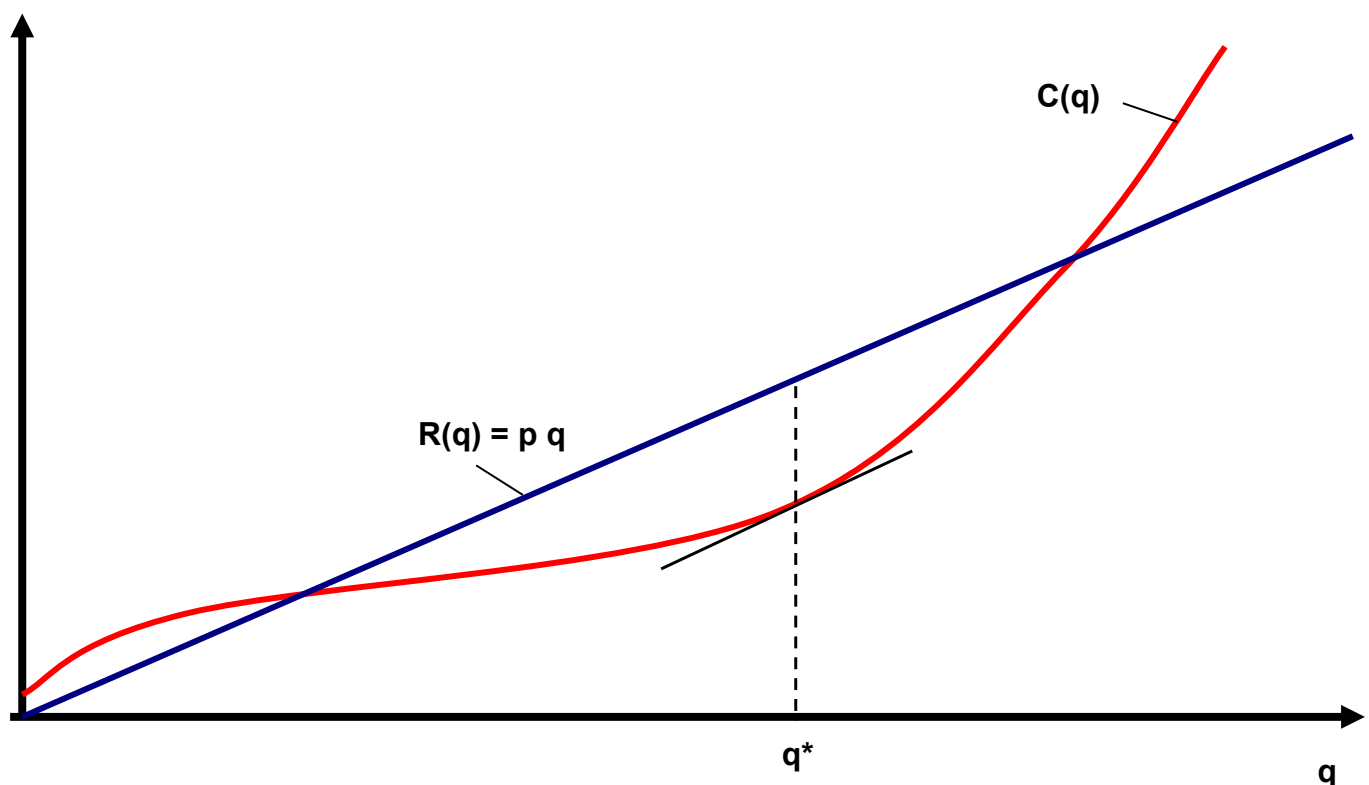
- Assumes marginal revenue = marginal cost
- Uses to predict activities of firms

- **Internal Analyst**

- Tries to see if the marginal revenue is equal to marginal cost, and adjusts levels of activities to bring two into equality.

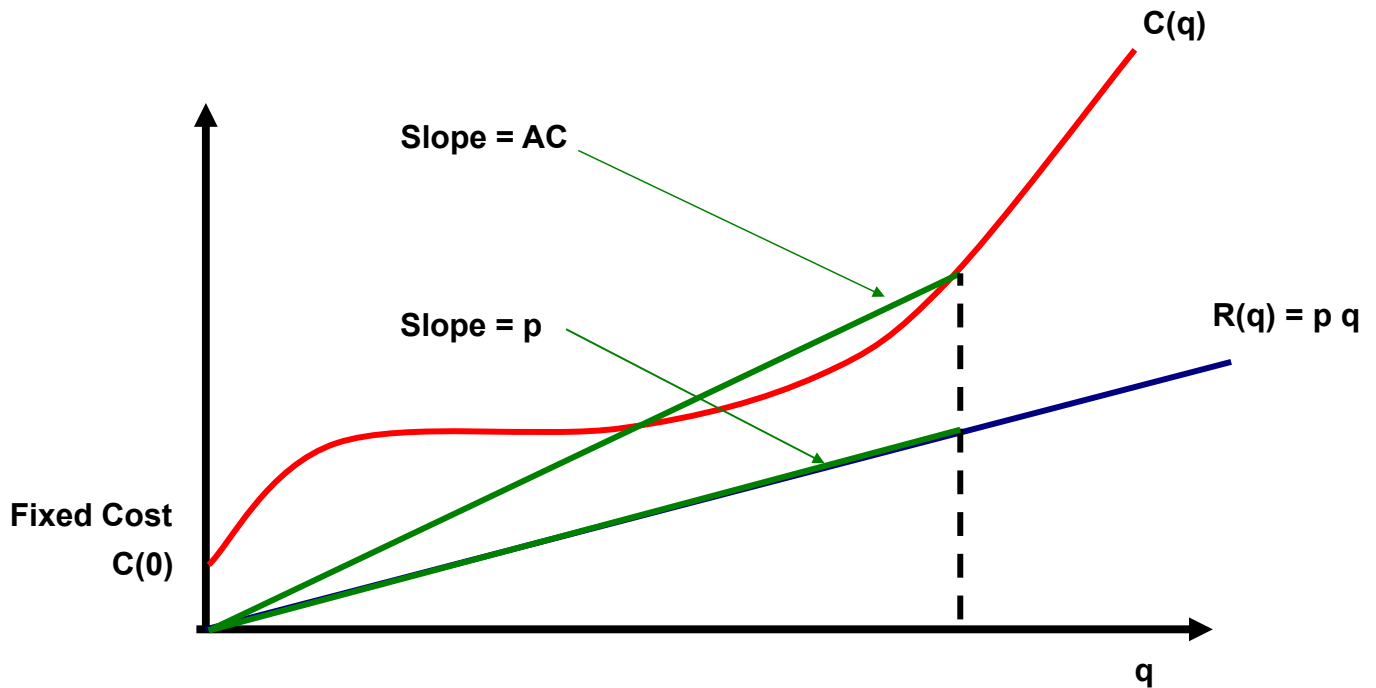
TOTAL COST AND REVENUE

Consider firm in a competitive market, where price p is a given constant



AVERAGE COST & AVERAGE REVENUE

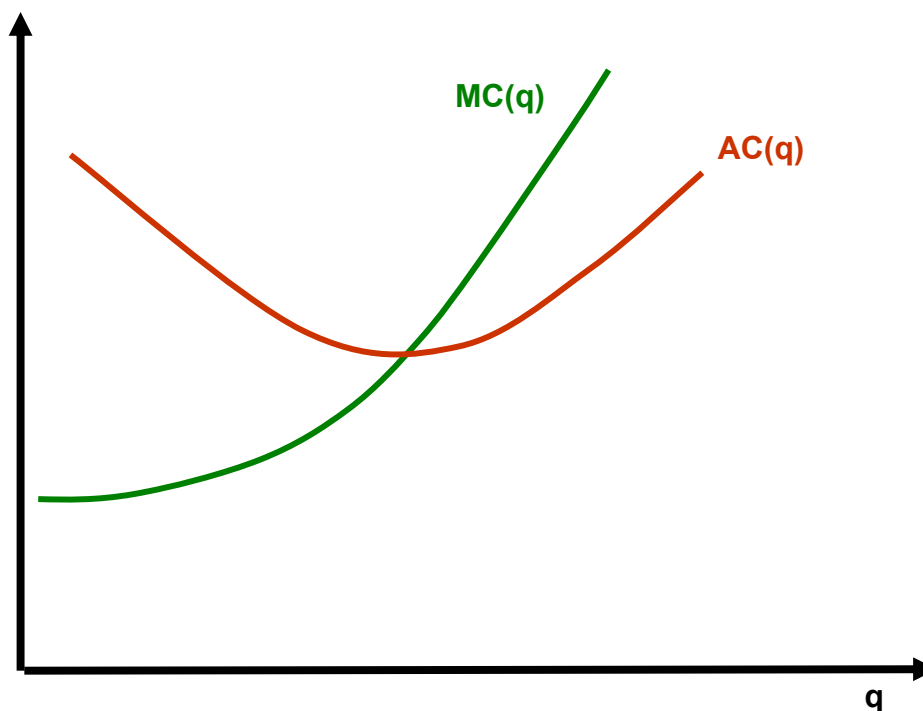
Average Cost: $AC(q) = C(q)/q$
 Average Revenue: $AR(q) = R(q)/q = p$



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AVERAGE COST AND MARGINAL COST



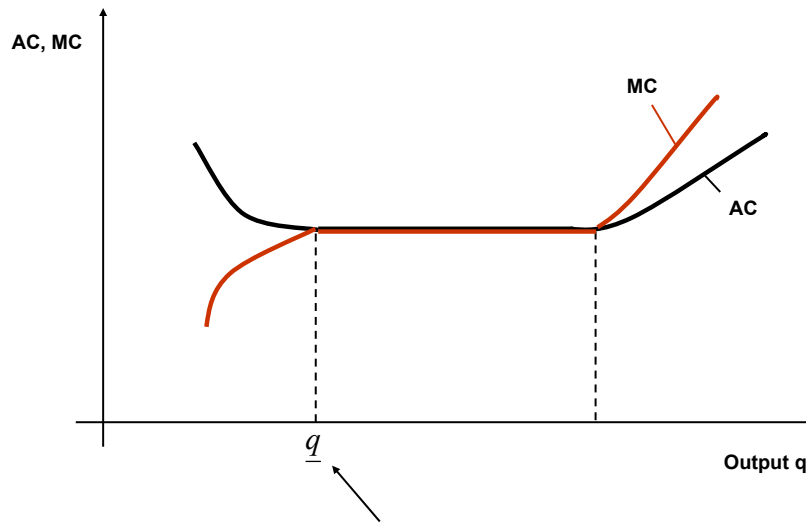
$MC(q) = AC(q)$ at the minimizer of the average cost (= "minimum efficient scale")
 You should be able to prove this relation yourself

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MINIMUM EFFICIENT SCALE

Let $C(q)$ be a firm's increasing, smooth, and convex cost function



$$AC(q) = C(q) / q$$

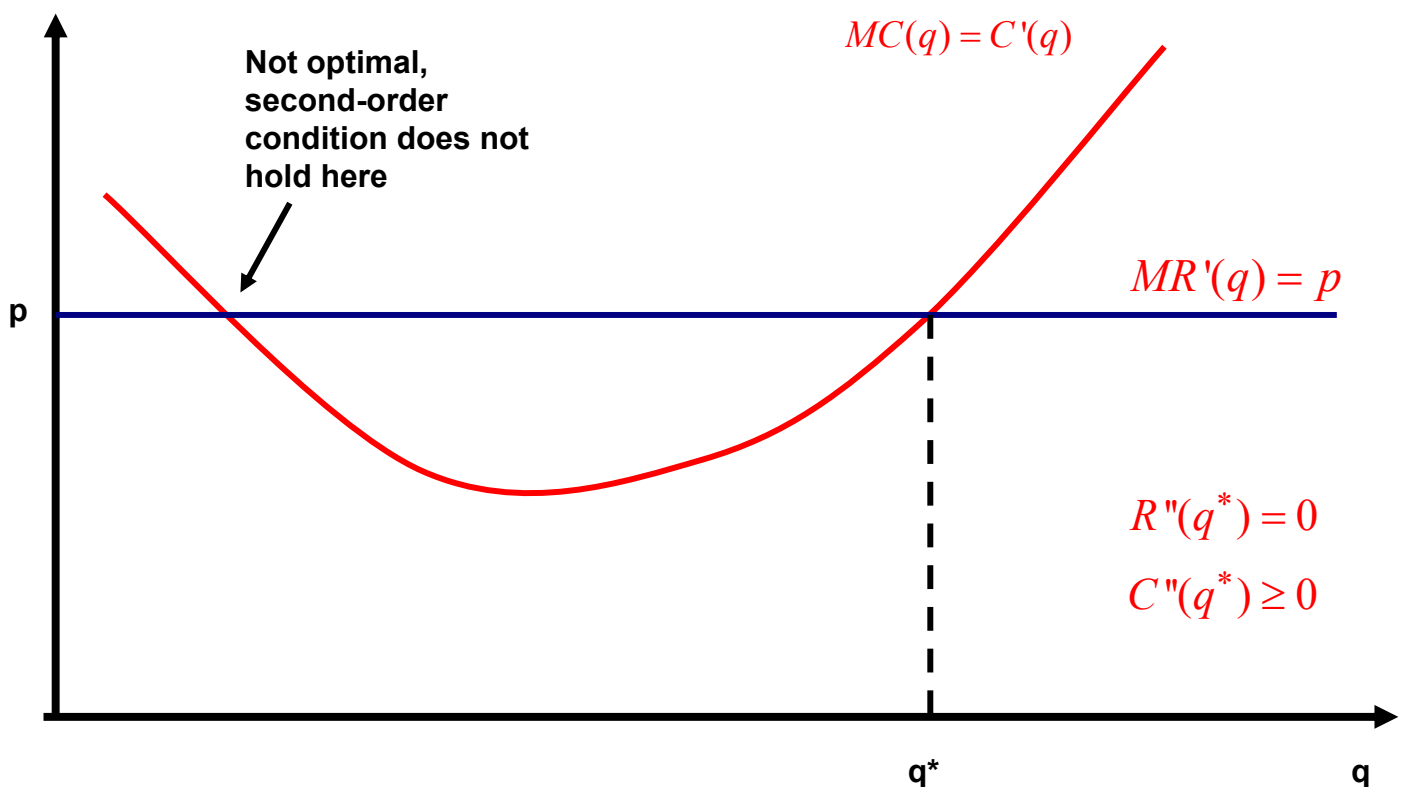
$$MC(q) = C'(q)$$

Minimum Efficient Scale ⁽¹⁾

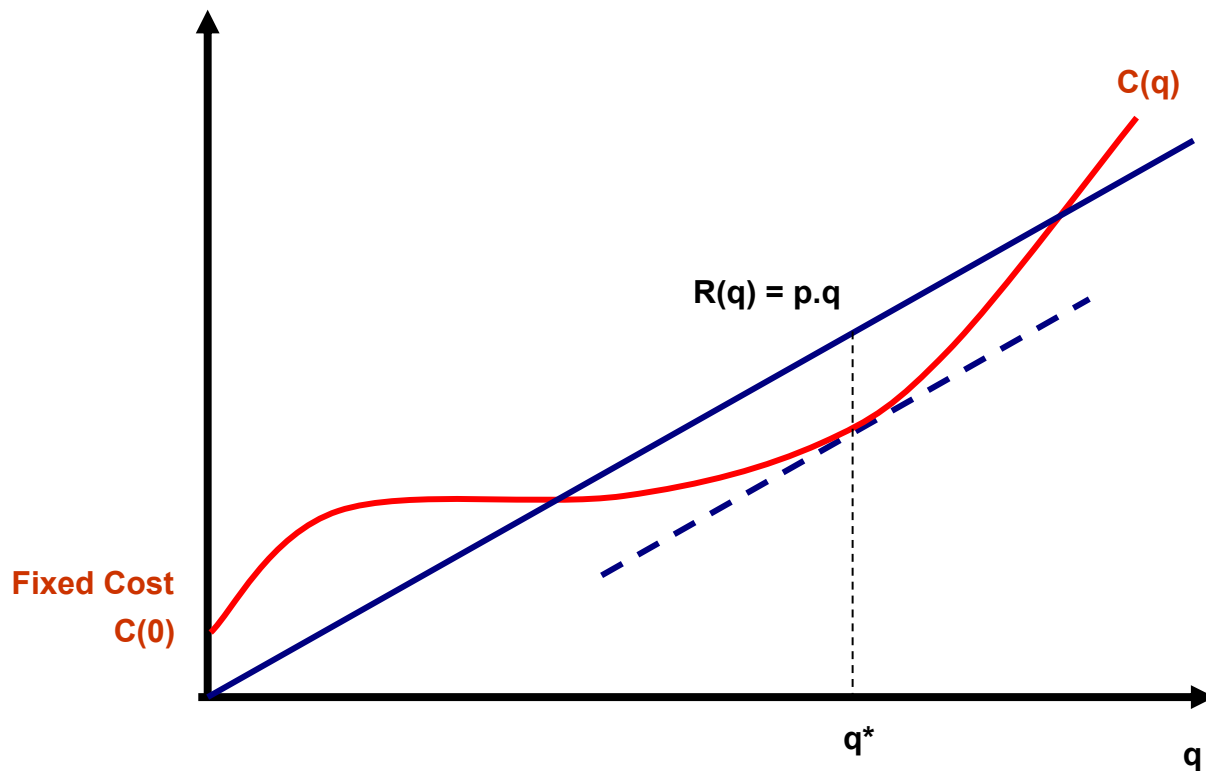
$$\underline{q} = \min \{q \geq 0 : AC'(q) = 0\}$$

(1): More precisely: $\underline{q} = \min \left\{ \arg \min_{q \geq 0} \{AC(q)\} \right\}$

OPTIMAL CHOICE OF THE FIRM'S OUTPUT LEVEL



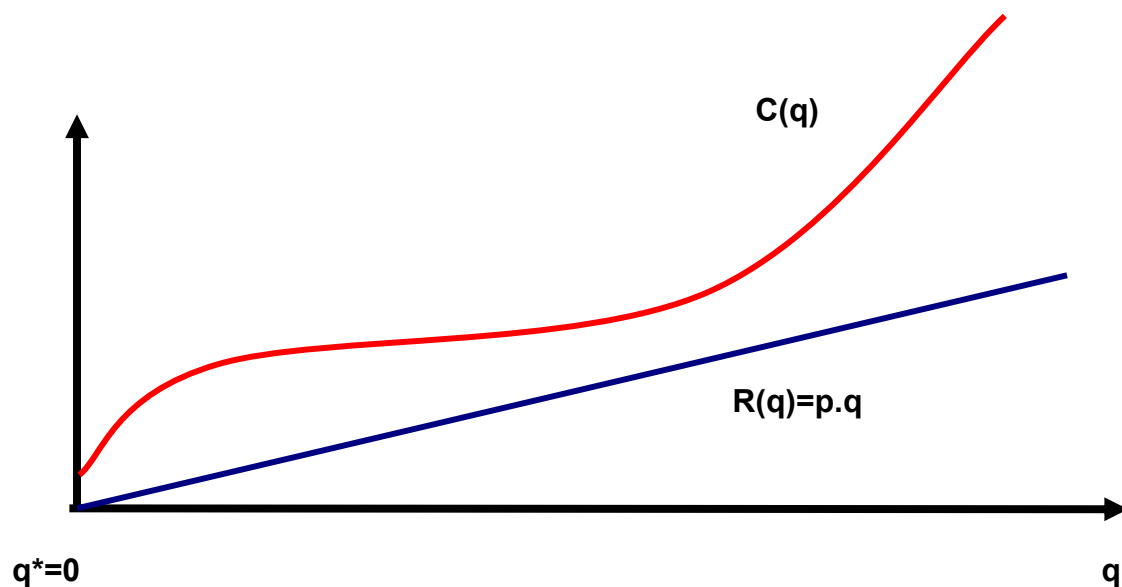
OPTIMAL CHOICE OF THE FIRM'S OUTPUT LEVEL (Cont'd)



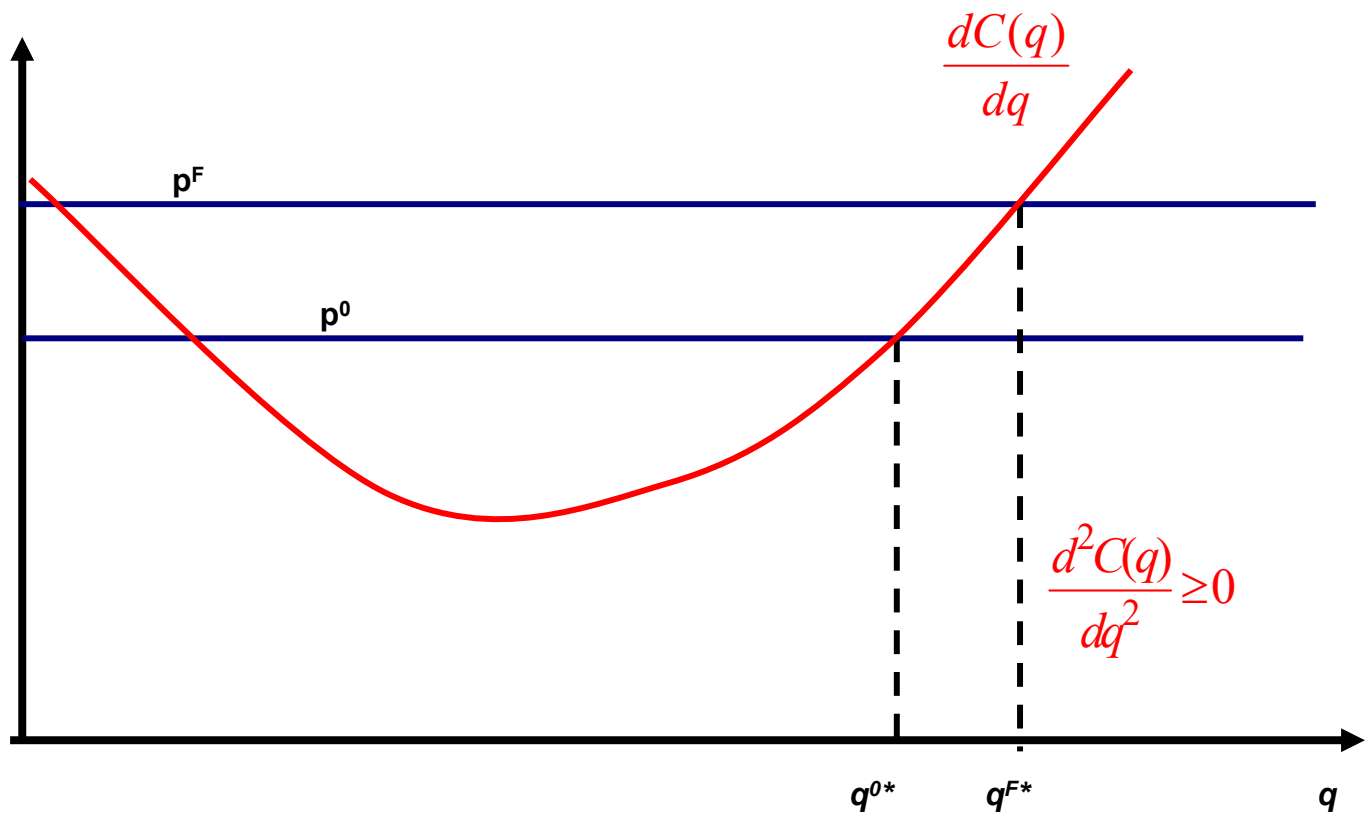
Remark. The firm's **variable cost** is $C(y) - C(0)$.

OPTIMAL CHOICE OF THE FIRM'S OUTPUT LEVEL (Cont'd)

First and second-order conditions are necessary for an interior optimum. Need to examine the boundaries of the action set as well (here: $q^* = 0$)



COMPARATIVE STATICS OF COMPETITIVE FIRM'S OUTPUT



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KEY CONCEPTS TO REMEMBER

- **Profit = Revenue – Cost**
- **(Net) Inputs vs. (Net) Outputs**
- **Profit Maximization Problem**
- **Production (Possibilities) Set**
- **Properties of Production Sets**
- **Production Function**
- **Increasing/Decreasing/Constant Returns to Scale**
- **Cost Function**
- **Cost Minimization Problem**
- **Average Cost, Marginal Cost**
- **Profit Maximization**
- **Marginal Revenue = Marginal Cost**

MGT 621 – MICROECONOMICS

Thomas A. Weber

5. Market Power

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

What is Market Power?

Monopoly

Monopsony

Price Discrimination

Key Concepts to Remember

MARKET POWER

Definition. **Market power** is the ability of a firm to *increase its output prices above the competitive level, and/or to reduce its input prices below the competitive level.*

- **Monopoly**
 - Single seller of a product
- **Oligopoly**
 - Small number of sellers of a product

Sellers' Market

- **Monopsony**
 - Single buyer of a product
- **Oligopsony**
 - Small number of buyers of a product

Buyers' Market

ANALYSIS OF MARKET POWER Initial Focus on Single Firm

We first examine the case where **one single firm has market power**, in a monopoly or a monopsony. Other market participants' actions are aggregated to a market demand (for monopoly) or a market supply (for monopsony).

- When more than one firm holds market power, it is necessary to model the interactions between those firms explicitly. For this, one needs the tools of *Game Theory*

Since actions of all non-market-power-holding entities (the 'other' side of the market) are aggregated into a demand curve (or a supply curve), this is often referred to as **partial equilibrium analysis**.

In **general equilibrium analysis**, the optimizing behavior of all market participants is explicitly taken into account (they could be price takers or not).

We first focus on partial equilibrium analysis of monopoly and monopsony.

AGENDA

What is Market Power?

Monopoly

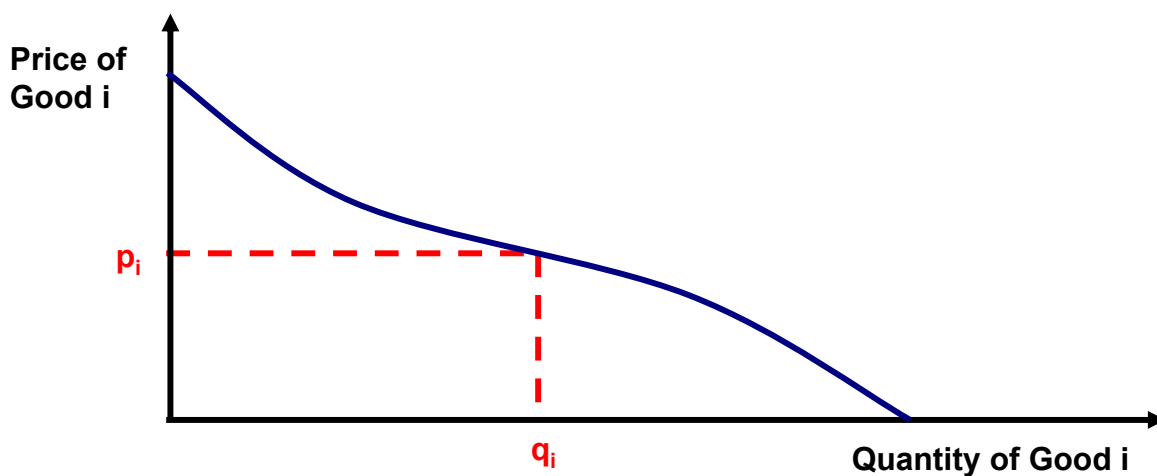
Monopsony

Price Discrimination

Key Concepts to Remember

DEMAND CURVE

The quantity of commodity i a monopolist can sell, its “demand” $D_i(p)$, is a decreasing function of the price p_i . Equivalently, the price at which the firm can sell the product, referred to as its “inverse demand” $p_i(q_i, q_{-i}; p_{-i})$, is a decreasing function of the quantity q_i .



$$q_i = D_i(p_i, p_{-i}) \quad \longleftarrow \quad \text{Demand Curve}$$

$$p_i = p_i(q_i, q_{-i}; p_{-i}) \quad \longleftarrow \quad \text{Inverse Demand Curve}$$

OPTIMAL CHOICE OF MONOPOLY OUTPUT

Assume that a monopolist produces a quantity q of a single output and that the market price at that output is given by the *downward-sloping* inverse market demand $p(q)$. The monopolist's cost function $C(q)$ is *increasing and convex*.

Monopolist's profit: $\Pi(q) = \underbrace{R(q)}_{\text{Revenue}} - \underbrace{C(q)}_{\text{Cost}} = p(q)q - C(q)$

First-order necessary optimality condition:

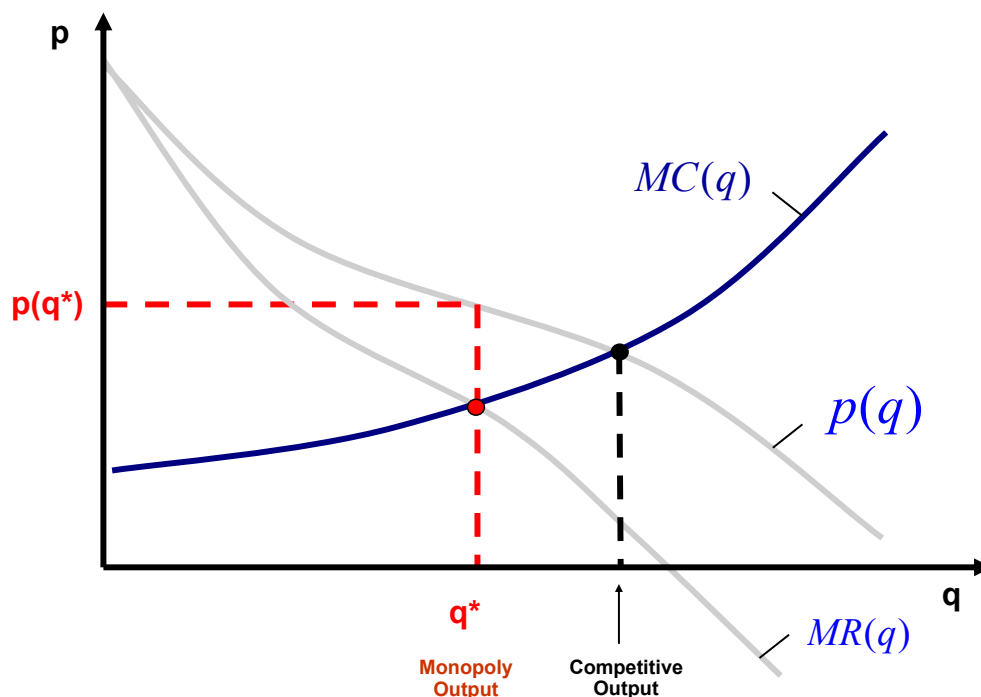
$$\frac{d\Pi(q)}{dq} = \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0 \quad \Leftrightarrow \quad \frac{dR(q)}{dq} = \frac{dC(q)}{dq}$$

Hence,

$$p(q) > p(q) + q \underbrace{\frac{dp(q)}{dq}}_{<0} = \frac{dC(q)}{dq}$$

In other words, the **market price in a monopoly exceeds marginal cost!**

OPTIMAL MONOPOLY OUTPUT (Cont'd)



MONOPOLY PRICING

Inverse Elasticity Rule

Consider the monopolist's choice of a profit-maximizing price p , given its (downward-sloping) demand function $D(p)$.

The **(own-price) demand elasticity** is $\varepsilon(p) = -\frac{p}{D(p)} \frac{dD(p)}{dp}$

Maximizing the monopolist's profit $\Pi(p) = pD(p) - C(D(p))$

yields the first-order necessary optimality condition

$$D(p) + p \frac{dD(p)}{dp} = \frac{dC(D(p))}{dq} \frac{dD(p)}{dp} \quad \text{or} \quad 1 = \left(-\frac{D'(p)}{D(p)} \right) (p - MC(D(p))) = \varepsilon(p) \frac{p - MC(D(p))}{p}$$

Hence, we obtain the **"inverse elasticity rule"** for monopoly pricing:

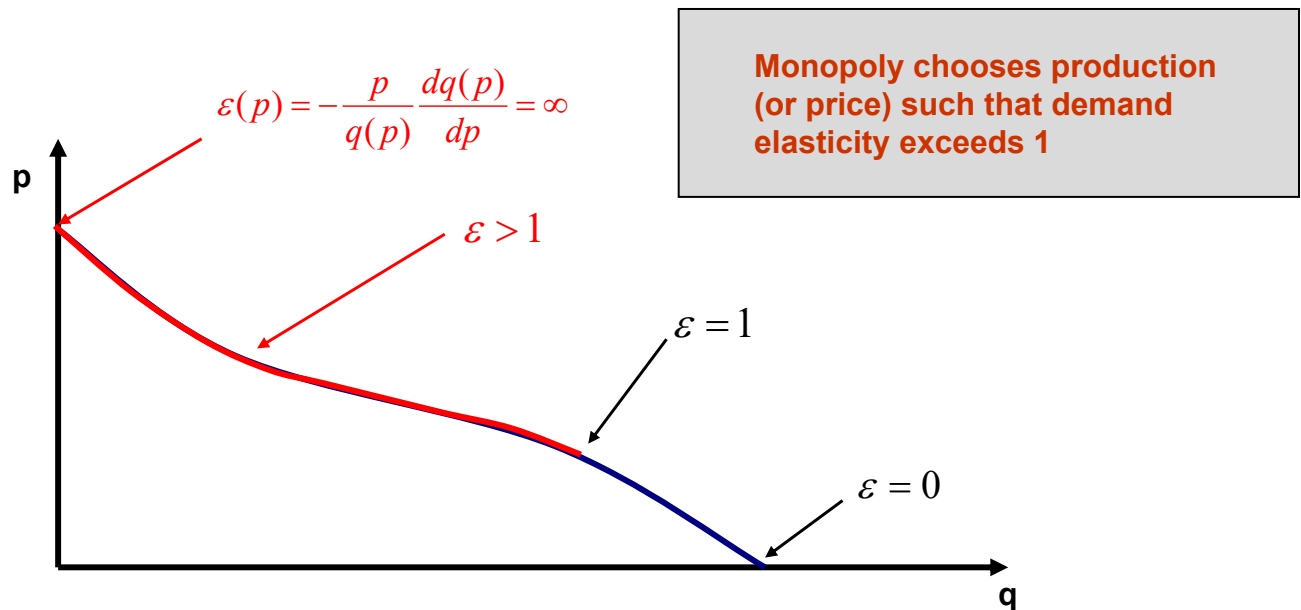
$$1 \geq \underbrace{\frac{p - MC(D(p))}{p}}_{\text{Relative Markup}} = \underbrace{\frac{1}{\varepsilon(p)}}_{\text{Inverse Demand Elasticity (Lerner Index)}}$$

RELATIVE MONOPOLY MARKUPS

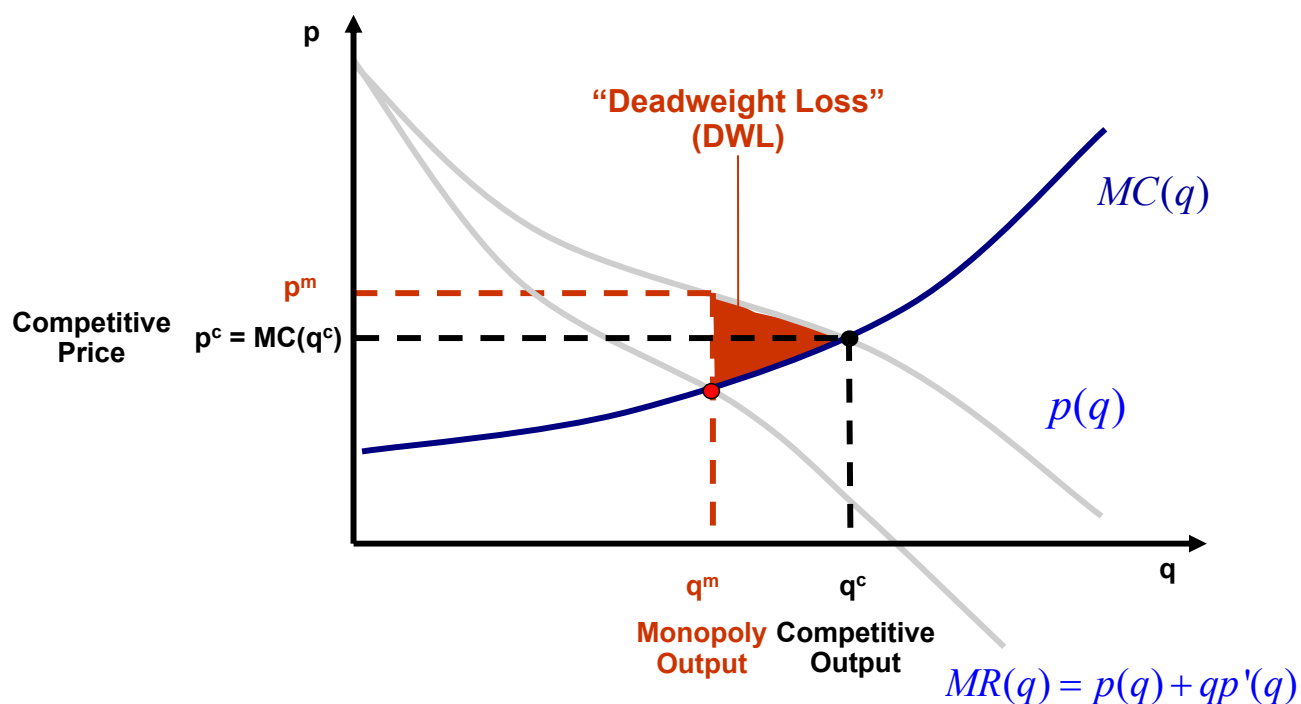
Demand Elasticity	Lerner Index: Markup as Percent of Price	Markup in Percent of Marginal Cost
50	2%	2%
20	5%	5%
10	10%	11%
2	50%	100%
1.5	67%	200%
1.1	91%	1,000%
1.01	99%	10,000%
1	100%	Infinity

DEMAND ELASTICITY CHANGES ALONG DEMAND FUNCTION

... typically from 0 to infinity



INEFFICIENCY CREATED BY MONOPOLY



$$DWL = \int_{q^m}^{q^c} (p(q) - MC(q)) dq = \int_{q^m}^{q^c} p(q) dq - (C(q^c) - C(q^m))$$

WHAT CAN A REGULATOR DO?

Price Caps

When trying to reduce the deadweight loss created by a monopolist, the typical difficulty a regulator faces, is that the marginal cost $MC(q)$ as a function of output belongs to the monopolist's private information.

Hence, when imposing a price-cap p^{reg} the regulator has no way of knowing if the regulated price corresponds to the efficient market price

Two exceptions:

- When $p^{\text{reg}} > p^m$, then the observed market price is below the price cap
- When $p^{\text{reg}} < p^c$, then one may be able to observe excess demand

In general, in order to set an efficient market price (improving the performance of the market by reducing deadweight loss) a regulator needs to find ways to elicit the monopolist's private information about its cost structure.

AGENDA

What is Market Power?

Monopoly

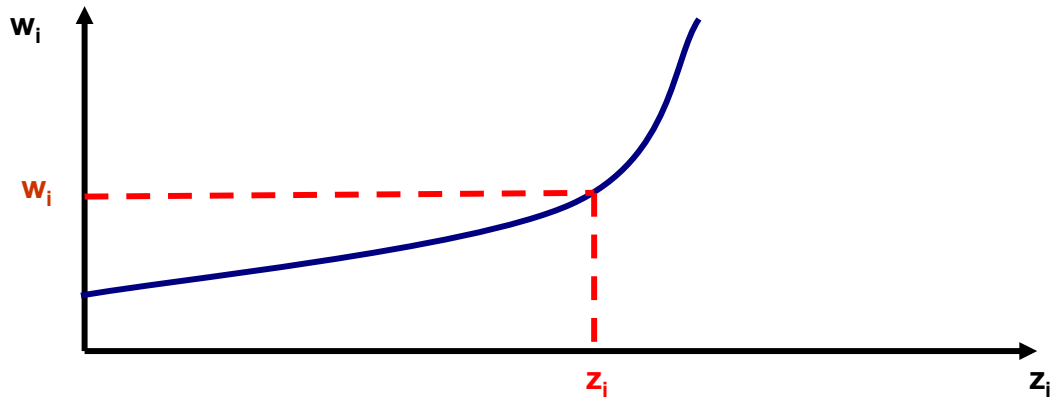
Monopsony

Price Discrimination

Key Concepts to Remember

SUPPLY CURVE

The quantity z_i of commodity i a monopsonist can buy, its “supply” $S_i(w_i, w_{-i}; z_{-i})$, is an **increasing function of the price w_i** . Equivalently, the price at which the firm can buy the product, referred to as its “inverse supply” $w_i(z_i, z_{-i}; w_{-i})$, is a decreasing function of the quantity z_i .



$z_i = S_i(w_i, w_{-i}; z_{-i})$	←	Supply Curve
$w_i = w_i(z_i, z_{-i}; w_{-i})$	←	Inverse Supply Curve

OPTIMAL MONOPSONY INPUT

Without loss of generality, consider input 1, and assume that the firm has one output q which is produced as a function of the input vector, i.e., $q(z)$ is the firm's production function.

Profit:

$$\Pi(z) = pq(z) - \sum_{i \neq 1} w_i z_i - w_1(z_1) z_1$$

FOCs:

$$\frac{\partial \Pi(z)}{\partial z_i} = p \frac{\partial q(z)}{\partial z_i} - w_i = 0 \quad \text{for } i \neq 1$$

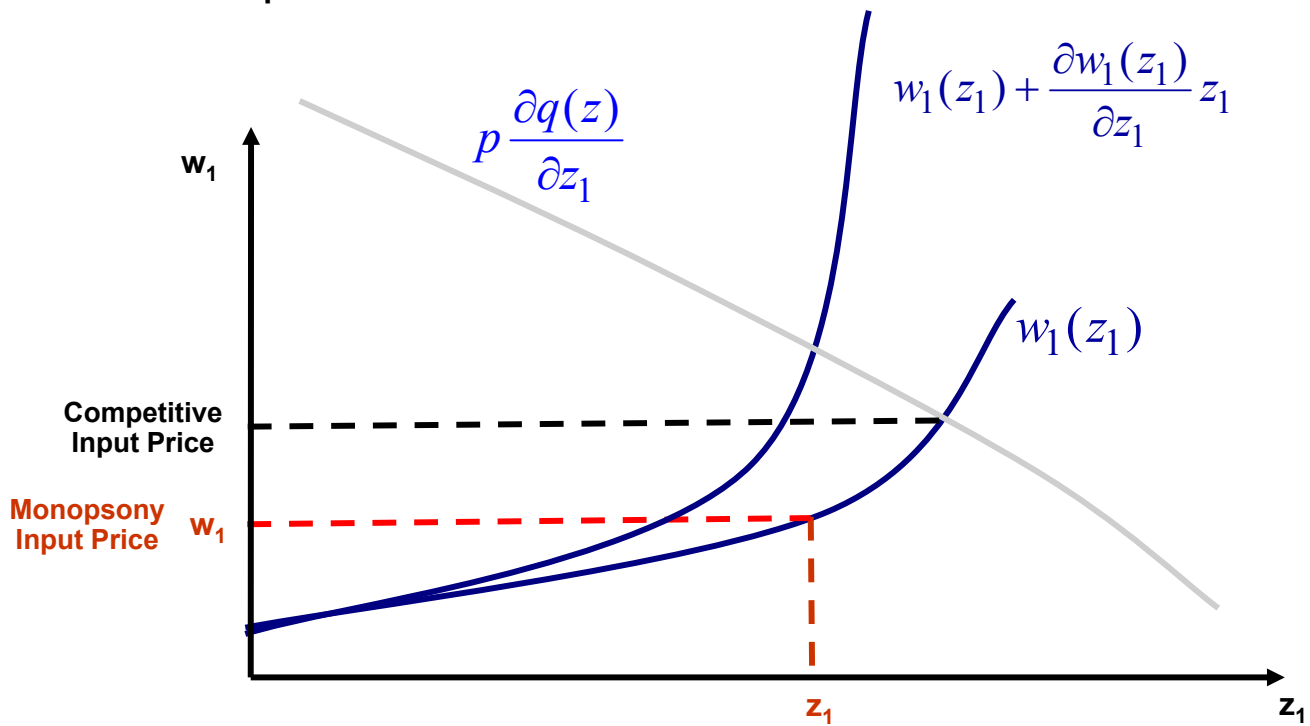
$$\frac{\partial \Pi(z)}{\partial z_1} = p \frac{\partial q(z)}{\partial z_1} - w_1(z_1) - \frac{\partial w_1(z_1)}{\partial z_1} z_1 = 0$$

$$p \frac{\partial q(z)}{\partial z_i} = w_i \quad \text{for } i \neq 1$$

$$p \frac{\partial q(z)}{\partial z_1} = w_1(z_1) + \frac{\partial w_1(z_1)}{\partial z_1} z_1 > w_1(z_1)$$

MONOPSONIST'S INPUT CHOICE

Consider input 1.



AGENDA

What is Market Power?

Monopoly

Monopsony

Price Discrimination

Key Concepts to Remember

WHAT IS PRICE DISCRIMINATION?

Definition. **Price discrimination** exists if *different units of the same good are sold at different prices to one or more consumers*.

One commonly distinguishes three different degrees of price discrimination.

- **First-Degree** Price Discrimination: the seller charges a *price for each unit corresponding to the maximum willingness to pay* over all available consumers of that unit. This is also referred to as **perfect price discrimination** as it maximizes the seller's revenues.
- **Second-Degree** Price Discrimination: the seller charges *different amounts for different numbers of units* bought by the same consumer. This is also referred to as **nonlinear pricing**.
- **Third-Degree** Price Discrimination: the seller charges *different prices to different consumer groups* based on observable differences between the groups.

FIRST-DEGREE PRICE DISCRIMINATION

If the maximum willingness to pay for each unit is available, then the seller can order these values so that the willingness to pay for additional units is nonincreasing. This yields a **nonincreasing inverse demand curve $p(q)$** as a function of the seller's output q .

The seller can choose the optimal output by maximizing

$$\Pi(q) = \int_0^q p(\hat{q}) d\hat{q} - C(q)$$

with respect to q . The first-order necessary optimality condition is

$$p(q) = MC(q)$$

In other words, the *seller should increase output until the maximum willingness to pay for the next unit exactly equals her marginal cost of producing that unit*.

Note that with perfect price discrimination, the **monopolist's deadweight loss vanishes**, and so does the consumers' surplus.

SECOND-DEGREE PRICE DISCRIMINATION

- Second-degree price discrimination (or “nonlinear pricing,” or “screening”) is a **mechanism-design problem**. It is more difficult than first-degree or third-degree price discrimination, but it is also more realistic.
- It operates under the assumption that the **seller knows that consumers have heterogeneous preferences but is unable to directly distinguish the different consumers**. Information about a given consumer’s preferences (his utility function) is assumed to be only privately available to that consumer.
- In order to **incentivize a consumer to reveal his utility function** (or his “type”) the seller needs to offer several options for the consumer to choose from. Through his choice the consumer “reveals” his preference, and the seller may thereby be able to charge different consumers (or groups of consumers) different prices.
- The **solution to the problem will naturally depend on the seller’s model of the consumer heterogeneity**.

EXAMPLE: SELLING A REFRIGERATOR Screening Model

Instead of quantities (which can vary continuously) we take a very simple shot at this generally difficult problem and examine a special case where the seller has two refrigerators (of qualities $q_1=1$ and $q_2=2$) to sell to consumers who are heterogeneous but indistinguishable to the seller.

Question. How much should the seller charge for the two refrigerators?

What needs to be considered?

1. Buyer’s private information:

- The seller does not know how much the buyer is willing to pay for the refrigerator
- She assumes that the buyer values a refrigerator of quality q at $u = \theta q$, where θ is unknown to her
- She assumes that the buyer might be of two types $\theta \in \{\theta_L, \theta_H\}$: with probability μ it is $\theta_H = 2$ and with probability $1-\mu$ it is $\theta_L = 1$

2. The buyer’s voluntary participation in the mechanism

- The seller cannot force the buyer to pay more than his WTP u
- The seller has to leave the choice of the refrigerator up to the buyer

SELLING A REFRIGERATOR (Cont'd)

Designing a mechanism amounts for the seller to choosing the best possible prices p_1 and p_2 for the two products (of qualities q_1 and q_2 respectively).

Seller's maximizes expected revenues and assuming that the high-type buys the product

Buyer's participation ("**individual rationality**"):

- Type $\theta_L = 1$: participates if and only if $p_1 \leq \theta_L q_1 \Leftrightarrow p_1 \leq 1$
- Type $\theta_H = 2$: participates if and only if $p_2 \leq \theta_H q_2 \Leftrightarrow p_2 \leq 4$

Buyer's choice ("**incentive compatibility**"):

- Type $\theta_L = 1$: chooses q_1 over q_2 if and only if
 $\theta_L q_2 - p_2 \leq \theta_L q_1 - p_1 \Leftrightarrow 2 - p_2 \leq 1 - p_1 \Leftrightarrow 1 \leq p_2 - p_1$
- Type $\theta_H = 2$: chooses q_2 over q_1 if and only if
 $\theta_H q_1 - p_1 \leq \theta_H q_2 - p_2 \Leftrightarrow 2 - p_1 \leq 4 - p_2 \Leftrightarrow p_2 - p_1 \leq 2$

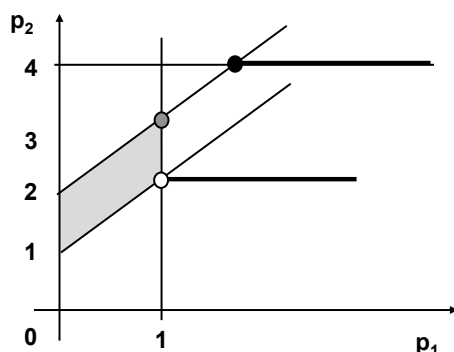
SELLING A REFRIGERATOR (Cont'd)

Hence, the seller solves the following revenue-maximization problem:

$$\max_{p_1, p_2} \{ \mu p_1 + (1 - \mu) p_2 \}$$

subject to $p_1 \leq 1, p_2 \leq 4$ (individual rationality)

and $1 \leq p_2 - p_1 \leq 2$ (incentive compatibility)



(*)

$p_1^* \geq 2, p_2^* = 4$ (only θ_H participates;
sell only product 2)
optimal for $\mu \leq 1/2$

$p_1^* = 1, p_2^* = 3$ (full participation;
sell both products)
optimal for $\mu = 1/2$

$p_1^* > 1, p_2^* = 2$ (full participation;
sell only product 2)
optimal for $\mu \geq 1/2$

CONCLUSIONS ABOUT THE SCREENING MODEL

Second-Degree Price Discrimination

Key Conclusions from the example: (generalizes to other nonlinear pricing models)

1. In the presence of asymmetric information, high consumer types typically obtain a positive surplus (“information rent”)
2. Low-type consumers exert a positive externality on high-type consumers
3. As low-type consumers become less frequent, it becomes optimal for the seller to exclude them from the market (“shut-down solution”)
4. When designing a good mechanism, the seller needs to take into account the consumers’ individual rationality and incentive compatibility constraints
5. As long as the seller can commit to her mechanism she can, without any loss in generality, restrict her attention to “truthful” mechanisms in which all participating agents report their types truthfully (“revelation principle”)

A MORE GENERAL EXAMPLE



Question. At what qualities and what prices should a company offer a “vertically differentiated” product, such as an espresso maker?

For simplicity, we restrict attention to a firm which offers at most two products.

THERE ARE MANY OTHER EXAMPLES
SD Card



?

THERE ARE MANY OTHER EXAMPLES
Parmesan Cheese



THERE ARE MANY OTHER EXAMPLES Parmesan Cheese (Cont'd)



CONSIDER A SIMPLE SCREENING MODEL

Model Features

- Two Types (“high” θ_H and “low” θ_L , with $\theta_H > \theta_L > 0$)
- Utility increasing in instrument and in type, quasi-linear in wealth
- Outside option valued at zero
- Risk-neutral seller, maximizes expected profit
- Prior beliefs of principal (corresponding to the probability μ of a consumer being a high type) given
- Instrument (i.e., product quality) costly to provide, $C(q) \geq 0$

What is missing? – **SORTING CONDITION** ...

$$\hat{q} \geq q \Rightarrow u(\hat{q}, \theta_H) - u(q, \theta_H) \geq u(\hat{q}, \theta_L) - u(q, \theta_L)$$

u exhibits “**increasing differences**” (or is “**supermodular**”)

The sorting condition enables the seller to separate high types from low types.

SELLER'S PROBLEM

The seller chooses the qualities and prices of the products such as to maximize her expected profits, i.e., she solves the constrained optimization problem

$$\max_{p_L, p_H, q_L, q_H \geq 0} \left\{ (1 - \mu)(p_L - C(q_L)) + \mu(p_H - C(q_H)) \right\}$$

subject to

$$\begin{aligned} u(q_L, \theta_L) - p_L &\geq 0 & (\text{IR-L}) \\ u(q_H, \theta_H) - p_H &\geq 0 & (\text{IR-H}) \\ u(q_L, \theta_L) - p_L &\geq u(q_H, \theta_L) - p_H & (\text{IC-L}) \\ u(q_H, \theta_H) - p_H &\geq u(q_L, \theta_H) - p_L & (\text{IC-H}) \end{aligned}$$

Individual Rationality

Incentive Compatibility

THE SELLER'S PROBLEM CAN BE SIMPLIFIED

Two constraints are **binding**.

1. (IR-L) is binding at optimum

Proof. Assume not. Then $u(q_L, \theta_L) - p_L > 0$, so that

$$\begin{aligned} u(q_H, \theta_H) - p_H &\geq u(q_L, \theta_H) - p_L \geq u(q_L, \theta_L) - p_L > 0 \\ &\quad \uparrow \quad \quad \quad \uparrow \\ &\quad (\text{IC-H}) \quad \quad \partial u / \partial \theta > 0 \end{aligned}$$

But this means that p_L cannot be optimal, a contradiction. QED

2. (IC-H) is binding at optimum

Proof. Assume not. Then

$$\begin{aligned} (*) \quad u(q_H, \theta_H) - p_H &> u(q_L, \theta_H) - p_L \geq u(q_L, \theta_L) - p_L = 0 \\ &\quad \uparrow \quad \quad \quad \uparrow \\ &\quad (\text{IC-H}) \quad \quad \partial u / \partial \theta > 0 \end{aligned}$$

But this means that the seller could increase p_H , a contradiction. QED

THE SELLER'S PROBLEM CAN BE SIMPLIFIED (Cont'd)

Two constraints are **redundant**.

3. (IC-L) can be neglected

Proof. Since (IC-H) is binding, it is $u(q_H, \theta_H) - p_H = u(q_L, \theta_H) - p_L$

Hence, $p_H - p_L = u(q_H, \theta_H) - u(q_L, \theta_H) \geq u(q_H, \theta_L) - u(q_L, \theta_L)$

Therefore $u(q_L, \theta_L) - p_L \geq u(q_H, \theta_L) - p_H$ QED

↑
(SC)

4. (IR-H) can be neglected

The proof follows directly from (*) in the proof of claim 2.

THE SIMPLIFIED PROBLEM

The seller's nonlinear pricing problem is equivalent to

$$\max_{p_L, p_H, q_L, q_H \geq 0} \left\{ (1 - \mu)(p_L - C(q_L)) + \mu(p_H - C(q_H)) \right\}$$

subject to

$$u(q_L, \theta_L) - p_L = 0 \quad (\text{IR-L})$$

$$u(q_H, \theta_H) - p_H = u(q_L, \theta_H) - p_L \quad (\text{IC-H})$$

The constraints (IR-L) and (IC-H) can be directly substituted into the objective function.

THE SIMPLIFIED PROBLEM

The **seller's nonlinear pricing problem** is equivalent to

$$\max_{q_L, q_H \geq 0} \left\{ (1 - \mu) (u(q_L, \theta_L) - C(q_L)) + \mu (u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L)) - C(q_H) \right\}$$

Hence, the seller's optimal quality levels obtain as follows (for $\mu > 0$):

$$q_L^* \in \arg \max_{q_L \geq 0} \left\{ (u(q_L, \theta_L) - C(q_L)) - \frac{\mu}{1 - \mu} (u(q_L, \theta_H) - u(q_L, \theta_L)) \right\} \quad \text{(Distorted Quality Level)}$$

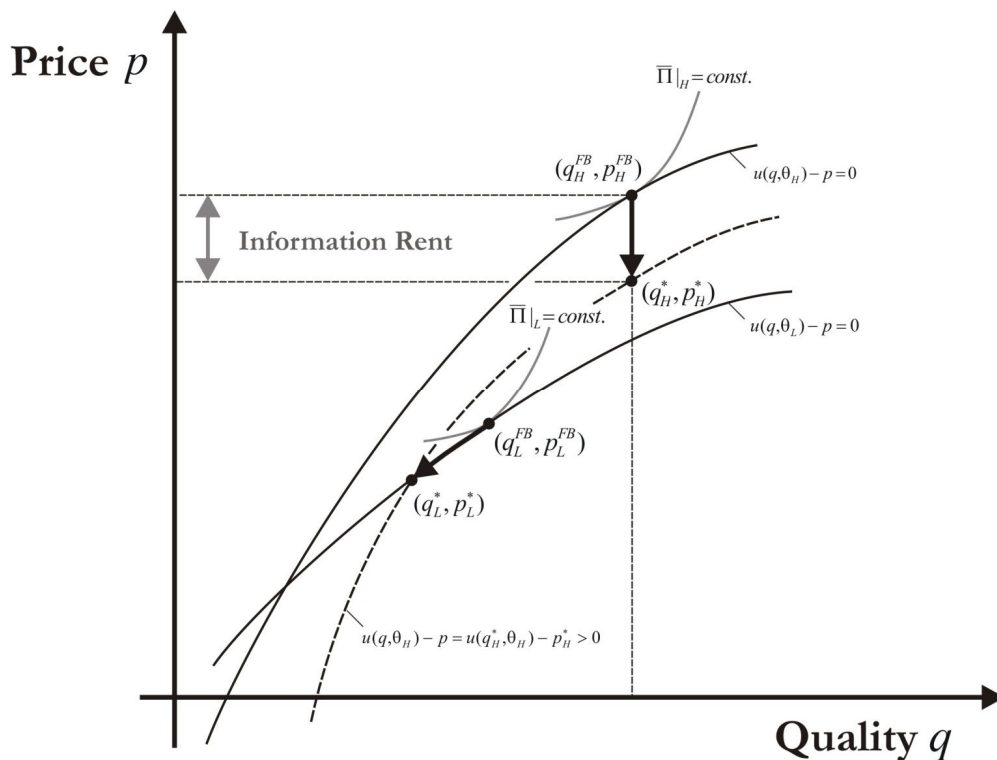
$$q_H^* \in \arg \max_{q_H \geq 0} \{ u(q_H, \theta_H) - C(q_H) \} \quad \text{(Efficient Quality Level)}$$

From (IR-L) and (IC-H) we then get

$$p_L^* = u(q_L^*, \theta_L) \quad \text{(Efficient Price Level)}$$

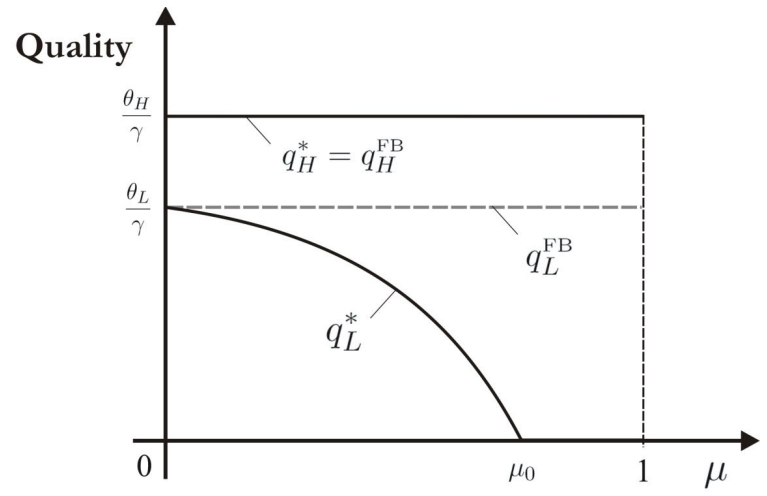
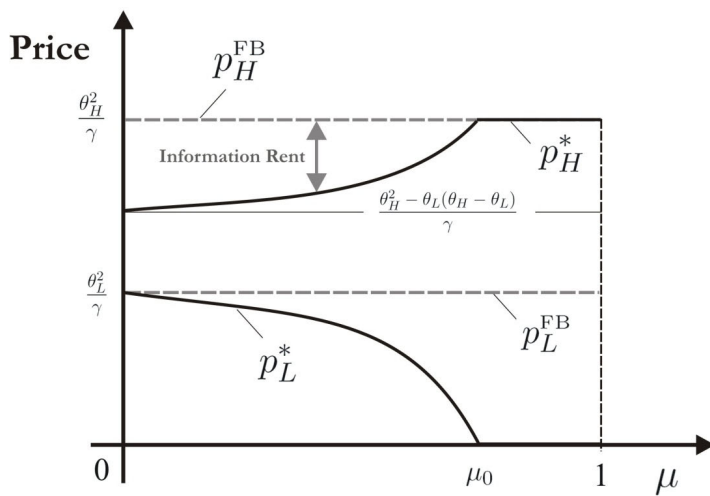
$$p_H^* = u(q_H^*, \theta_H) - \underbrace{(u(q_L^*, \theta_H) - u(q_L^*, \theta_L))}_{\text{Information Rent } (\geq 0)} \quad \text{(Distorted Price Level)}$$

FIRST-BEST AND SECOND-BEST SOLUTIONS IN THE SCREENING MODEL



PRICE AND QUALITY IN THE TWO-TYPE SCREENING MODEL

Example: $u(q, \theta) = \theta q$ $C(q) = \gamma q^2 / 2, \gamma > 0$



THIRD-DEGREE PRICE DISCRIMINATION

For simplicity, let us assume that there are two different consumer groups, 1 and 2, that the seller can distinguish and which can legally be charged different prices for the same product. Let the inverse demand curve of consumer group $i \in \{1, 2\}$ be given by $p_i(q_i)$, where q_i is the amount consumed by that group.

Given a standard (increasing, convex) cost function $C(q)$, the monopolist then solves the profit-maximization problem

$$\max_{q_1, q_2 \geq 0} \{p_1(q_1)q_1 + p_2(q_2)q_2 - C(q_1 + q_2)\}$$

which for $q_1, q_2 > 0$ leads to the first-order necessary optimality conditions

$$\begin{aligned} p_1(q_1) + q_1 p_1'(q_1) &= C'(q_1 + q_2) \\ p_2(q_2) + q_2 p_2'(q_2) &= C'(q_1 + q_2) \end{aligned}$$

Hence, at an optimum, the marginal revenues from the two consumer groups are equal to each other and equal to the marginal cost at the combined output.

THIRD-DEGREE PRICE DISCRIMINATION (Cont'd)

More generally, the two consumer groups may not be fully separable. Each group's demand may be influenced by the amount sold to the other group. Then the inverse demand curve of consumer group $i \in \{1, 2\}$ is given by $p_i(q_1, q_2)$, where q_i is the amount consumed by that group.

Given an increasing, jointly convex cost function $C(q_1, q_2)$, the monopolist then solves the profit-maximization problem

$$\max_{q_1, q_2 \geq 0} \{p_1(q_1, q_2)q_1 + p_2(q_1, q_2)q_2 - C(q_1, q_2)\}$$

which for $q_1, q_2 > 0$ leads to the first-order necessary optimality conditions

$$p_1(q_1, q_2) + q_1 \frac{\partial p_1(q_1, q_2)}{\partial q_1} + q_2 \frac{\partial p_2(q_1, q_2)}{\partial q_1} = \frac{\partial C(q_1, q_2)}{\partial q_1}$$

$$p_2(q_1, q_2) + q_1 \frac{\partial p_1(q_1, q_2)}{\partial q_2} + q_2 \frac{\partial p_2(q_1, q_2)}{\partial q_2} = \frac{\partial C(q_1, q_2)}{\partial q_2}$$

At an optimum, the marginal revenue from each of the two consumer groups is equal to the marginal cost of increasing the output for that group (sometimes equal to the marginal cost of increasing output for the other group, e.g., when the cost depends only on $q_1 + q_2$).

AGENDA

What is Market Power?

Monopoly

Monopsony

Price Discrimination

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Market Power**
- **Monopoly/Monopsony**
- **(Own-)Price Elasticity**
- **Inverse Elasticity Pricing Rule**
- **Lerner Index**
- **Deadweight Loss**
- **Price Caps**
- **Price Discrimination (first/second/third degree)**

MGT 621 – MICROECONOMICS

Thomas A. Weber

6. Oligopoly

Autumn 2026

École Polytechnique Fédérale de Lausanne
College of Management of Technology

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OLIGOPOLY THEORY Introduction

So far in this course we have not emphasized strategic interactions between firms.

- We have seen that externalities can lead to significant distortions of the market outcome, even if all firms are price takers
- When a monopolist has market power, it can use second-degree price discrimination to segment a heterogeneous consumer base. For that analysis we did consider strategic interactions, but obtained a pure optimization problem, since the monopolist is able to move first by committing to a pricing scheme, anticipating the consumers' actions

When multiple firms select their actions simultaneously, and those actions directly influence each others' payoffs (i.e., there are externalities), then we need **game theory** to produce reasonable predictions about the outcome of the interaction.

Game theory is a fundamental tool in the analysis of strategic interactions between multiple firms with market power.

AGENDA

What is Game Theory?

Building Blocks and Key Assumptions

Market Structure & Strategy Analysis

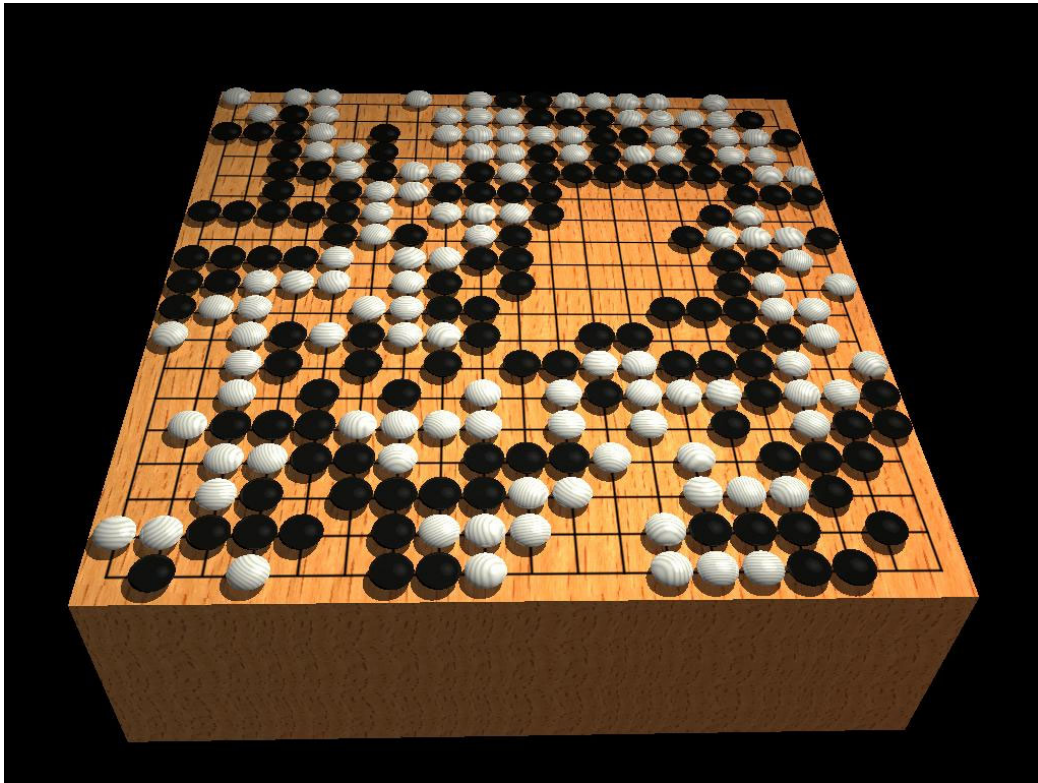
- Cournot Quantity Competition
- Bertrand Price Competition

Key Concepts to Remember

GAME THEORY



GAME THEORY



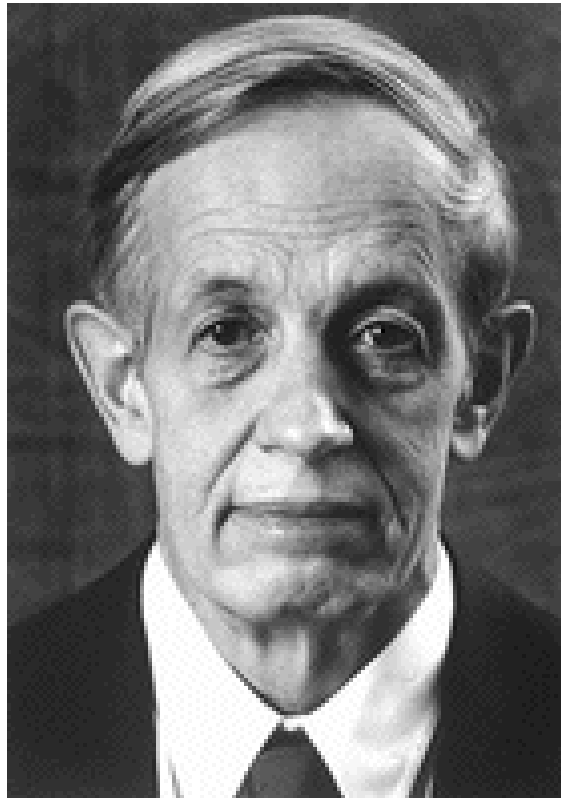
JOHN VON NEUMANN (1903 – 1957)



Oskar Morgenstern
(1902 – 1976)



JOHN FORBES NASH (1928 – 2015)



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- 7 -

GAME THEORY

Game Theory is the **analysis of strategic interactions among agents**.

A **strategic interaction** is a situation in which each agent, when selecting his or her most preferred action, takes into account the likely decisions of the other agents.

Example: War

"In war the will is directed at an animate object that reacts."

- Carl von Clausewitz, *On War*

The **objective** of game theory is **to provide predictions** about the behavior of agents (players) in strategic interactions. The more precise these predictions are, the higher their **"predictive power."**

(1) Cf. von Clausewitz, C. (1976) *On War*, Princeton University Press, Princeton, NJ. Clausewitz lived from 1780 to 1831; for more details about his life and work, see <http://www.clausewitz.com/>. The first systematic academic treatment of game theory is von Neumann, J., Morgenstern (1944) *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

AGENDA

What is Game Theory?

Building Blocks and Key Assumptions

Market Structure & Strategy Analysis

- Cournot Quantity Competition
- Bertrand Price Competition

Key Concepts to Remember

NORMAL-FORM GAME

Building Blocks

- **Players**, $i \in N = \{1, \dots, n\}$
- **Action Sets (Strategy Spaces)**, A_i , with elements $a_i \in A_i$
- **Individual Payoffs**⁽¹⁾, $u_i(a)$, where $a = (a_i, a_{-i}) \in A = A_1 \times \dots \times A_n$ and $a_{-i} \in A_{-i} = A_1 \times \dots \times A_{i-1} \times A_{i+1} \times \dots \times A_n$
- **(Mixed) Strategies**,⁽²⁾ $\sigma_i \in \Delta(A_i)$ and $\sigma_{-i} \in \Delta(A_{-i})$

Definition: A Normal-Form Game Γ_N is a collection of players, action sets, and payoffs,

$$\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$$

(1) The payoff functions $u_i(\cdot)$ are generally taken to be **von Neumann-Morgenstern utility functions**. You can think of them as bounded measurable functions (functionals) mapping actions (probability distributions over actions) into real values. The **payoff of a mixed strategy profile** is simply the expected value over the random action profile, $u_i(\sigma) = u_i(\sigma_i, \sigma_{-i}) = \sum_{a \in A} [\sigma_1(a_1) \cdot \dots \cdot \sigma_n(a_n)] u_i(a)$

(2) Instead of **pure strategies** (elements of A_i) we allow **"mixed" strategies** (elements of $\Delta(A_i)$). If A_i is finite containing m_i elements, then $\Delta(A_i)$ corresponds to the $(m_i - 1)$ -dimensional simplex over A_i .

PRISONER'S DILEMMA Example

Two suspects, 1 and 2, are being interrogated separately about a crime

- If **both confess**, each is sentenced to five years in prison
- If **both deny** their involvement, each is sentenced to one year in prison
- If **just one confesses**, he is released but the other one is sentenced to ten years in prison

Assume that each player's payoffs are proportional to the length of time of his prison sentence.

Formulate this game in normal form.

PRISONER'S DILEMMA (Cont'd) Example

Normal-Form Representation

- **Players**, $i \in N = \{1, 2\}$
- **Action Sets**, $A_i = \{Deny, Confess\}$
- **Individual Payoffs**, $u(a_1, a_2)$, defined by **"payoff matrix"**
- **(Mixed) Strategies**, $\sigma_i = (\sigma_i(Deny), \sigma_i(Confess)) \geq 0$, with $\sigma_i(Deny) + \sigma_i(Confess) = 1$

Payoff Matrix⁽¹⁾

		<u>Player 2</u>	
		Confess	Deny
<u>Player 1</u>	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

PRISONER'S DILEMMA (Cont'd) Example

Find Prediction about Outcome of this Game

		<u>Player 2</u>	
		Confess	Deny
<u>Player 1</u>	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

- Consider player 1's "best response" when fixing player 2's strategy
- Consider player 2's "best response" when fixing player 1's strategy

Hence, each player has a **dominant strategy**: no matter what the other player does, it is optimal (i.e., payoff-maximizing) for player i to select $a_i = \text{Confess}$.

Note also that the outcome is **inefficient** (i.e., does not maximize social surplus).

FUNDAMENTAL ASSUMPTIONS

Question: What assumptions are necessary to arrive at predictions about outcomes of normal-form games?

Assumption 1: All **players are rational**, i.e., they maximize (expected) payoffs.

Assumption 2: The players' payoff functions and action sets are **common knowledge**, i.e.,⁽¹⁾

- Each player knows the rules of the game
- Each player knows that each player knows the rules
- Each player knows that each player knows that each player knows the rules
- Each player knows that each player knows that each player knows that each player knows the rules
- Each player knows that each player knows that each player knows that each player knows that each player knows the rules
- ...

Assumptions 1 and 2 imply a unique prediction in the Prisoner's Dilemma game; we will maintain these assumptions throughout this course

(1) For a formal definition of common knowledge, see Osborne, M.J., Rubinstein, A. (1994) *A Course in Game Theory*, MIT Press, Cambridge, MA, pp. 73—75.

WHAT HAPPENS IF PLAYERS ARE NOT RATIONAL?

Relaxing Assumption 1

Relaxing the rationality assumption leads to **boundedly rational agents**, which is compatible with empirical observations. Some features of real-world agents which violate the rationality assumption are:

- Overconfidence
- Sensitivity to framing of the problem
- Satisficing behavior
- Intransitive preferences over outcomes (e.g., Allais Paradox, Ellsberg Paradox)
- Limited information-processing capabilities
- Availability heuristic
- Status-quo bias (e.g., endowment effect, regret avoidance, cognitive dissonance)
- ...

There is a fast growing literature on **“behavioral game theory”** ⁽¹⁾

(1) See e.g., Camerer, C.F. (2003) *Behavioral Game Theory: Experiments in Strategic Interaction*, Princeton University Press, Princeton, NJ. For more on behavioral decision making, see Kahnemann, D., Tversky, A. (2000) *Choices, Values, and Frames*, Cambridge University Press, Cambridge, UK.

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UNDERSTANDING RATIONALITY

Consider the following normal-form game (for which we just provide the payoff matrix):

		<u>Player 2</u>	
		L	R
<u>Player 1</u>	U	(4 ,4)	(-1000,3.9)
	D	(3.9, 3.9)	(4 ,3.8)

Player 2 has a **strictly dominant strategy**; his **dominated strategy can thus be eliminated**. This leads to a unique prediction of the outcome (U,L) in this game.

Note though that player 1 has to be **absolutely sure** of the rationality of player 2!

PURE-STRATEGY NASH EQUILIBRIUM

Definition: For any normal-form game $\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$ a **pure-strategy Nash equilibrium** is a strategy profile $a^* = (a_i^*, a_{-i}^*)$, such that for every $i \in N$:

$$a_i^* \in \arg \max_{a_i \in A_i} u_i(a_i, a_{-i}^*)$$

In other words,

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad \forall a_i \in A_i, i \in N$$

or equivalently

$$a_i^* \in B_i(a_{-i}^*) \quad \forall i \in N$$

Examples: The Prisoner's Dilemma game has a unique pure-strategy Nash equilibrium (NE), in the Matching Penny game such an equilibrium does not exist

MIXED-STRATEGY NASH EQUILIBRIUM

To increase the predictive power in games such as Matching Pennies, we extend the definition of Nash Equilibrium to include mixed strategy profiles of the form $\sigma \in \Delta(A_1) \times \dots \times \Delta(A_n)$.

Definition: For any normal-form game $\Gamma_N = \{N, \{\Delta(A_i)\}, \{u_i(\cdot)\}\}$ a **mixed-strategy Nash equilibrium** is a strategy profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$, such that for every $i \in N$:

$$\sigma_i^* \in \arg \max_{\sigma_i \in \Delta(A_i)} u_i(\sigma_i, \sigma_{-i}^*)$$

where

$$u_i(\sigma) = u_i(\sigma_i, \sigma_{-i}) = \sum_{a \in A} [\sigma_1(a_1) \cdot \dots \cdot \sigma_n(a_n)] u_i(a)$$

MATCHING PENNIES

In the Prisoner's Dilemma game the assumptions of common knowledge and rationality were enough to generate a unique prediction about the outcome, the reason being that each player found it strictly dominant to confess.

As we see below, rationality and common knowledge, are generally not enough to generate a prediction for the outcome of a normal-form game.

Example: **Matching Pennies**

In a game of Matching Pennies, Ann and Bert, show each other a penny with either heads (H) or tails (T) up. If they choose the same side of the penny, Ann gets both pennies, otherwise Bert gets them.

(Note that this is a **zero-sum game**, as are most games people play for leisure.)

MATCHING PENNIES (Cont'd)

Normal-Form Representation

$$N = \{Ann, Bert\}$$

$$A_i = \{H, T\}, i \in N$$

$u_i(\cdot)$ defined by the following **payoff matrix**

		<u>Bert</u>	
		H	T
<u>Ann</u>	H	(1,-1)	(-1,1)
	T	(-1,1)	(1,-1)

Question: **What is the outcome of this game?**

MATCHING PENNIES (Cont'd)

Consider each player's **best-response correspondence**

$$B_i(a_{-i}) = \arg \max_{a_i \in A_i} u(a_i, a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(\hat{a}_i, a_{-i}), \forall \hat{a}_i \in A_i\}$$

$$B_{Ann}(H) = H$$

$$B_{Ann}(T) = T$$

$$B_{Bert}(H) = T$$

$$B_{Bert}(T) = H$$

		<u>Bert</u>	
		H	T
<u>Ann</u>	H	(1 , -1) → (-1, 1)	
	T	(-1, 1) ← (1, -1)	

Result: The players' best-response correspondences do not "intersect."

MATCHING PENNIES (Cont'd)

Let us try to find a mixed-strategy Nash equilibrium in the Matching Pennies game. For simplicity set Ann = Player 1 and Bert = Player 2, so that $N = \{1, 2\}$

The player's mixed-strategy spaces are

$$\Delta(A_i) = \{(\sigma_i(H), \sigma_i(T)) : \sigma_i(H), \sigma_i(T) \geq 0 \text{ and } \sigma_i(H) + \sigma_i(T) = 1\}$$

Without loss of generality, let $\sigma_1(H) = p$ and $\sigma_2(H) = q$.

Then

$$\begin{aligned} u_1(\sigma) &= p(qu_1(H, H) + (1-q)u_1(H, T)) + (1-p)(qu_1(T, H) + (1-q)u_1(T, T)) \\ &= p(q - (1-q)) + (1-p)(-q + (1-q)) \\ &= p(2q - 1) + (1-p)(1 - 2q) \\ &= (1 - 2p)(2q - 1) \\ &= -u_2(\sigma) \end{aligned}$$

MATCHING PENNIES (Cont'd)

This is a **linear optimization problem** for each player. Note that player 1 has only control over p and player 2 has only control over q .

Player 1 can make player 2 **indifferent** about any of his strategies by choosing $p = .5$ i.e., $\hat{\sigma}_1 = (p, 1-p) = (0.5, 0.5)$ and thus

$$\Delta(A_2) = \arg \max_{\sigma_2 \in \Delta(A_2)} u_2(\sigma_2, \hat{\sigma}_1)$$

If player 1 chooses a different strategy, player 2 is not indifferent and strictly prefers to play either $q = 0$ (for $p > .5$) or $q = 1$ (for $p < .5$).

On the other hand, if player 2 chooses anything other than $q = .5$, player 1 is not indifferent about her actions and will strictly prefer to play a pure strategy.

As a result, $\sigma^* = (\sigma_1^*, \sigma_2^*)$ with $\sigma_i^* = (.5, .5)$ is the **unique mixed-strategy Nash equilibrium** of the Matching Pennies game.

ROLE OF INDIFFERENCE

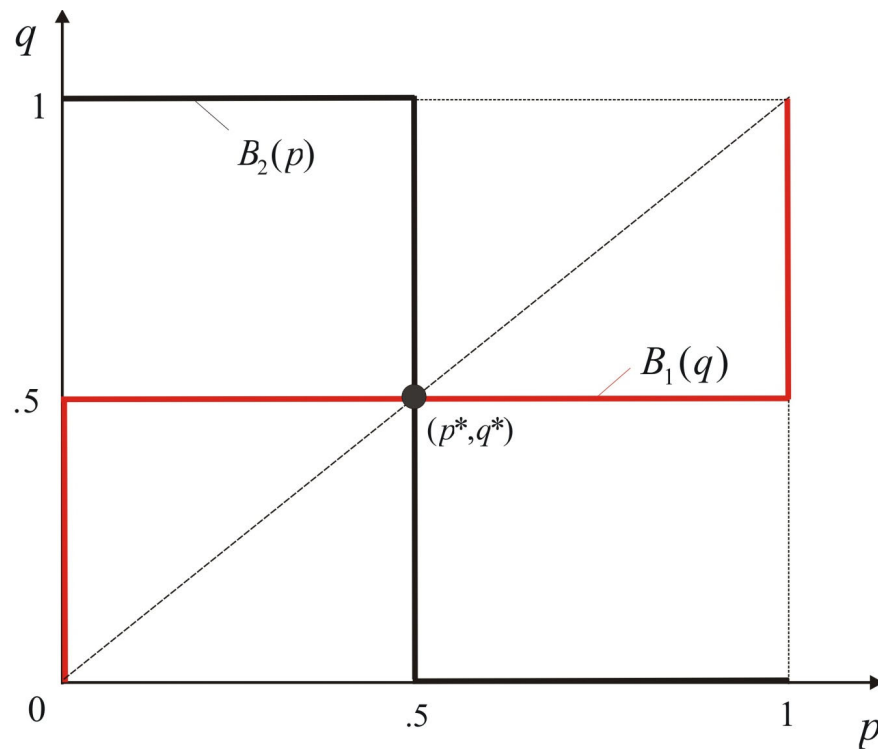
We emphasize the role that the players' indifference played in determining the NE in the Matching Pennies game. The following assumption is maintained for the rest of the course.

Assumption: provided indifference between two or more actions in a player's (mixed-strategy) best-response correspondence, this player will select an action that is part of a (mixed-strategy) Nash equilibrium.⁽¹⁾

(1) In other words, if a player is indifferent between a strategy that is not a part of a particular Nash equilibrium and a strategy that is part of a particular Nash equilibrium, we assume that this player plays the strategy that is part of the equilibrium (i.e., he makes the equilibrium happen).

MATCHING PENNIES (Cont'd)

It is possible to **graph the players' best-response correspondences**. The unique intersection is at $(p^*, q^*) = (.5, .5)$.



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AGENDA

What is Game Theory?

Building Blocks and Key Assumptions

Market Structure & Strategy Analysis

- Cournot Quantity Competition
- Bertrand Price Competition

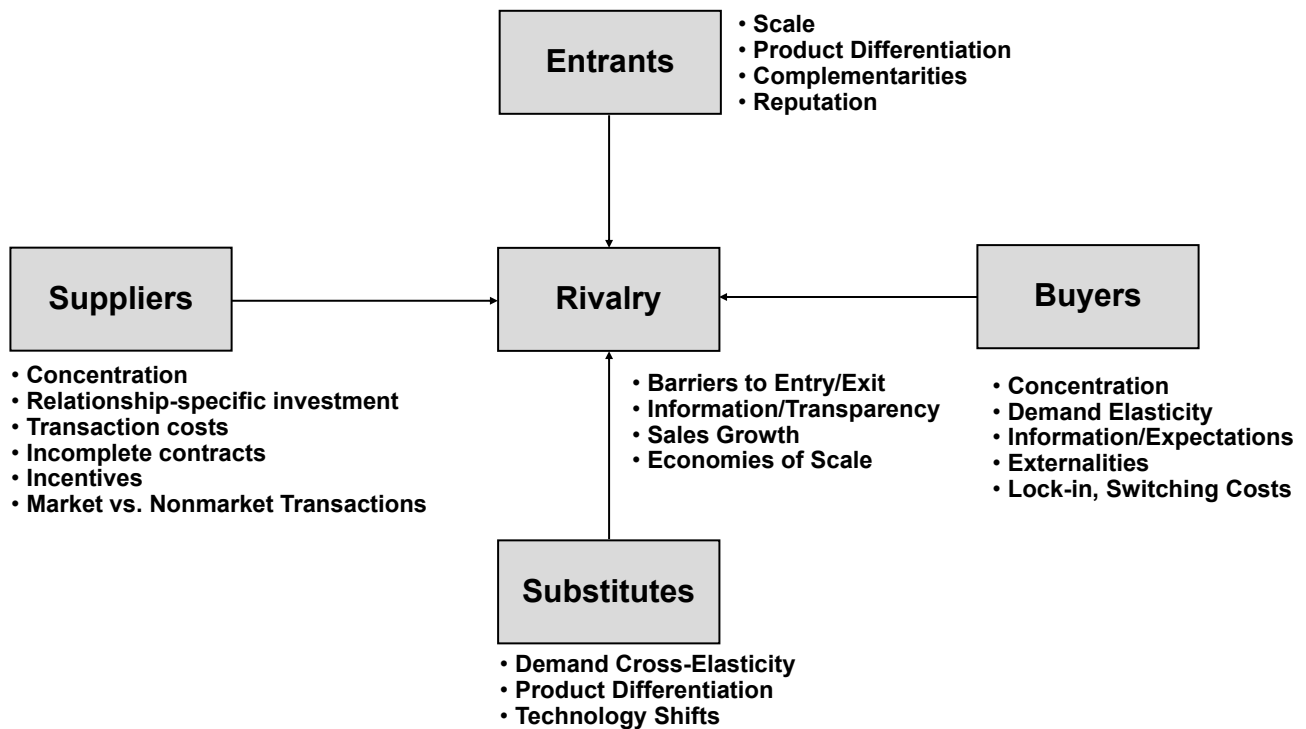
Key Concepts to Remember

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PORTER'S FIVE FORCES

... and what influences them



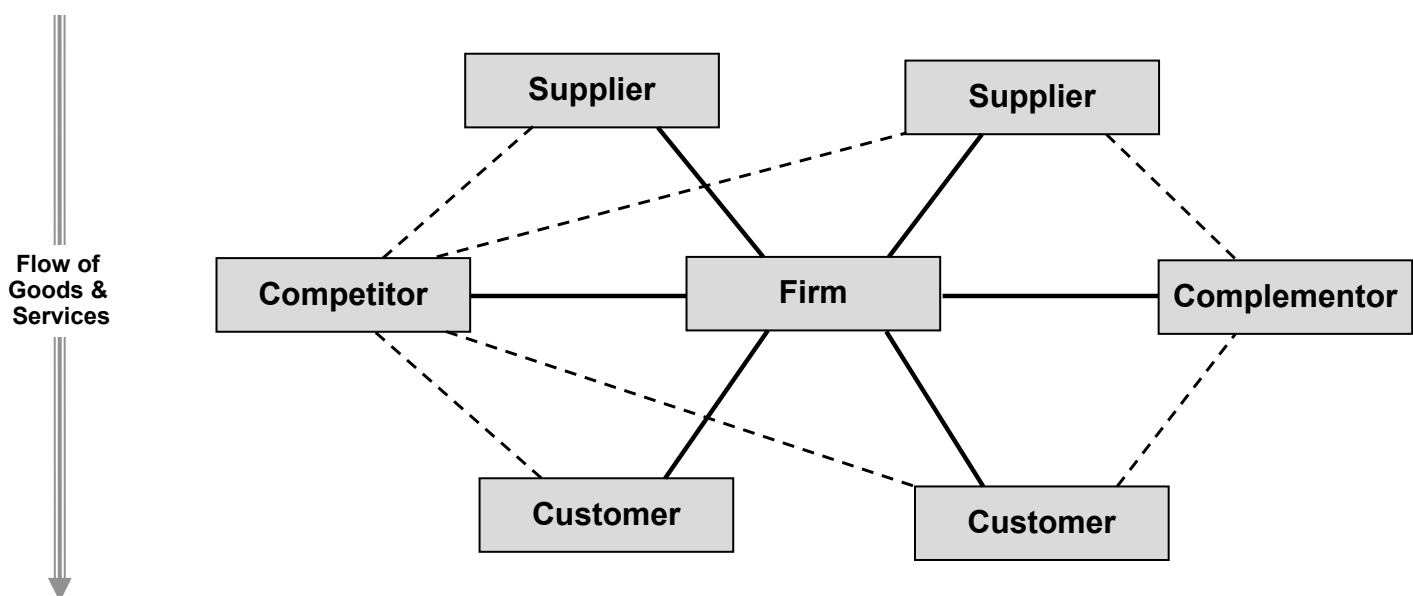
Note: For the original presentation of the Five-Forces Model, see Porter, M.E. (1980) *Competitive Strategy*, Free Press, New York, NY.

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BRANDENBURGER AND NALEBUFF'S VALUE NET

The Firm and Its Network of Transaction Relationships

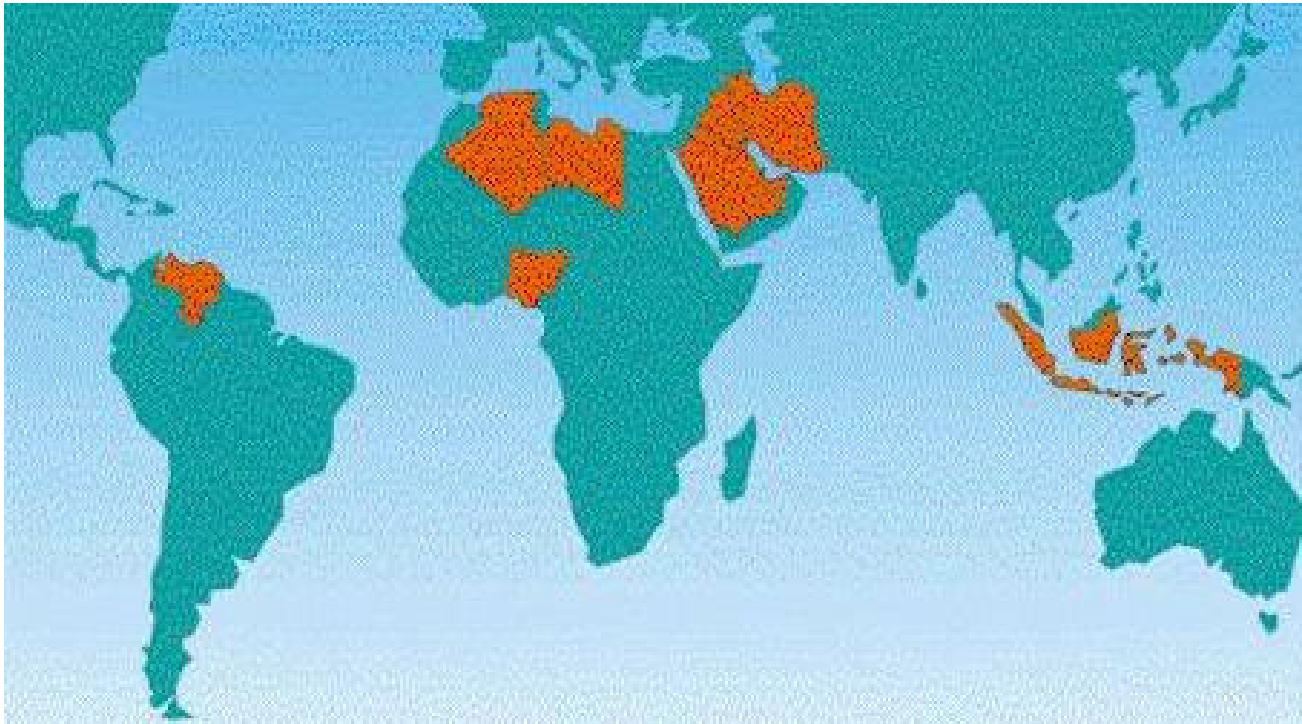


Note that the firm and its competitors/complementors can have relationships in different markets at the same time ("multimarket contact")

Note: For the original presentation of the value net, see Brandenburger, A.M., Nalebuff, B.J. (1995) "The Right Game: Use Game Theory to Shape Strategy," *Harvard Business Review*, Vol. 73, No. 4, pp. 57—71. The presentation here is close to the one in McAfee, R.P. (2002) *Competitive Solutions: The Strategist's Toolkit*, Princeton University Press, Princeton, NJ, p. 25.

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CHOOSING QUANTITIES: COURNOT DUOPOLY

Consider **two firms**, 1 and 2, choosing their production outputs q_1 and q_2 simultaneously. Each firm has a unit production cost of c (with $0 < c < 1$).

- The market (inverse) demand is given by $p(q_1, q_2) = 1 - q_1 - q_2$

Question. Determine a Nash equilibrium of this game.

Solution.

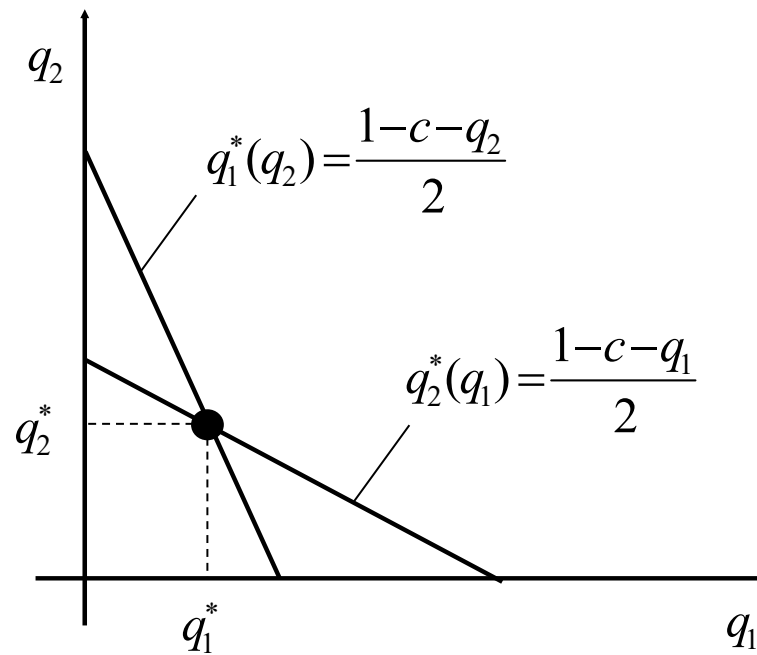
Firm i 's profit is $\Pi_i(q_1, q_2) = (p(q_1, q_2) - c)q_i = (1 - c - q_1 - q_2)q_i$

- Its optimality condition is $\frac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = 1 - c - 2q_i - q_j \stackrel{!}{=} 0$

- Its **best-response** to q_j is therefore $q_i^*(q_j) = \frac{1 - c - q_j}{2}$

- Symmetry implies that at the Nash equilibrium $q_i^* = \frac{1 - c - q_i^*}{2}$

COURNOT DUOPOLY (Cont'd)



Unique Nash Equilibrium:

$$q_1^* = q_2^* = \frac{1-c}{3}$$

COURNOT OLIGOPOLY Generalization of Previous Example

Consider the **symmetric linear model** and perfect substitutability, where

$$p_i(q) = p(Q) = a - bQ$$

with $Q = q_1 + \dots + q_n$, $a, b > 0$, and

$$C_i(q_i) = cq_i$$

where $c \in (0, a)$.

- We find that at the unique NE **each firm produces** $q_i^* = \frac{a-c}{(n+1)b}$
- The **total supply** in a Cournot NE is thus $Q^* = nq_i^* = \frac{a-c}{(1+1/n)b}$
- The **Cournot NE price** is $p^* = a - bQ^* = \frac{a/n+c}{1+1/n}$
- The **industry Cournot profits** are $\Pi_T^* = (p^* - c)Q^* = \frac{n}{b} \left(\frac{a-c}{n+1} \right)^2$

COURNOT OLIGOPOLY (Cont'd)

Comparison with Perfect Competition

The market power of each firm can be measured using the **Lerner index** L_i ,⁽¹⁾ which corresponds to the inverse of the **own demand elasticity** ε_i in equilibrium

$$\varepsilon_i(n) = - \left. \frac{\partial \log q_i}{\partial \log p_i} \right|_* = - \left. \frac{p_i}{q_i} \frac{dq_i}{dp_i} \right|_* = - \frac{p^*}{q_i^*} \left(- \frac{1}{b} \right) = \frac{a + nc}{a - c}$$

Let p^c and Q^c denote the price and total output in a symmetric equilibrium under perfect competition. Under perfect competition we have that necessarily $p^c = c$ and Q^c therefore solves $c = a - bQ^c$

We have that

$$\lim_{n \rightarrow \infty} Q^*(n) = \lim_{n \rightarrow \infty} \frac{a - c}{(1 + 1/n)b} = \frac{a - c}{b} = Q^c$$

Note also:

$$\lim_{n \rightarrow \infty} L_i(n) = \lim_{n \rightarrow \infty} \frac{a - c}{a + nc} = 0$$

(1) Lerner, A.P. (1934) "The Concept of Monopoly and the Measurement of Monopoly Power," *Review of Economic Studies*, Vol. 1, No. 3, pp. 157—175.

STACKELBERG QUANTITY LEADERSHIP GAME

Consider firms in industries producing goods that are perfect or at least close substitutes

- Duopoly
- Oligopoly with one dominant firm
- Dominant firm and 'competitive fringe'

Examples

- OPEC or Saudi Arabia
- Certain airlines or particular hubs

STACKELBERG GAME: COURNOT WITH LEADER

Suppose there are **two symmetric firms**. Firm 1 is the leader and gets to choose its quantity at $t = 0$. Firm 2 is the **follower** and chooses its quantity at $t = 1$.

Any **SPNE** (the “**Stackelberg Equilibrium**”) can be found using backward induction, i.e., we start at $t = 1$. Firm 2 solves

$$q_2^*(q_1) = \arg \max_{\hat{q}_2 \geq 0} \{(a - c - b(q_1 + \hat{q}_2))\hat{q}_2\}$$

so that the **best-response** for firm 2 given the leader's output choice q_1 becomes

$$q_2^*(q_1) = \frac{a - c}{2b} - \frac{q_1}{2}$$

STACKELBERG GAME (Cont'd)

Let us now examine the **leader's optimal policy at $t = 0$** . Firm 1's **residual demand** is given by

$$\hat{p}(q_1) = a - b(q_1 + q_2^*(q_1)) = \frac{a + c - bq_1}{2}$$

The **elasticity of firm 1's residual demand curve** is

$$\hat{\varepsilon}_1 = -\frac{\hat{p}_1(q_1)}{q_1} \frac{dq_1}{d\hat{p}_1} = -\frac{a + c - bq_1}{2q_1} \frac{1}{\frac{d\hat{p}_1(q_1)}{dq_1}} = \frac{a + c}{bq_1} - 1 > \frac{a}{bq_1} - 1$$

↖ Demand elasticity in the absence of followers

Firm 1 maximizes its profits with respect to residual demand,

$$q_1^* = \arg \max_{\hat{q}_1 \geq 0} \{(a - c - b\hat{q}_1)\hat{q}_1 / n\} = \arg \max_{\hat{q}_1 \geq 0} \{(a - c - b\hat{q}_1)\hat{q}_1\} = \frac{a - c}{2b} = q^m$$

↖ Monopoly quantity!

Hence, the follower produces

$$q_2^* = \frac{a - c}{2b} - \frac{q^m}{2} = \frac{a - c}{4b}$$

STACKELBERG GAME (Cont'd)

Total Stackelberg equilibrium output of all firms is therefore

$$Q^* = q_1^* + q_2^*(q_1^*) = \left(1 - \frac{1}{4}\right) \frac{a-c}{b}$$

and **equilibrium market price** is

$$p^* = a - bQ^* = c + \frac{a-c}{4}$$

The **leader's equilibrium profit** is

$$\Pi_1^* = \frac{(a-c)^2}{8b}$$

while the follower obtains in equilibrium

$$\Pi_2^* = \frac{(a-c)^2}{16b}$$

BERTRAND DUOPOLY

Consider two firms selling a homogeneous product at a unit cost $c_i \in [0,1], i \in N = \{1,2\}$

- The firms simultaneously set their prices $p_i \in [0, \infty) = A_i$
- Let the total number of consumers be normalized to one. All consumers buy from the cheaper firm and randomize evenly between the two firms if their prices are equal
- The value of the consumers' (common) outside option is zero; their (net) value from the product if they buy from firm i is Y (Assume that $Y > c_i$)

Question: Determine the NE of this simultaneous-move game.

Answer: 1. Determine the firms' **payoff functions**, $u_i(\cdot)$:

$$u_i(p_i, p_{-i}) = \begin{cases} p_i - c_i, & p_i < p_{-i}, p_i \leq Y, \\ (p_i - c_i)/2, & p_i = p_{-i}, p_i \leq Y, \\ 0, & \text{otherwise.} \end{cases}$$

BERTRAND DUOPOLY (Cont'd)

2. Determine the firms' best-response correspondences

- Assume that $c_1 < c_2$
 - Find the set of strategies that *survive iterated deletion of strategies which are never a best response*:
 - For player i , the strategies $p_i < c_i$ and $p_i > Y$ are dominated by $p_i = Y$
 - All strategies $p_i \in [c_i, Y]$ *could* be rationalizable $\rightarrow$ not very useful
 - Find best-response correspondences
 - Start with $p_2 \in [c_2, Y]$: then $p_1 \in [p_2, Y]$ is **strictly dominated** by any $p_1 \in (c_1, p_2)$
 - Player 1's payoffs are strictly increasing in $p_1 \in (c_1, p_2)$. Thus, **there is no best-response for player 1**, since the payoff from any particular strategy in (c_1, p_2) can be strictly improved upon
- However, if **increments are finite**, of arbitrarily small size $\varepsilon > 0$, then⁽¹⁾

$$B_i(p_{-i}) = \begin{cases} Y, & p_{-i} > \max\{c_i, Y + \varepsilon\}, \\ \max\{c_i, p_{-i} - \varepsilon\}, & p_{-i} \in (c_i, Y + \varepsilon], \\ \{c_i + k\varepsilon\}_{k=0}^{\infty}, & p_{-i} = c_i, \\ \{p_{-i} + k\varepsilon\}_{k=1}^{\infty}, & p_{-i} \leq c_i. \end{cases}$$

MGT-621-Spring-2026-TAW (1) We approximate the Bertrand game at this point by a series of discrete games $\{\Gamma_N(\varepsilon_n)\}_{n=1}^{\infty}$ with $\varepsilon_n \rightarrow 0+$, as $n \rightarrow \infty$.

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BERTRAND DUOPOLY (Cont'd)

3. Find the intersection of the best-response correspondences

Continuum of Nash equilibria:

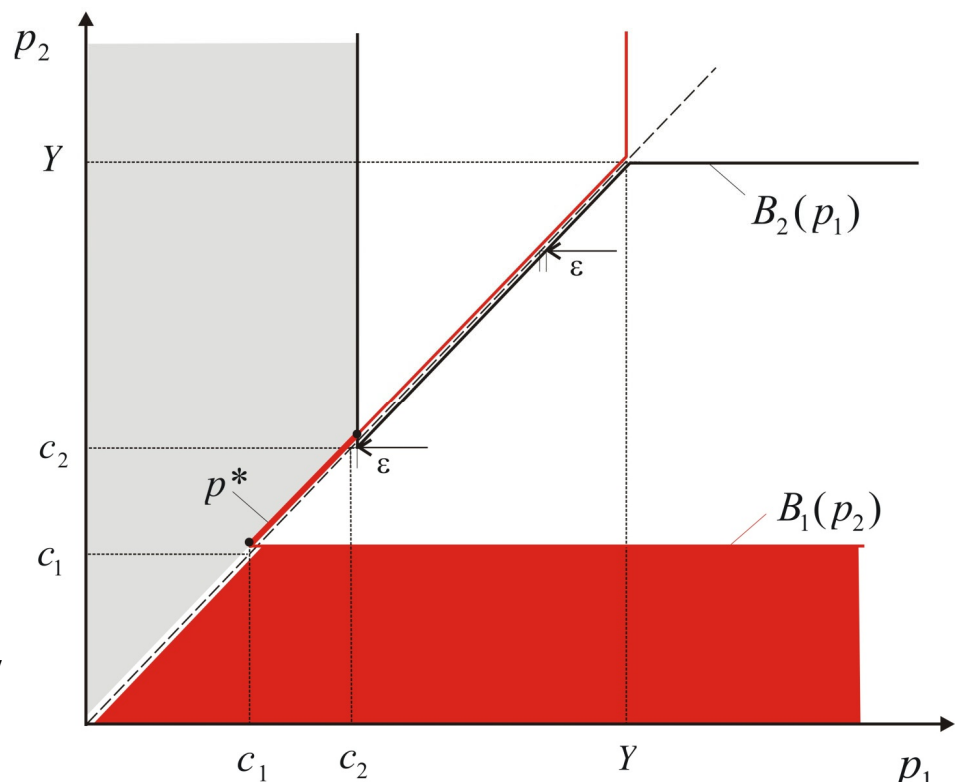
$$p^* = (p_1^*, p_2^*)$$

with

$$p_1^* \in [c_1, c_2]$$

$$p_2^* = p_1^* + \varepsilon$$

Note that all NE involve at least one player playing a weakly dominated strategy



BERTRAND DUOPOLY (Cont'd)

Additional Notes

- In equilibrium with $c_1 < c_2$, firm 2 plays a **weakly dominated strategy**
- A **tiebreaking rule** that assigns all profits to firm 1 in case of equal prices guarantees a set of NE p^* for $\varepsilon \rightarrow 0+$:

“In a Bertrand equilibrium, firms charge a price between the first-
the second-most efficient firm’s costs.”⁽¹⁾

It is possible that all firms play a weakly dominated strategy in equilibrium.

DIFFERENTIATION SOFTENS PRICE COMPETITION

Generalization: Imperfect Substitutes

Given: Demands for products of firm 1 and firm 2: $q_1(p_1, p_2)$ and $q_2(p_1, p_2)$
[& the firms' cost functions: $C_1(q_1)$ and $C_2(q_2)$]

$$\max_{p_2} \{ p_2 q_2(p_1, p_2) - C_2(q_2(p_1, p_2)) \}$$

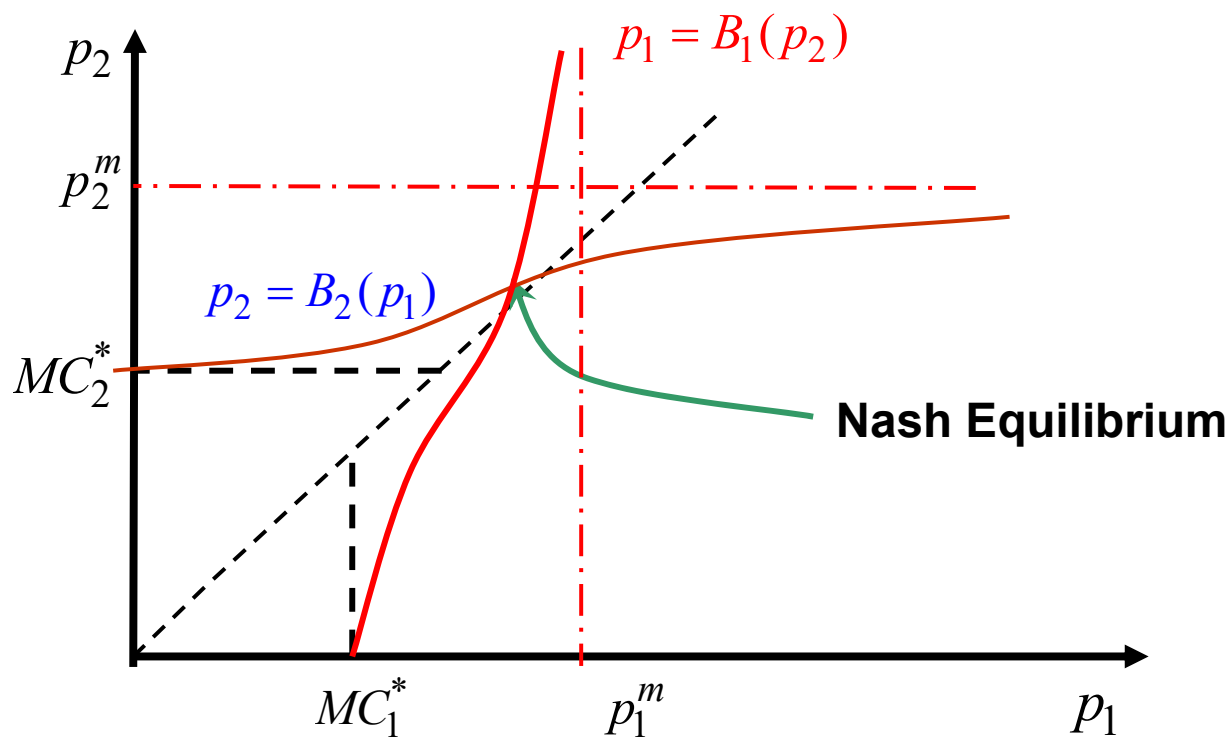
$$FOC: \quad MR_2(q_2) = MC_2(q_2) \quad \Rightarrow \quad p_2 = B_2(p_1)$$

$$\max_{p_1} \{ p_1 q_1(p_1, p_2) - C_1(q_1(p_1, p_2)) \}$$

$$FOC: \quad MR_1(q_1) = MC_1(q_1) \quad \Rightarrow \quad p_1 = B_1(p_2)$$

$$\left[p_1 - \frac{dC_1(q_1)}{dq_1} \right] = - \frac{q_1(p_1, p_2)}{\frac{\partial q_1(p_1, p_2)}{\partial p_1}}$$

BERTRAND WITH IMPERFECT SUBSTITUTES (Cont'd)



AGENDA

What is Game Theory?

Building Blocks and Key Assumptions

Market Structure & Strategy Analysis

- Cournot Quantity Competition
- Bertrand Price Competition

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Predictive Power**
- **Payoff Matrix**
- **Pure/Mixed Strategy**
- **Dominant Strategy**
- **Best-Response**
- **Nash Equilibrium**
- **Cournot and Bertrand Game**
- **Stackelberg Sequential-Move Games**

MGT 621 – MICROECONOMICS

Thomas A. Weber

7. Externalities & Regulation

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

What are externalities?

Example: Production Externalities

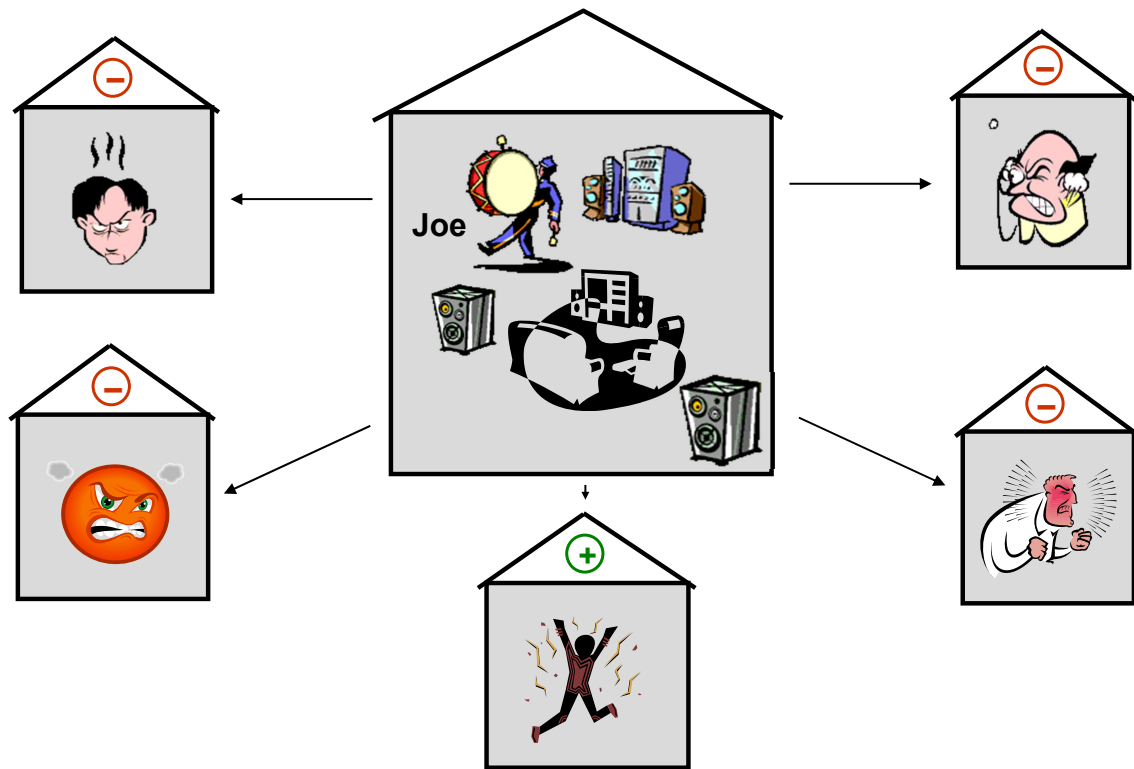
Some Regulatory Options

Uncertainty Matters: Prices vs. Quantities

Key Concepts to Remember

WHAT ARE EXTERNALITIES?

Example



WHAT ARE EXTERNALITIES? (Cont'd)

Definition. An **externality** exists whenever the well-being (utility) of a consumer or the production possibility set of a firm are *directly* affected by the action of another agent in the economy.

Externalities can be “**positive**” or “**negative**.”

WHAT ARE EXTERNALITIES? (Cont'd)

Some Examples

- **Drivers' cars release pollutants that deteriorate the air quality**
- **Cigarette smoke increases the probability of lung cancer for smokers and others**
- **Chemical plant releases wastes in river; fishing industry becomes less productive**
- **Fish caught by one fishing boat cannot be caught by another fishing boat**
- **High-tech patents lead to public disclosure of inventions that can be used by other firms (be they complementors or competitors)**

AGENDA

What are externalities?

Example: Production Externalities

Some Regulatory Options

Uncertainty Matters: Prices vs. Quantities

Key Concepts to Remember

EXAMPLE: PRODUCTION EXTERNALITIES

One firm's decisions may have a direct impact on another firm's payoff.

- Our discussion of game theory shows (see, e.g., the Prisoners' Dilemma) that, in general, if each firm individually maximizes its profits, then the sum of both firms' profits may **not** be maximal.

Consider the following situation for two firms, 1 and 2:

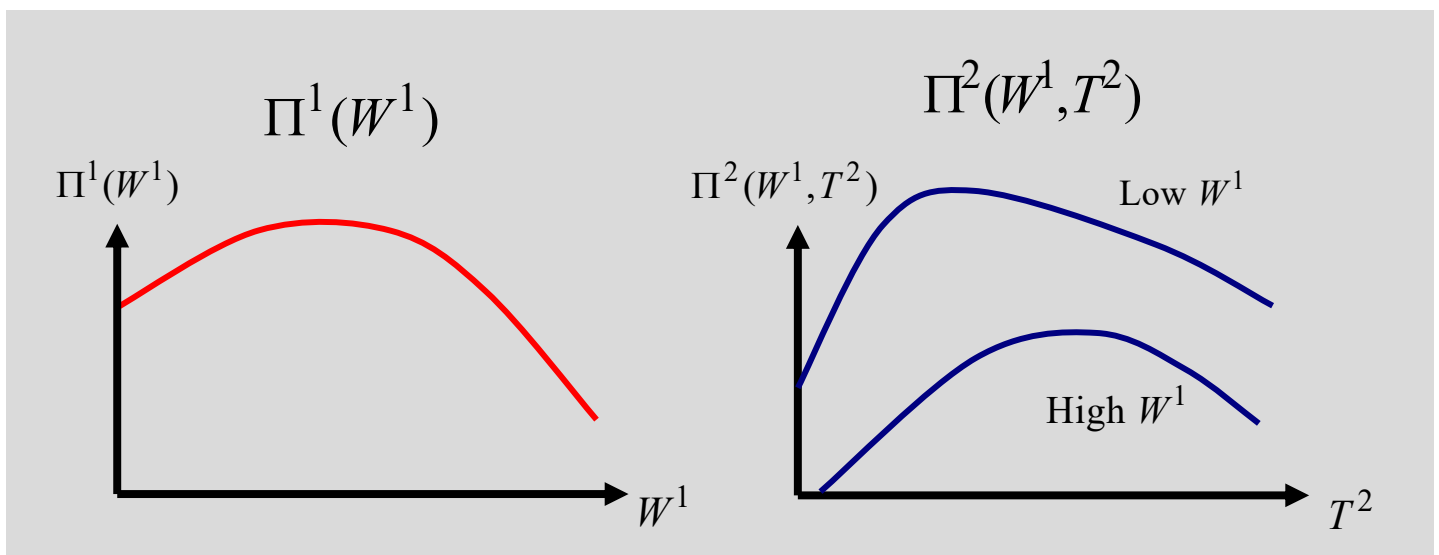
- Firm 1's production produces wastewater, resulting in an externality for a downstream fishing company
- Firm 2, impacted by firm 1's waste production could reduce harmful effects, say by treatment of water, but at a cost

PRODUCTION EXTERNALITIES (Cont'd)

Firm 1 produces a nonnegative amount of waste, W^1

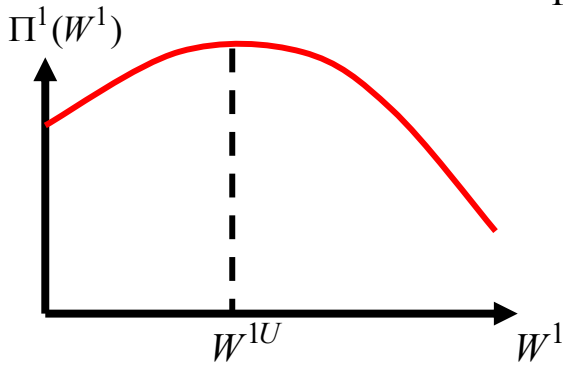
Firm 2, negatively impacted by firm 1's waste production, could reduce the harmful effects through treatment T^2 , which comes at a cost.

Payoff Functions:

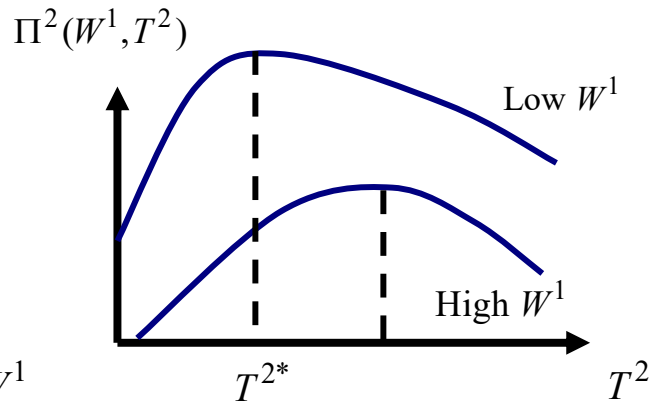


PRODUCTION EXTERNALITIES (Cont'd)

$$\frac{d\Pi^1(W^1)}{dW^1} = 0$$



$$\frac{\partial \Pi^2(W^1, T^2)}{\partial T^2} = 0$$



Optimal treatment T^{2*}
Depends on W^1

PRODUCTION EXTERNALITIES (Cont'd)

Efficient outcome maximizes total profits,

$$\max_{W^1, T^2 \geq 0} \left\{ \Pi^1(W^1) + \Pi^2(W^1, T^2) \right\}$$

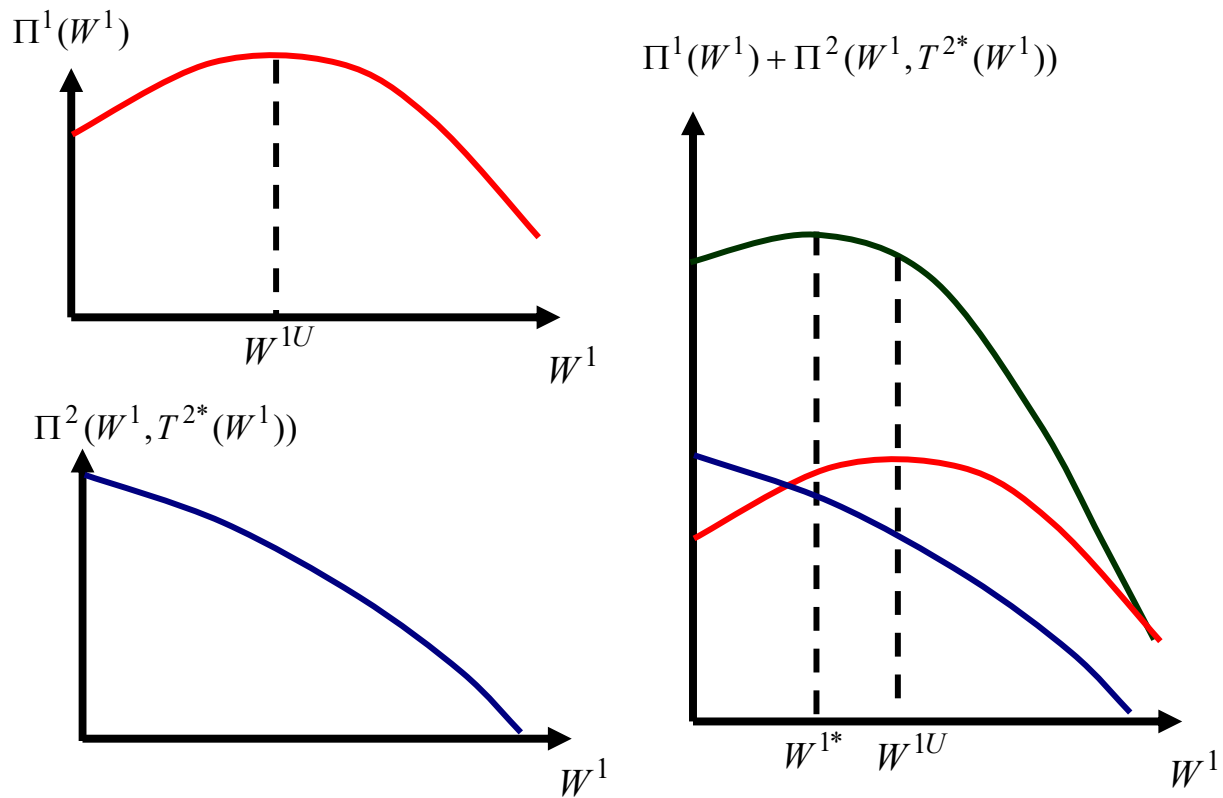
Firm 1: optimality condition for socially optimal W^1 differs from the individual optimality condition,

$$\frac{d\Pi^1(W^1)}{dW^1} + \frac{\partial \Pi^2(W^1, T^2)}{\partial W^1} = 0$$

Firm 2: optimality condition for socially optimal T^2 is same as individually optimal, for any given W^1

$$\frac{\partial \Pi^2(W^1, T^2)}{\partial T^2} = 0$$

PRODUCTION EXTERNALITIES (Cont'd)



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PRODUCTION EXTERNALITIES (Cont'd)

Define Externalities in Terms of Harm

Define harm to firm 2 as a function of W^1 (corresponds to the externality that firm 1 exerts on firm 2)

$$h(W^1) = \Pi^2(0, T^{2*}(0)) - \Pi^2(W^1, T^{2*}(W^1))$$

Choose W^1 to maximize profit of firm 1, minus harm:

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - h(W^1) \right\}$$

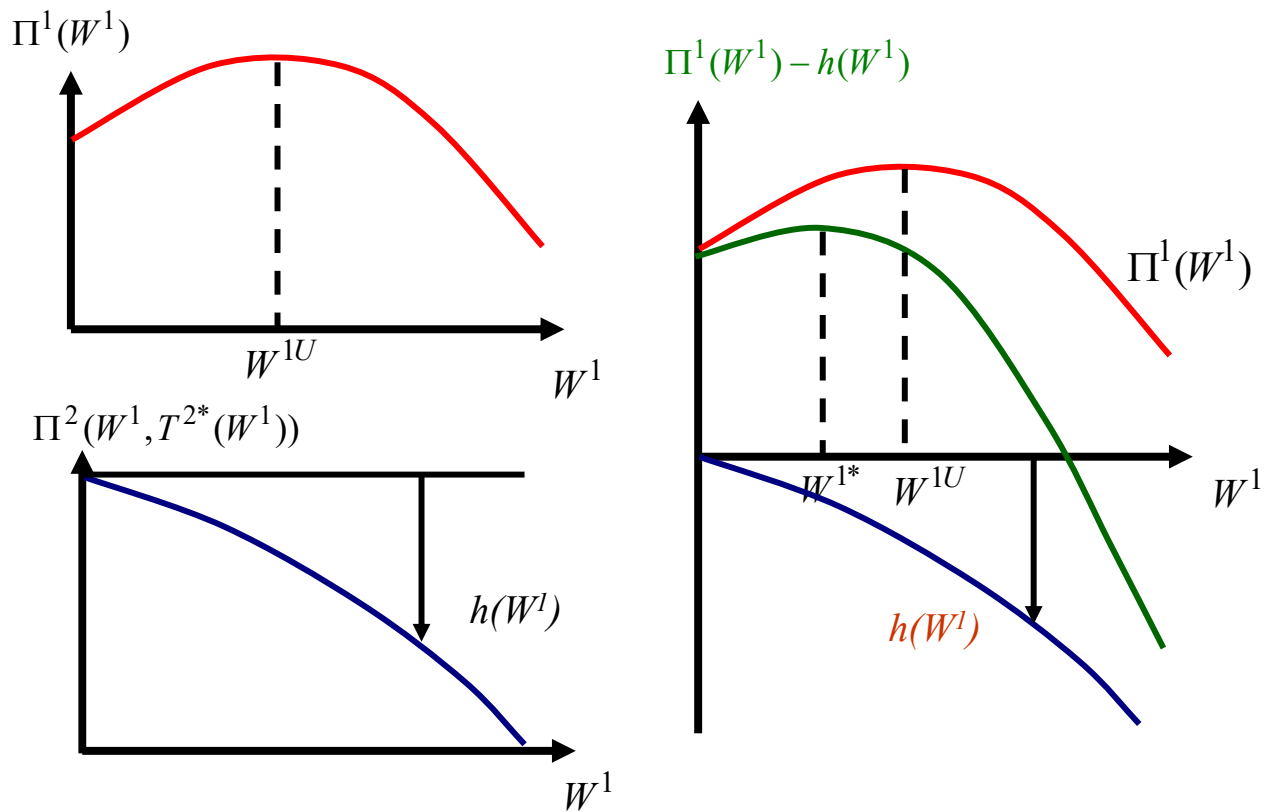
Choose T^2 to maximize profit of firm 2:

$$\max_{T^2 \geq 0} \Pi^2(W^{1*}, T^2)$$

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PRODUCTION EXTERNALITIES (Cont'd)



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PRODUCTION EXTERNALITIES (Cont'd)

Gives FOC for T^2 identical to optimal

$$\frac{\partial \Pi^2(W^1, T^2)}{\partial T^2} = 0$$

Gives FOC for W^1 identical to optimal

$$\begin{aligned} \frac{d\Pi^1(W^1)}{dW^1} - \frac{\partial h(W^1)}{\partial W^1} &= 0 \\ \frac{d\Pi^1(W^1)}{dW^1} - \left\{ -\frac{\partial \Pi^2(W^1, T^{2*})}{\partial W^1} \right\} - \underbrace{\left\{ -\frac{\partial \Pi^2(W^1, T^{2*})}{\partial T^{2*}} \right\} \left\{ \frac{\partial T^{2*}}{\partial W^1} \right\}}_0 &= 0 \\ \frac{d\Pi^1(W^1)}{dW^1} + \frac{\partial \Pi^2(W^1, T^{2*})}{\partial W^1} &= 0 \end{aligned}$$

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AGENDA

What are externalities?

Example: Production Externalities

Some Regulatory Options

Uncertainty Matters: Prices vs. Quantities

Key Concepts to Remember

REMEDIES FOR MARKET FAILURE

Idea: How can we correct market to move back toward competitive norm?

- **“Polluter Pays” Principle**
- **Litigation to recover harms: “damages”**
- **Tax per unit on externality: “pollution tax”**
- **Marketable emissions rights**
 - **Create market for rights to produce the externality**
- **Regulation of emissions or other waste**
 - **Restriction against hazardous waste**
 - **Limits on emissions rate**
- **Assign property rights and allow negotiation (Coase Theorem)**

“POLLUTER PAYS” PRINCIPLE

Polluter may be required to pay a fee to the government, offsetting the entire damage. In that case, the polluter (firm 1) solves the problem

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - h(W^1) \right\}$$

If firm 1 maximizes its profit minus harm, it will choose optimal waste (W^1). No money paid to harmed firm (firm 2), who solves the following problem:

$$\max_{T^2 \geq 0} \Pi^2(W^{1*}, T^2)$$

If firm 2 is not compensated for damages, it will choose optimal treatment.

LITIGATION RECOVERS DAMAGES

Polluter may be required to pay fee to harmed firm equal to the damage. Then the polluter solves problem:

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - h(W^1) \right\}$$

If firm 1 maximizes its profit minus harm, it will choose optimal waste (W^1) for whatever is the level of T^2 .

Firm 1 has an incentive to select a socially efficient waste level.

LITIGATION RECOVERS DAMAGES

Damages paid to harmed firm, with damages determined for optimal level of T^2 . Then firm 2 chooses T^2 to solve problem:

$$\max_{T^2 \geq 0} \left\{ \Pi^2(W^1, T^2) + h(W^1) \right\}$$

Monetary damages – harm – do not depend on actual choice of T^2 , so $h(W^1)$ is a constant, from perspective of firm 2.

If firm 2 is compensated a fixed amount for damages, it will maximize its before-harm-payment profit and will choose optimal treatment.

$$h(W^1) = \Pi^2(0, T^{2*}(0)) - \Pi^2(W^1, T^{2*}(W^1))$$

LITIGATION RECOVERS DAMAGES (Cont'd)

Damages paid to harmed firm, with damages determined for actual level of T^2 . Then firm 2 solves the problem:

$$\max_{T^2 \geq 0} \left\{ \Pi^2(W^1, T^2) + h(W^1) \right\}$$

Monetary damages – harm – do depend on actual choice of T^2 , so $h(W^1)$ is not a constant, from perspective of firm 2, in this case.

$$h(W^1) = \Pi^2(0, T^2(0)) - \Pi^2(W^1, T^2)$$

$$\max \underbrace{\left\{ \Pi^2(W^1, T^2) - \Pi^2(W^1, T^2) \right\}}_0 + \Pi^2(0, T^2(0))$$

Profit after damage payment is independent of T^2 ; firm 2 has no incentive to choose optimal treatment.

POLLUTION TAX

The polluter may be required to pay a fee to government equal to a fixed amount t per unit of pollution. Per-unit amount is set equal to the marginal harm. Then polluter solves problem:

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - t W^1 \right\}$$

Firm 1, maximizing its own after-tax profit, leads to efficient level of waste ($W^1 = W^{1*}$) :

$$t = \left. \frac{\partial h(W^1)}{\partial W^1} \right|_{W^1 = W^{1*}}$$

$$\frac{d\Pi^1(W^1)}{dW^1} = \frac{\partial h(W^{1*})}{\partial W^1}$$

MARKETABLE EMISSIONS RIGHTS

Government determines optimal total waste, and issues emissions rights. The number of rights equal to optimal waste. Rights can be bought and sold. Market clearing price will equal marginal cost of waste reduction. Thus, if the optimal total waste is produced, then the price will be equal to marginal harm,

$$p_{pr} = \frac{\partial h(W^1)}{\partial W^1}$$

Firm 1's optimization problem:

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - p_{pr} W^1 \right\}$$

Profit maximization condition leads Firm 1, maximizing its own after-tax profit, to an efficient level of waste:

$$\frac{d\Pi^1(W^1)}{dW^1} = p_{pr} = \frac{\partial h(W^1)}{\partial W^1}$$

DIRECT REGULATIONS OF EMISSIONS

Government sets maximum allowable waste regulation, after determining the solution of the problem:

$$\max_{W^1 \geq 0} \left\{ \Pi^1(W^1) - h(W^1) \right\}$$

Firm required to meet regulation or else face a penalty higher than the cost of meeting the regulation. Incentive to just meet the regulation.

PROPERTY RIGHTS AND RENEGOTIATION

Generally applicable only with a very small number of firms

General approach

- **Assign property rights to either party**
 - **Right to pollute or Right to no pollution**
- **Allow negotiation**
- **Efficient outcome either way**
- **Distribution of profits differs**

Coase Theorem

PROPERTY RIGHTS AND RENEGOTIATION

Give property rights to either (→ Case 1 and 2)

Case 1. Assume give right to polluter for some high pollution level. Then impacted firm will pay to firm 1 some amount of money, B, to reduce pollution.

$$\Pi^1(W^{1*}) + \Pi^2(W^{1*}, T^{2*}) > \Pi^1(W^{1U}) + \Pi^2(W^{1U}, T^{2U})$$

$$\Pi^2(W^{1*}, T^{2*}) - \Pi^2(W^{1U}, T^{2U}) > \Pi^1(W^{1U}) - \Pi^1(W^{1*}) > 0$$

Then B can be chosen such that

$$\Pi^2(W^{1*}, T^{2*}) - \Pi^2(W^{1U}, T^{2U}) > B > \Pi^1(W^{1U}) - \Pi^1(W^{1*}) > 0$$

whence

$$\Pi^2(W^{1*}, T^{2*}) - \Pi^2(W^{1U}, T^{2U}) - B > 0 \quad \text{Firm 2 increases profit}$$

$$\Pi^1(W^{1*}) - \Pi^1(W^{1U}) + B > 0 \quad \text{Firm 1 increases profit}$$

PROPERTY RIGHTS AND RENEGOTIATION

Case 2. Assume give to affected firm right to face no pollution. Then polluting firm will pay firm 2 some amount of money, F, to allow pollution. (Assume no abatement, i.e., $T^2 = 0$, when there is no pollution)

$$\Pi^1(W^{1*}) + \Pi^2(W^{1*}, T^{2*}) > \Pi^1(0) + \Pi^2(0, 0)$$

$$0 > \Pi^2(W^{1*}, T^{2*}) - \Pi^2(0, 0) > \Pi^1(0) - \Pi^1(W^{1*})$$

Then F can be chosen such that

$$0 > \Pi^2(W^{1*}, T^{2*}) - \Pi^2(0, 0) > -F > \Pi^1(0) - \Pi^1(W^{1*})$$

whence

$$\Pi^2(W^{1*}, T^{2*}) - \Pi^2(0, 0) + F > 0 \quad \text{Firm 2 increases profit}$$

$$\Pi^1(W^{1*}) - \Pi^1(0) - F > 0 \quad \text{Firm 1 increases profit}$$

COASE THEOREM

Coase Theorem. If all parties can negotiate with each other costlessly and with perfect information, then bargaining will lead to an efficient outcome.

The outcome will be efficient, no matter how the initial property rights are determined.

A Caveat

If property rights are not firmly established, so that agents spend resources trying to reallocate property rights, then the final outcome, including the costs of reallocating property rights, will not be efficient.

AGENDA

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Example: Production Externalities

Some Regulatory Options

Uncertainty Matters: Prices vs. Quantities

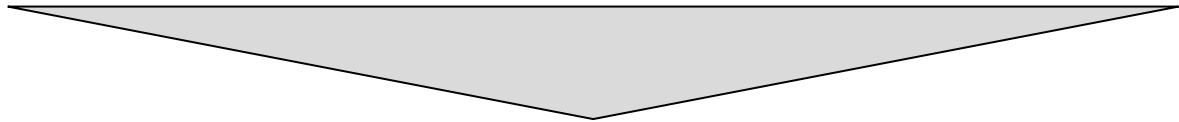
Key Concepts to Remember

PRACTICAL PROBLEM: REDUCING GLOBAL CARBON OUTPUT

To bound global warming to less than 2 degrees Celsius by 2050, worldwide carbon emissions need to be reduced by 50% in that timeframe (IPCC 2008).

Hence, need to provide incentives for ...

- **carbon abatement** by implementing an efficient carbon pricing policy
- **technological innovation** by encouraging the necessary investments



Question: Design a simple (i.e., implementable) regulatory scheme which *jointly* accounts for innovation and abatement.

Remark: For details on the material in the following slides, see Weber, T.A., Neuhoﬀ, K. (2010) “Carbon Markets and Technological Innovation,” *Journal of Environmental Economics and Management*, Vol. 60, No. 2, pp. 115—132; an earlier version is also available at <http://ssrn.com/abstract=1333244>

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THE MODEL Primitives

- Unit mass of firms, indexed by $\theta \in \Theta$, distributed on type space Θ such that

$$\mu = \int_{\Theta} \theta dF(\theta) < \infty \quad \text{and} \quad \sigma_{\theta}^2 = \int_{\Theta} (\theta - \mu)^2 dF(\theta) < \infty$$

- Each firm θ has business-as-usual (BAU) level of emissions $e_0(\theta) > 0$
- BAU emissions levels of all firms are subject to a common macroeconomic shock, modeled by the additive zero-mean noise $\tilde{\varepsilon}$ such that

$$\sigma_{\varepsilon}^2 = \int_{\Theta} \varepsilon^2 dG(\varepsilon) < \infty$$

- Expected total emissions:

$$e_0 = E\left[\int_{\Theta} (e_0(\theta) + \tilde{\varepsilon}) dF(\theta)\right] < \infty$$

THE MODEL (Cont'd) Timing

The actions take place in three periods, indexed by $t \in \{0,1,2\}$

- **Regulation Stage (t=0)**

- Regulator commits to a regulatory policy in the form of a cap-and-trade scheme with price controls, denoted by $R = (E, L, U)$

E : Emissions cap (e.g., set by number of issued emissions permits)

L : Price floor in market for emissions permits

U : Price cap in market for emissions permits

- **Innovation Stage (t=1)**

- Each firm θ decides about its innovation activity $y \geq 0$ at the cost of $K(y) = cy^2 / 2$ where $c \geq 0$ determines the slope of the marginal cost
- An innovation activity of y results in the realization ρ of a random cost improvement $\tilde{\rho}(y) > 0$, where $y = E[\max\{\tilde{\rho}(y), 1\} | y] - 1$

$\rho \leq 1$: Innovation unsuccessful – current practice is (weakly) better

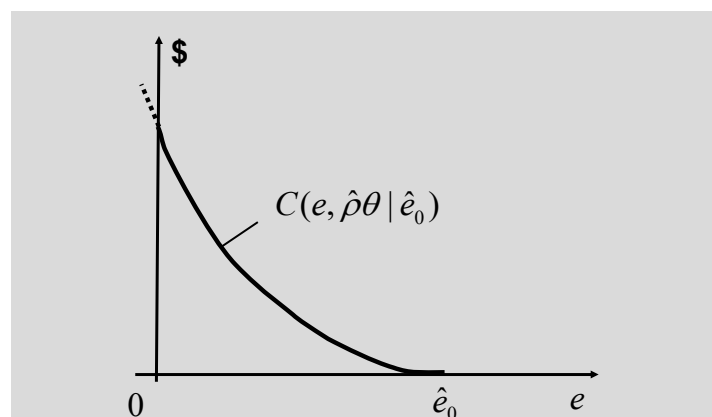
$\rho > 1$: Innovation successful – firm exercises option of using it

THE MODEL (Cont'd) Timing

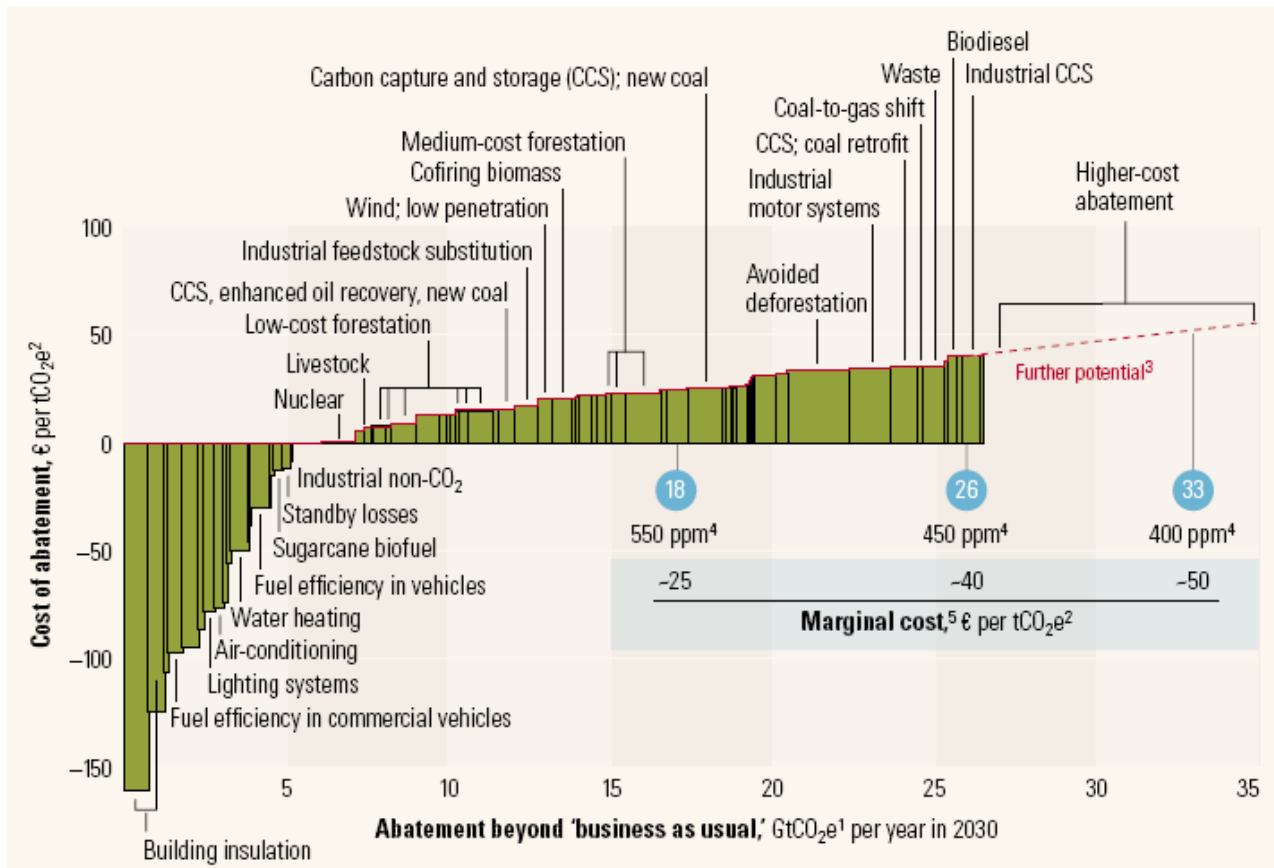
- **Implementation Stage (t=2)**

- The macroeconomic uncertainty ε realizes
- Each firm θ , based on the outcome $\hat{\rho} = \max\{\rho, 1\}$ of its innovation activity in the last stage and the current price p for carbon emissions, decides about its emission level $e(p, \hat{\theta}) = e(p, \hat{\rho}\theta) \geq 0$
- Firm θ 's total cost of abating its emissions to a level $e \leq \hat{e}_0 = e_0(\theta) + \varepsilon$ is

$$TC(e, p, \hat{\rho}\theta | \hat{e}_0) = C(e, \hat{\rho}\theta | \hat{e}_0) + pe = \frac{(\hat{e}_0 - e)^2}{2\hat{\rho}\theta} + pe$$



MARGINAL ABATEMENT COST ALMOST LINEAR



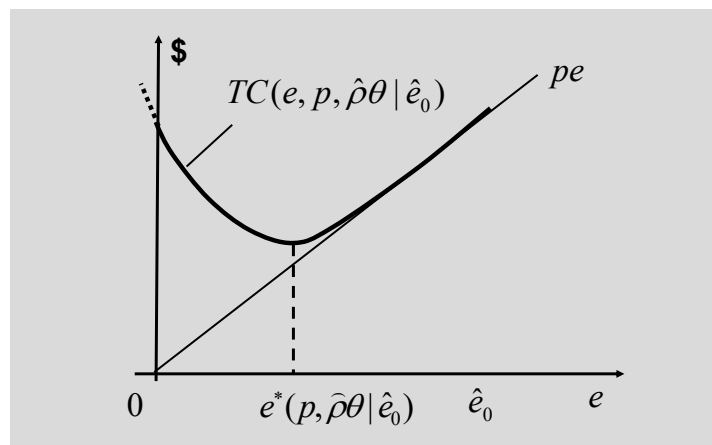
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Source: Enkvist et al. (2007)

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MODEL SOLUTION Implementation (t = 2)

Each firm θ chooses emissions so as to minimize its total emissions cost.



$$e^*(p, \hat{\rho}\theta | \hat{e}_0) = \hat{e}_0 - \hat{\rho}\theta p$$

$$TC^*(p, \hat{\rho}\theta | \hat{e}_0) = \hat{e}_0 p - \frac{\hat{\rho}\theta p^2}{2}$$

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MODEL SOLUTION (Cont'd) Innovation (t = 1)

Each firm θ chooses a level of innovation y to maximize its expected net payoff,

$$\pi(p, y, \theta) = \frac{\theta y p^2}{2} - K(y) = \frac{\theta y p^2}{2} - \frac{c y^2}{2}$$

resulting in the optimal innovation of

$$y^*(p, \theta) = \frac{\theta p^2}{2c}$$

and the positive expected payoff

$$\pi^*(p, \theta) = \frac{\theta^2 p^4}{8c}$$

MODEL SOLUTION (Cont'd) Regulation (t = 0)

The set of feasible cap-and-trade schemes is

$$\mathfrak{R} = \{(E, L, U) \in \mathbf{R}_+^3 : L \leq U\}$$

The total carbon emissions output in the economy, conditional on the market price for carbon and the macroeconomic condition, is

$$Q(p, \varepsilon) = \int_{\Theta} e^*(p, (1 + y^*(p, \theta))\theta | e_0(\theta) + \varepsilon) dF(\theta) = e_0 + \varepsilon - \underbrace{\mu p \left(1 + \frac{\mu^2 + \sigma_\theta^2}{2\mu\kappa} p^2\right)}_{\beta}$$

Environmental damages (measured in \$) are assumed to be quadratic in total emissions,

$$D(Q) = \frac{dQ^2}{2}$$

describes innovation
(for $\beta=0$: no innovation)

MODEL SOLUTION (Cont'd) Regulation (t = 0)

Given a feasible cap-and-trade scheme $R = (E, L, U)$, the **market-clearing condition** set by the regulator,

$$H(p, \varepsilon, R) = (U - p)(p - L)(E - Q(p, \varepsilon)) = 0$$

determines the price $p \in [L, U]$ for carbon

Hence, expected environmental damages are

$$\bar{D}(R) = E[D(Q(\tilde{p}, \tilde{\varepsilon})) | H(\tilde{p}, \tilde{\varepsilon}, R) = 0]$$

Similarly, expected abatement cost are

$$\bar{C}(R) = E[C(e^*(\tilde{p}, (1 + y^*(\tilde{p}, \tilde{\theta}))\tilde{\theta}) | e_0 + \tilde{\varepsilon}) | H(\tilde{p}, \tilde{\varepsilon}, R) = 0]$$

MODEL SOLUTION (Cont'd) Regulation (t = 0)

In addition, the regulator may want to consider the firms' cost of innovation

$$\bar{K}(R) = E[\lambda K(y^*(\tilde{p}, \tilde{\theta})) | H(\tilde{p}, \tilde{\varepsilon}, R) = 0]$$

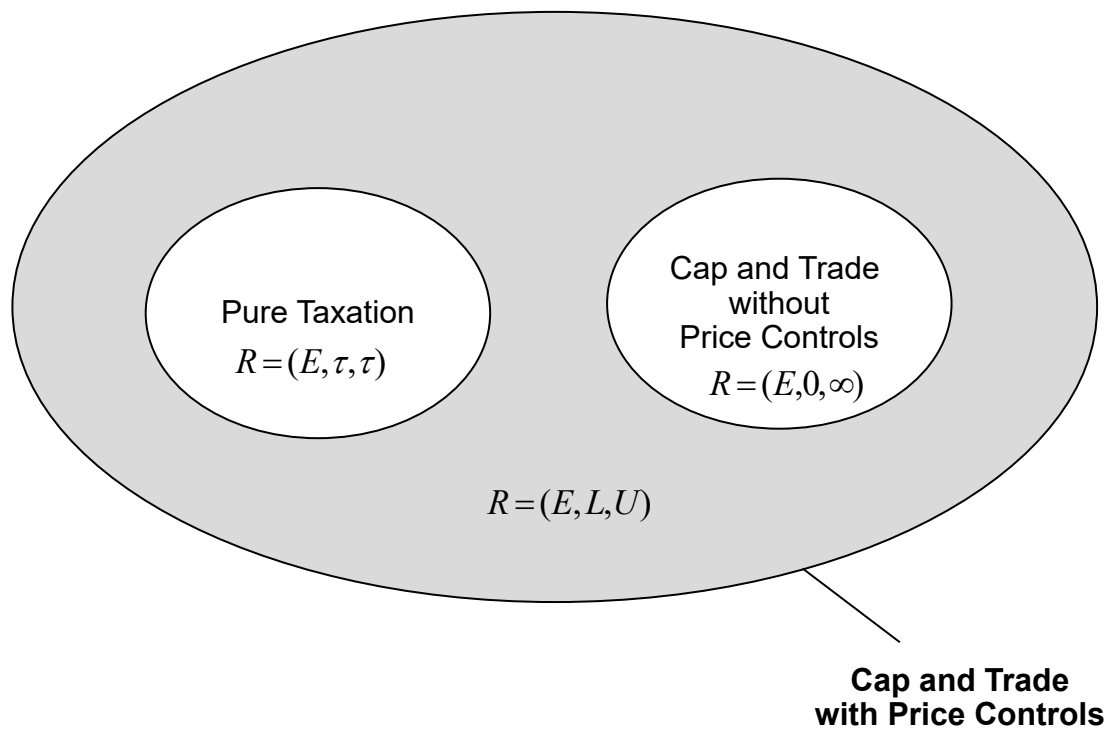
The regulator's objective function is

$$\bar{W}(R) = -(\bar{C}(R) + \bar{D}(R) + \bar{K}(R)) = -\bar{SC}(R)$$

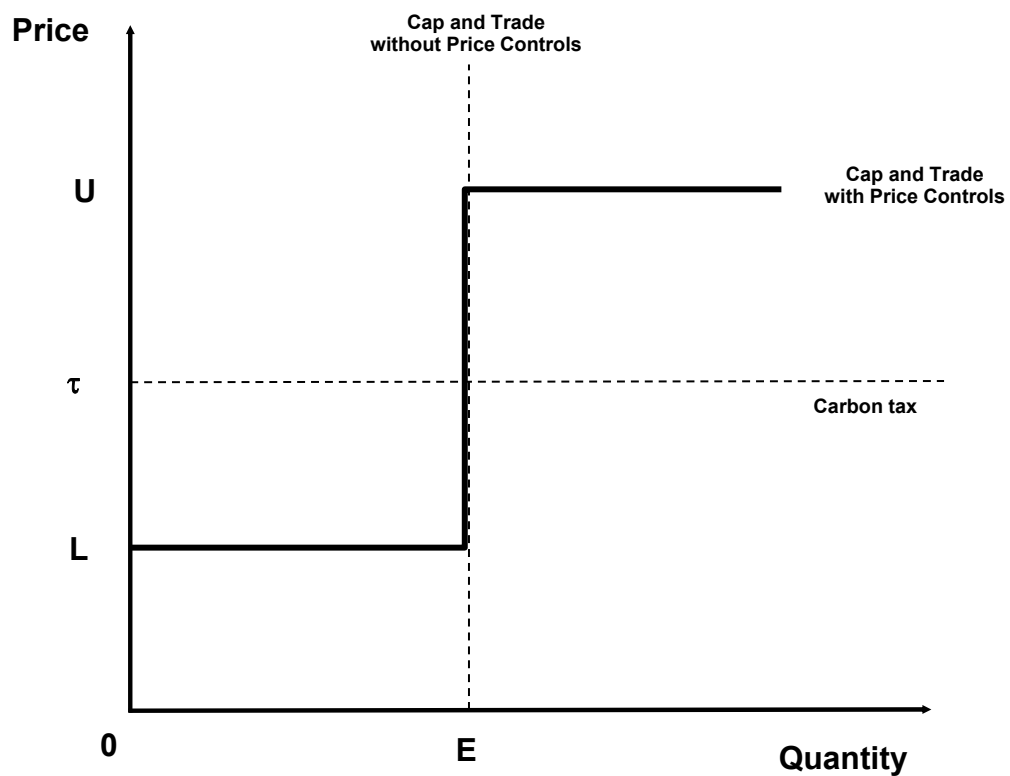
Problem: Find optimal $R^* \in \mathfrak{R}$

RELATION BETWEEN COMMON REGULATORY SCHEMES

Cap and Trade with Price Control = True Generalization



COMMON REGULATORY SCHEMES (Cont'd)



PURE TAXATION

Under pure taxation, there is no price uncertainty, but there is uncertainty about the environmental damage.

Let $\bar{W}(\tau)$ be the planner's objective function;

$$\bar{W}(\tau) = -\frac{\tau^2}{2} - \frac{d}{2}(\sigma_\varepsilon^2 + (e_0 - \tau)^2)$$

and the optimal tax becomes

$$\tau^* = \frac{de_0}{1+d}$$

Optimal carbon tax – numerical example

Assume:

Without innovation $\tau = 40 \$/tCO_2$

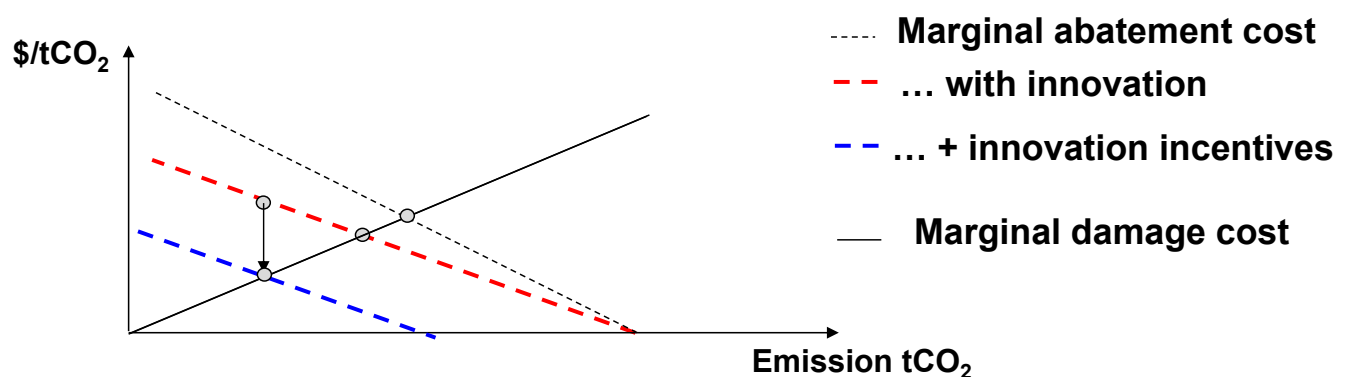
$E_0 = 13.5$ GT (OECD)

10% reduction from existing technologies: $\mu = 33 \cdot 10^6 tCO_2^2/\$^2$

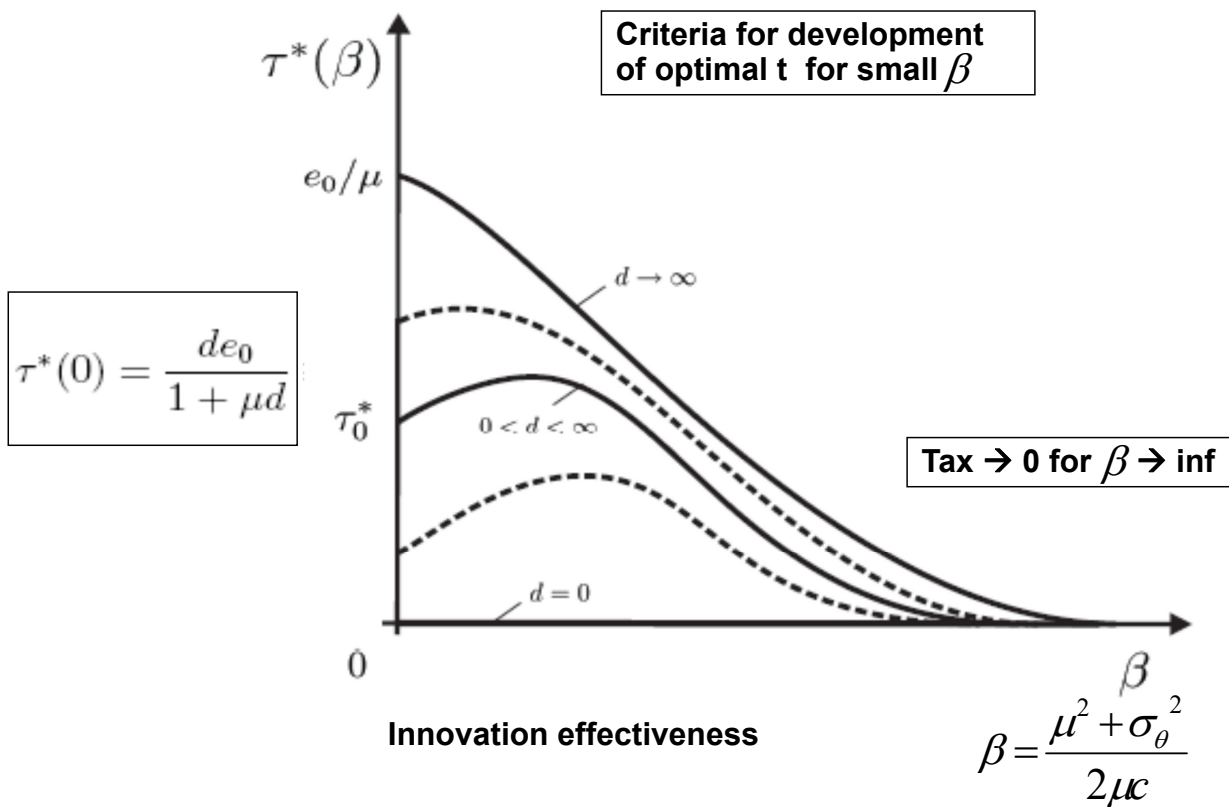
Innovation delivers additional 33% reductions: $c = 100\$ \cdot 10^9$

Result

($\lambda = 0$) $\tau = 46 \$/t CO_2$



Optimal Carbon Tax



BASIC CAP AND TRADE (WITHOUT PRICE CONTROLS)

Under basic cap and trade, there is no output uncertainty, but there is price uncertainty.

Let $\bar{W}(E)$ be the planner's objective function;

$$\bar{W}(E) = -\frac{(e_0 - E)^2 + \sigma_\varepsilon^2}{2} - \frac{dE^2}{2}$$

and the optimal emissions cap becomes

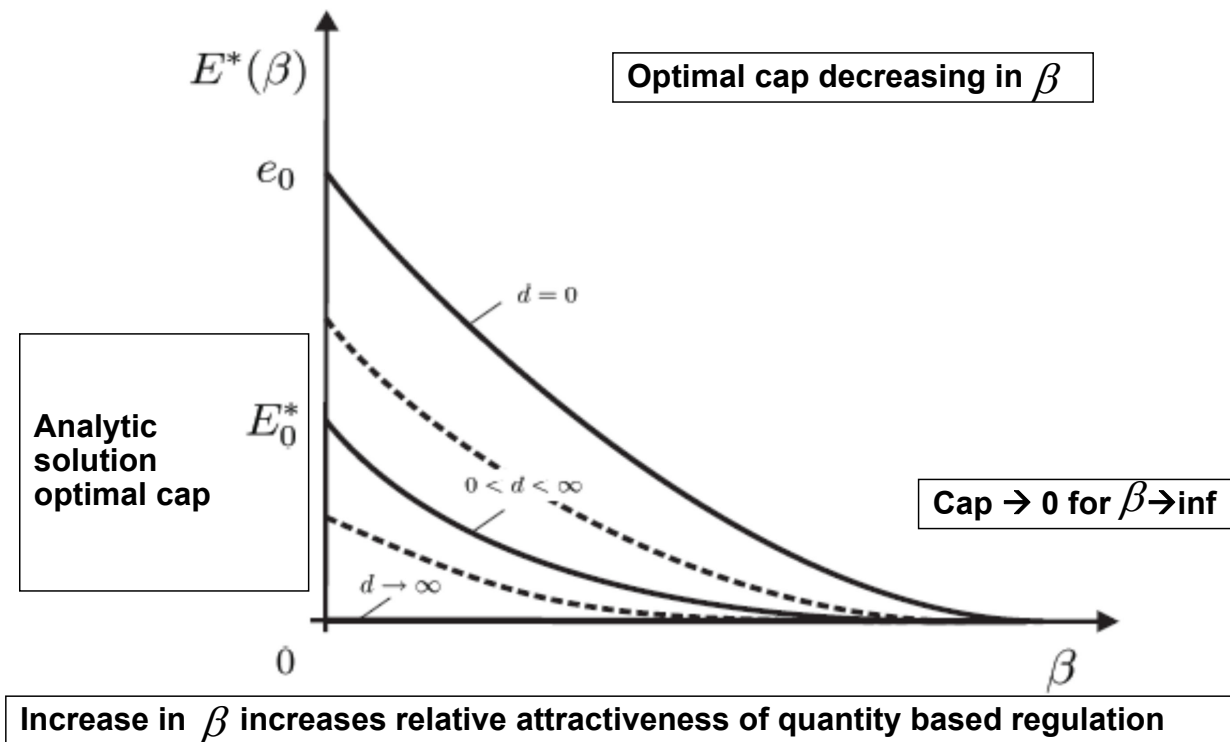
$$E^* = \frac{e_0}{1 + d}$$

Note that expected price under this cap is the same as the optimal tax, i.e.,

$$\bar{p}^* = E[p(\tilde{\varepsilon}, E^*) | E^*] = \tau^*$$

BUT, this does not mean that the two are equivalent!

OPTIMAL CAP



PRICES (TAXES) VS. QUANTITIES (EMISSIONS CAP) Weitzman (1974)

Depending on the relative magnitude of the marginal abatement cost (1) and the environmental damages (d), it may be better to either impose a pure tax or a basic cap-and-trade scheme:

$$\bar{W}_{Tax}^* - \bar{W}_{Basic\ C\&T}^* = (1-d) \frac{\sigma_\varepsilon^2}{2}$$

Tax is strictly better if and only if $d < 1$

Small damage cost d

$\rightarrow$

Pure Tax

Large damage cost d

$\rightarrow$

Basic Cap and Trade

CAP AND TRADE WITH PRICE CONTROLS A Simple Example

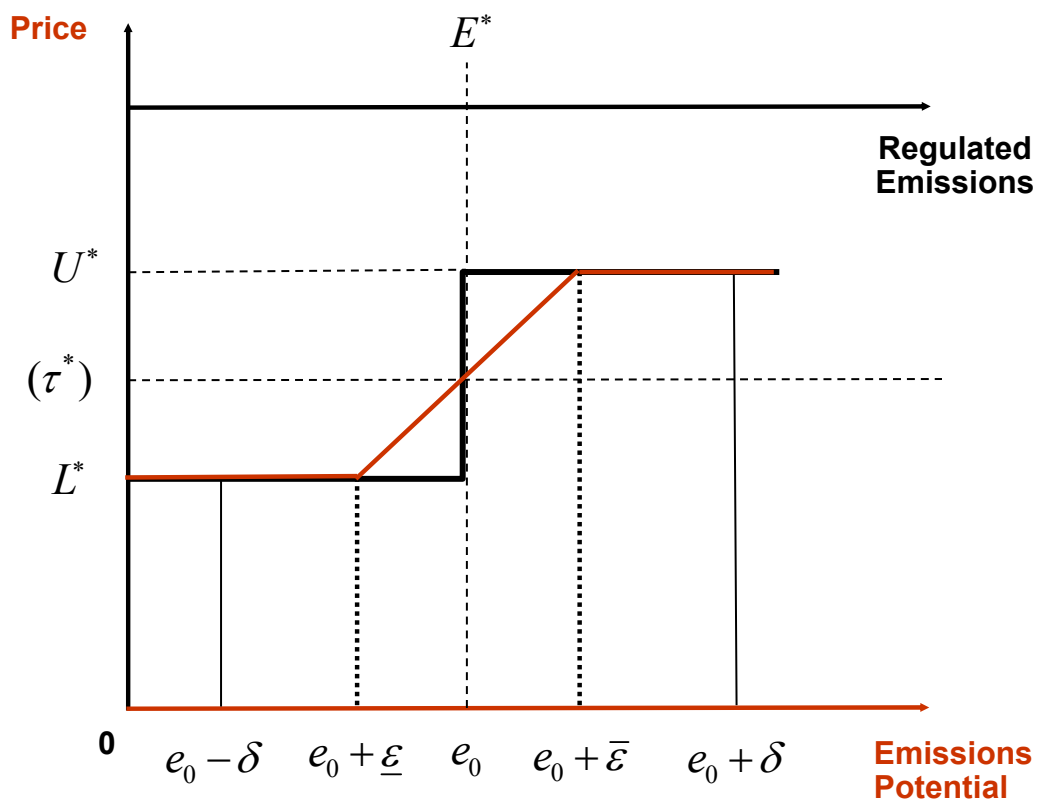
Example. Assume that the macroeconomic shock $\tilde{\varepsilon}$ is uniformly distributed on $[-\delta, \delta]$, where $\delta > 0$. Then,

$$E^* = \frac{e_0}{1+d}$$

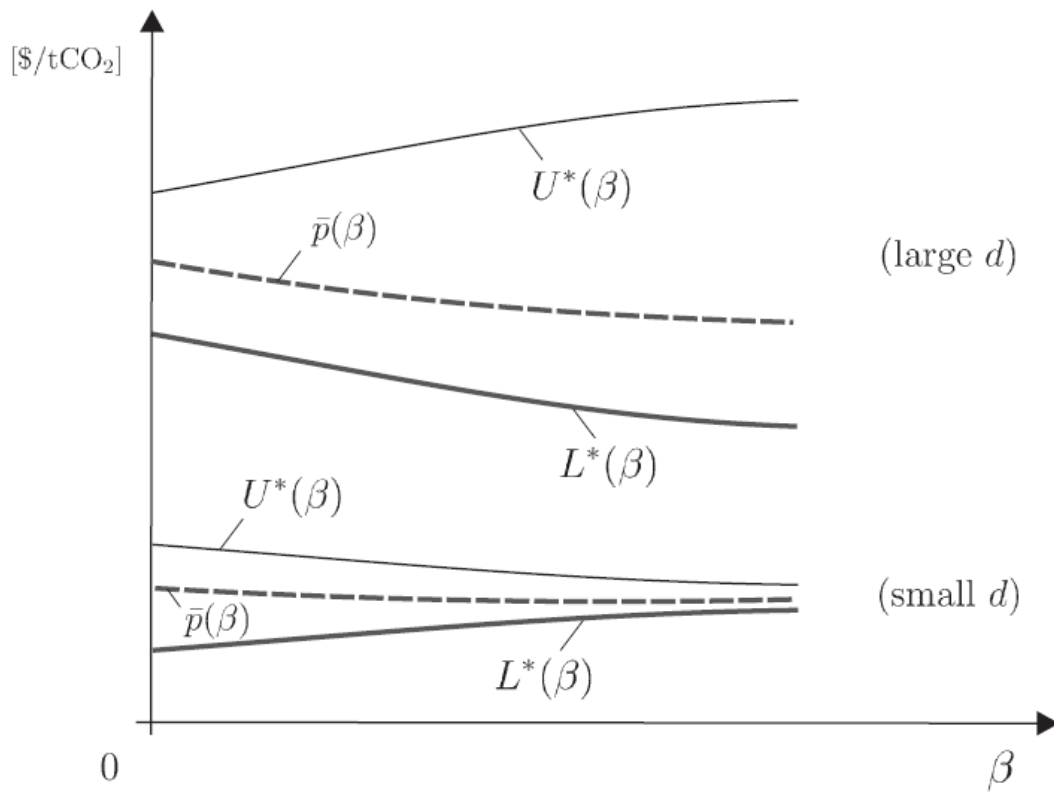
$$L^* = d \left(\frac{e_0}{1+d} - \frac{\delta}{2+d} \right)$$

$$U^* = d \left(\frac{e_0}{1+d} + \frac{\delta}{2+d} \right)$$

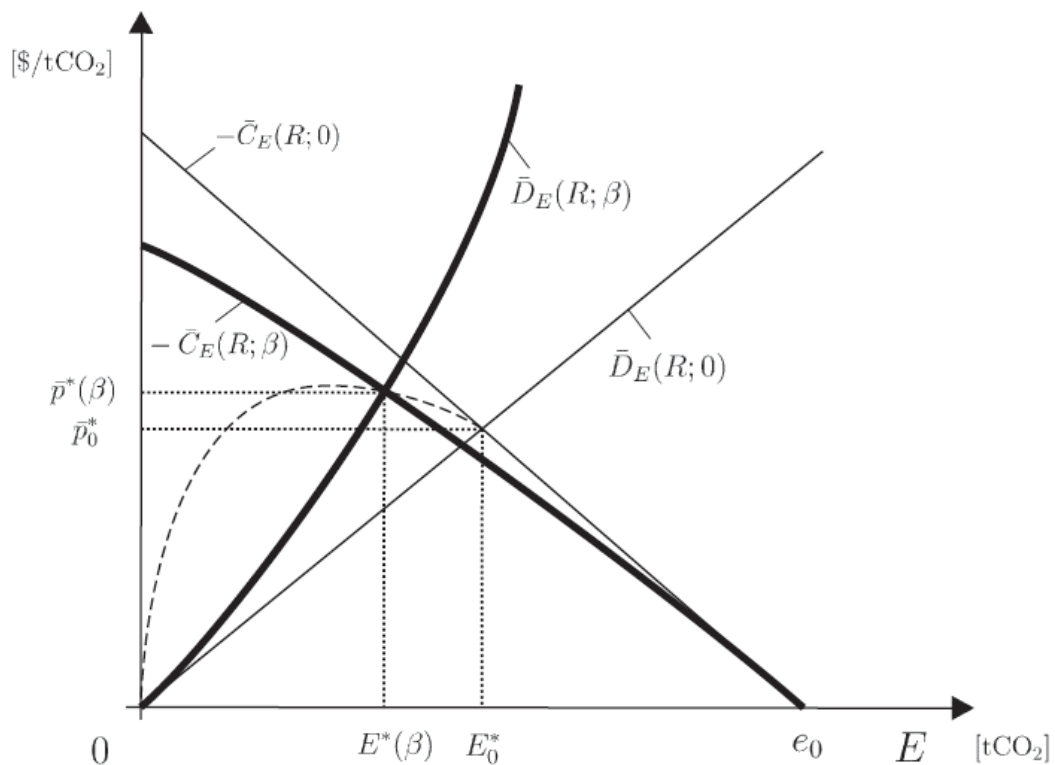
CAP AND TRADE WITH PRICE CONTROLS A Simple Example (Cont'd)



OPTIMAL HYBRID SCHEME

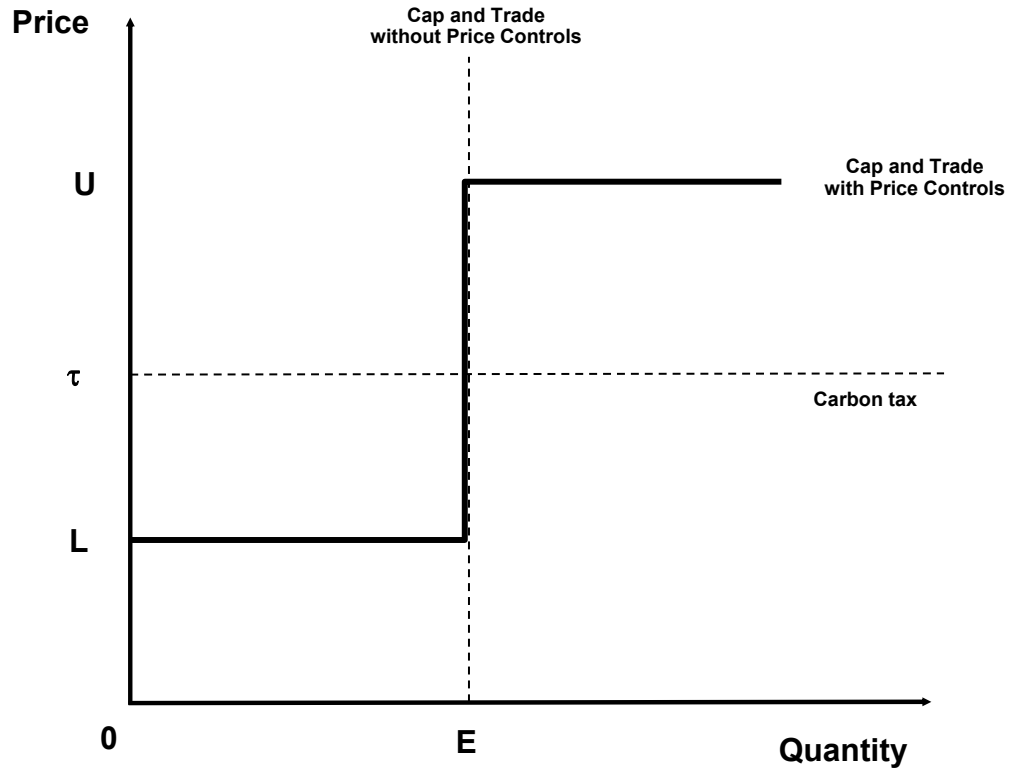


EXPECTED MARKET PRICE VARIES WITH INNOVATION EFFECTIVENESS



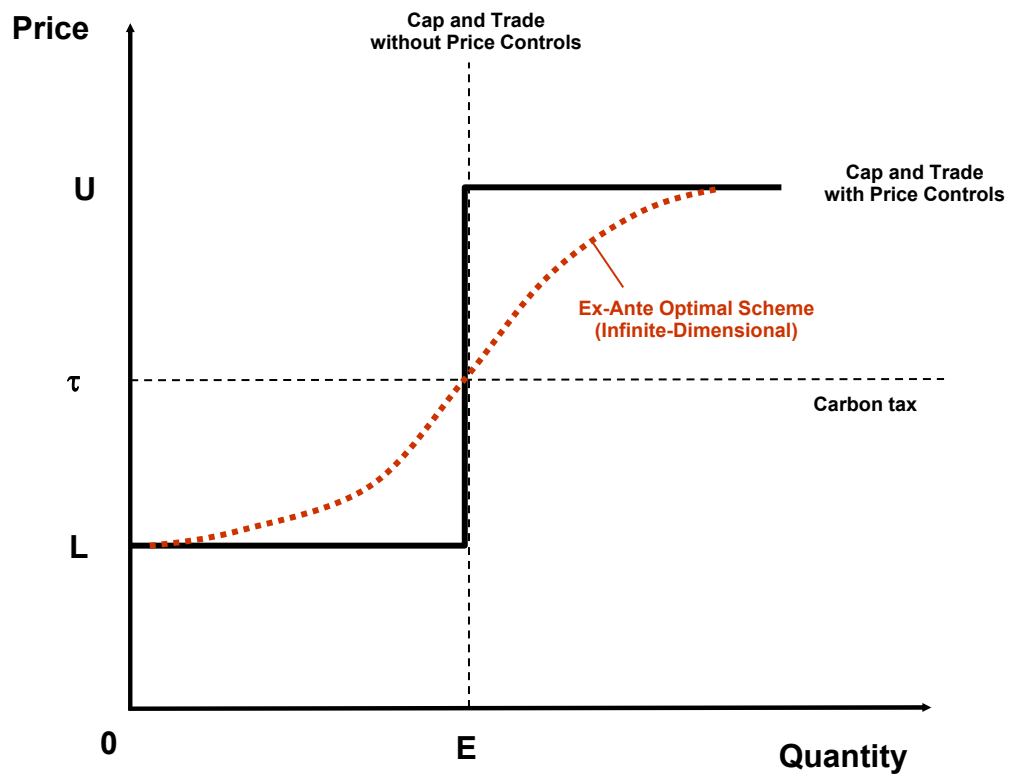
OPTIMAL REGULATION

Is Cap and Trade with Price Controls Really the Best One Can Do?



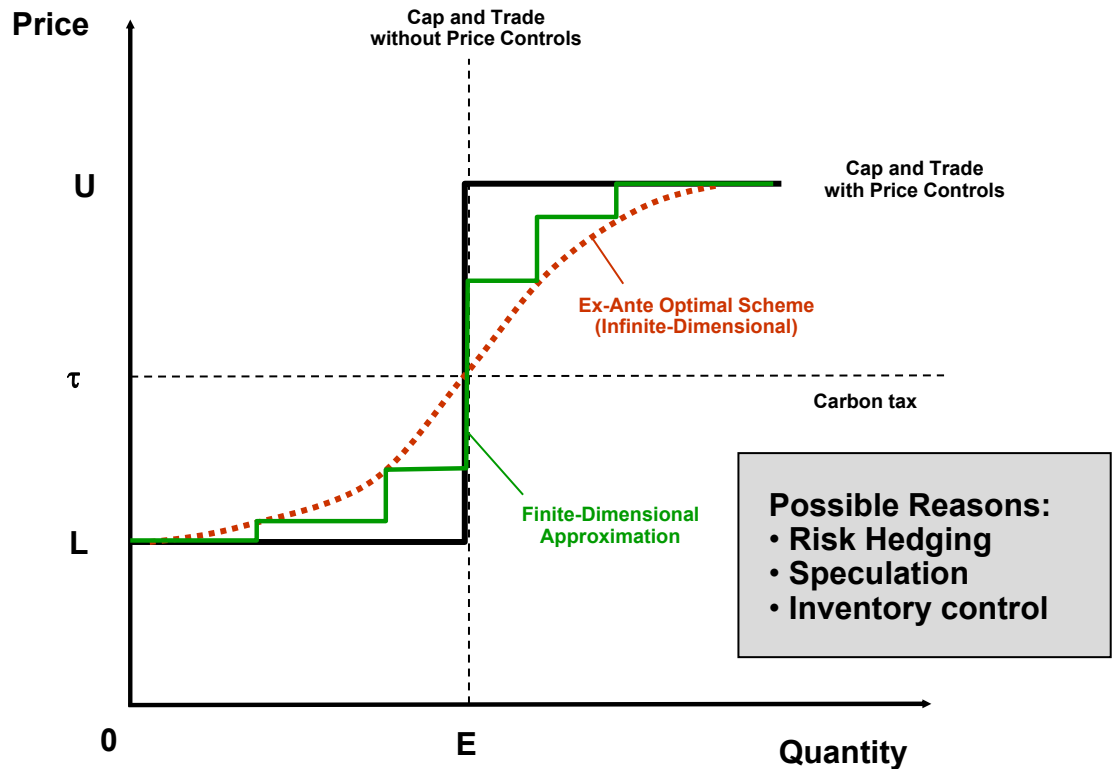
OPTIMAL REGULATION

Ex-Ante (Infinite-Dimensional) Solution Might Be Quite Different



MULTI-CAP AND TRADE

Implementation (e.g.) via Derivative Securities such as Options



ANOTHER IMPORTANT QUESTION: REGULATORY COMMITMENT

When the macroeconomic uncertainty has realized the regulator may want to deviate from his announced regulatory policy R and deviate to R'

- Is it good for a regulator to commit *ex ante* to a scheme R?
- What are *credible* commitment devices?
- What *degree* of commitment is optimal?

Additional policy instruments available at the implementation stage

- Incentives
- Supplementary regulation
- Emissions banking
- Mode of permit allocation

DYNAMIC POLICY ISSUES

A consistent policy mix is credible

Carbon pricing

- **Short-term** – address risk from extreme carbon prices (e.g., via price controls)
- **Medium-term** – flexible price response to deliver target (e.g., national targets, minimize leakage)
- **Long-term** – global mechanism with joint carbon price, where equity can be implemented via “green fund” (e.g., on per-capita-emission basis)

Complementary policies

- **Common trajectory** to ensure action across governments
- **Fairness** in the design of regional and global mechanism

Technology policy

- **Innovation incentives** (e.g., provided by law, technology competitions)
- **Aggressive standard setting**
- **Certification** (e.g., green labelling)

AGENDA

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Uncertainty Matters: Prices vs. Quantities

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Positive/Negative Externalities**
- **Production Externalities**
- **Market Failure**
- **Regulatory options to deal with market failure due to externalities**
- **Coase Theorem**
- **Prices vs. Quantities, and how to regulate both!**

MGT 621 – MICROECONOMICS

Thomas A. Weber

8. General Equilibrium, Part I

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

General Equilibrium: The Standard Model

Pure Exchange

Production Economies

Key Concepts to Remember

THE STANDARD MODEL

Basic Assumptions:

- N **goods**, $i \in \{1, \dots, N\}$
- C **consumers**, $c \in \{1, \dots, C\}$
 - Each consumer c has a **rational** (i.e., complete and transitive) **preference ordering** over $\mathfrak{R}_+^N$ representable by a **continuous utility function** $u_c : \mathfrak{R}_+^N \rightarrow \mathfrak{R}$. Consumers are **price takers**
- F **firms**, $f \in \{1, \dots, F\}$
 - Each firm f has a **production set** $Y_f \subset \mathfrak{R}^N$
 - **Nonempty and closed**
 - **No free lunch** ($Y_f \cap \mathfrak{R}_+^N \subset \{0\}$; can't produce something from nothing)
 - **Possibility of inaction** ($0 \in Y_f$)
 - **Free disposal** ($Y_f - \mathfrak{R}_+^N \subset Y_f$)
 - **Irreversibility** ($y \in Y_f \setminus \{0\} \Rightarrow -y \notin Y_f$)
- **Initial endowments**: each consumer c starts with an endowment vector $\omega^c \in \mathfrak{R}_+^N$ and a fractional share distribution $\theta^c = (\theta_1^c, \dots, \theta_F^c)$, where $\theta_f^c \in [0, 1]$ for each firm f with

$$\sum_{c=1}^C \theta_f^c = 1 \quad (\text{Private Ownership Economy})$$

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WALRASIAN EQUILIBRIUM

Definition: A **Walrasian equilibrium** (WE) is a specification of a **price vector** $p \in \mathfrak{R}_+^N$, a **demand vector** $x^c \in \mathfrak{R}_+^N$ for each consumer c , and a **supply vector** $y^f \in Y_f$ for each firm f , such that

- **profit maximization**, i.e., $y^f \in \arg \max_{y \in Y_f} p \cdot y$
- **utility maximization**, i.e., $x^c \in \arg \max_{x \in B(p, I^c)} u_c(x)$

where consumer c 's **budget set** is given by $B_c = B(p, I^c) = \{x \in \mathfrak{R}_+^N : p \cdot x \leq I^c\}$

with **total income**
$$I^c = p \cdot \omega^c + \sum_{f=1}^F \theta_f^c (p \cdot y^f)$$

- **demand = supply**,
$$\sum_{c=1}^C x^c = \sum_{c=1}^C \omega^c + \sum_{f=1}^F y^f \quad (\text{i.e., allocation is feasible})$$

hold.

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AGENDA

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Key Concepts to Remember

PURE EXCHANGE IS A SPECIAL CASE OF THE STANDARD MODEL

Consider an “**exchange economy**” *without firms and without production*

- **Two goods**, 1 and 2
- **M consumers** of each of **two types**, 1 and 2
- Each **consumer of type** $c \in \{1,2\}$ **begins with an allocation** $\omega^c \in \mathbb{R}_+^2$ (his endowment) and solves a utility maximization problem given a price vector $p \in \mathbb{R}_+^2$, which defines his “**offer curve**”,

$$x^c(p) \in \arg \max_{x \in B(p, p \cdot \omega^c)} u_c(x)$$

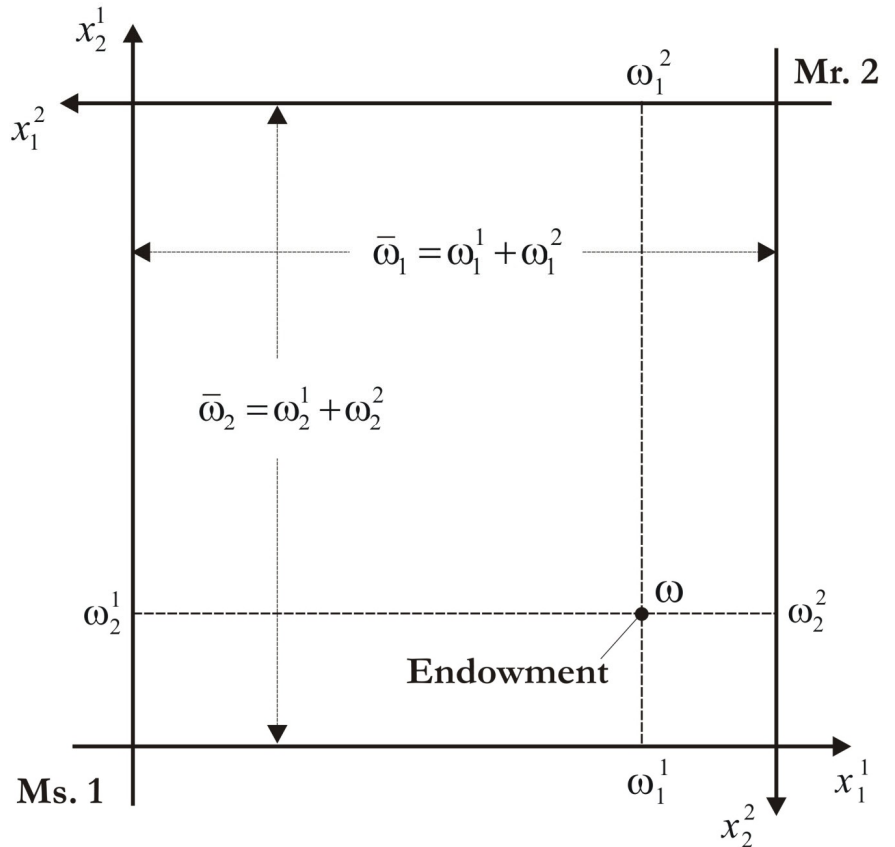
- Any **feasible allocation** satisfies $x^1 + x^2 = \omega^1 + \omega^2$

This two-consumer two-good exchange economy can be represented graphically using a “**Edgeworth box**” (sometimes also referred to as “Edgeworth-Bowley Diagram”).

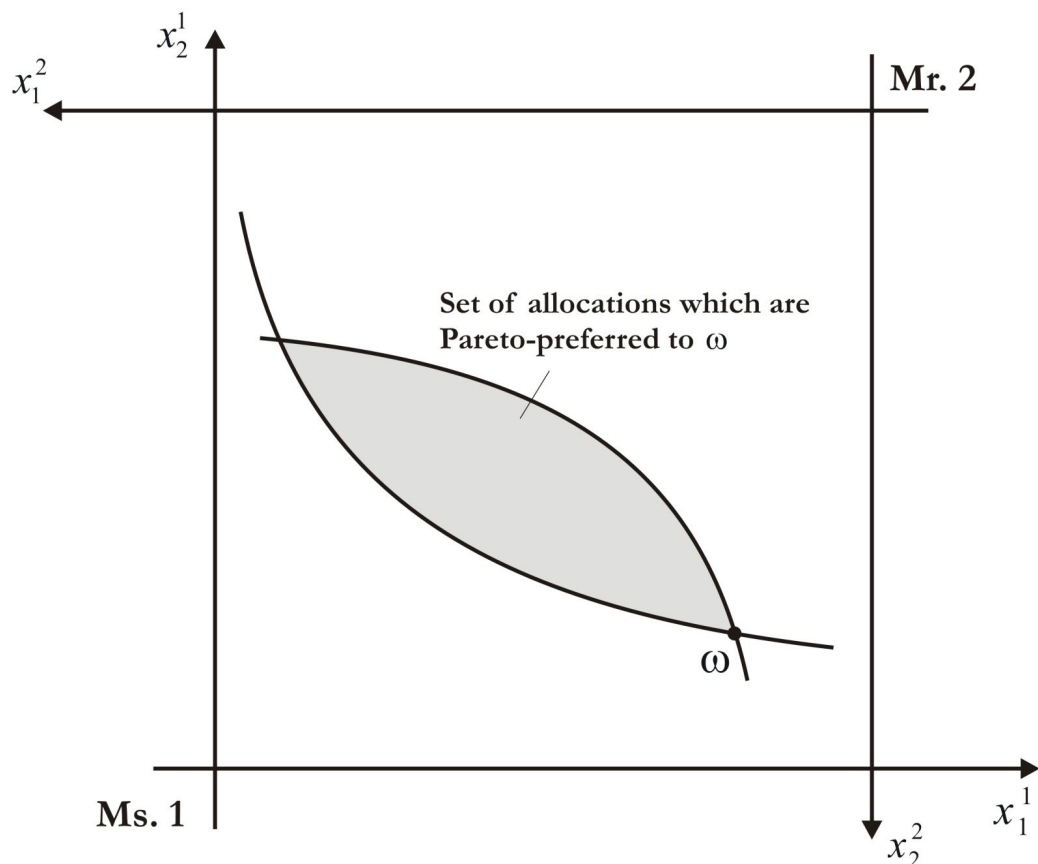
EDGEWORTH BOX

Consider **one consumer of each type**, Ms. 1 and Mr. 2

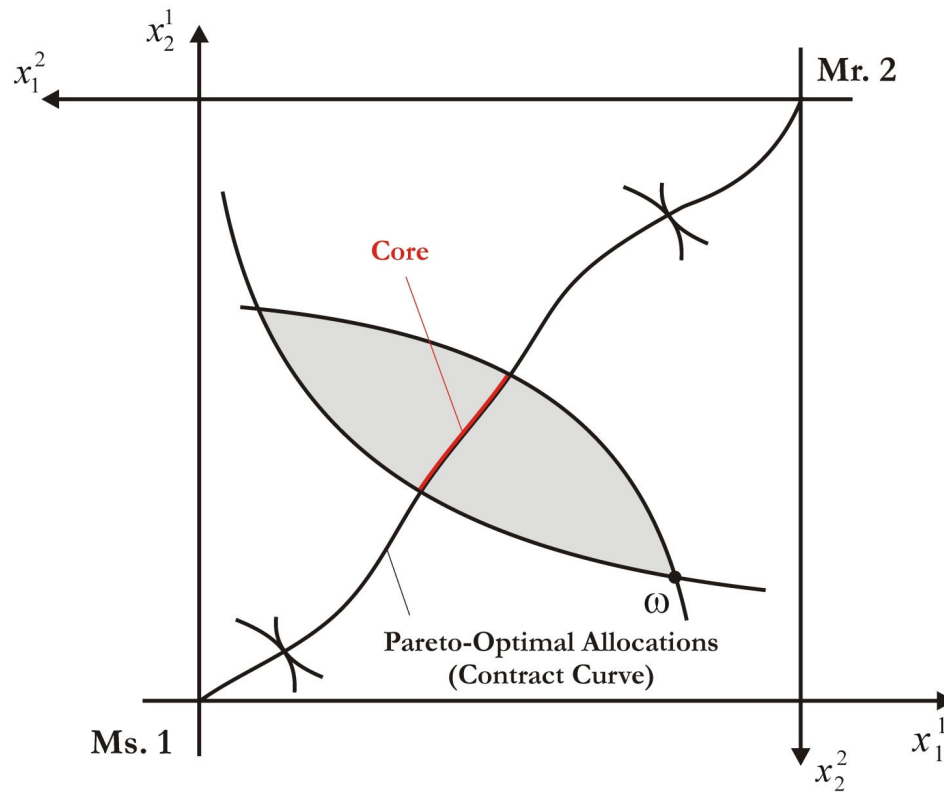
Any allocation of the **totally available quantities** $\bar{\omega}_i = \omega_i^1 + \omega_i^2$ of good i can be represented as a point in the Edgeworth box



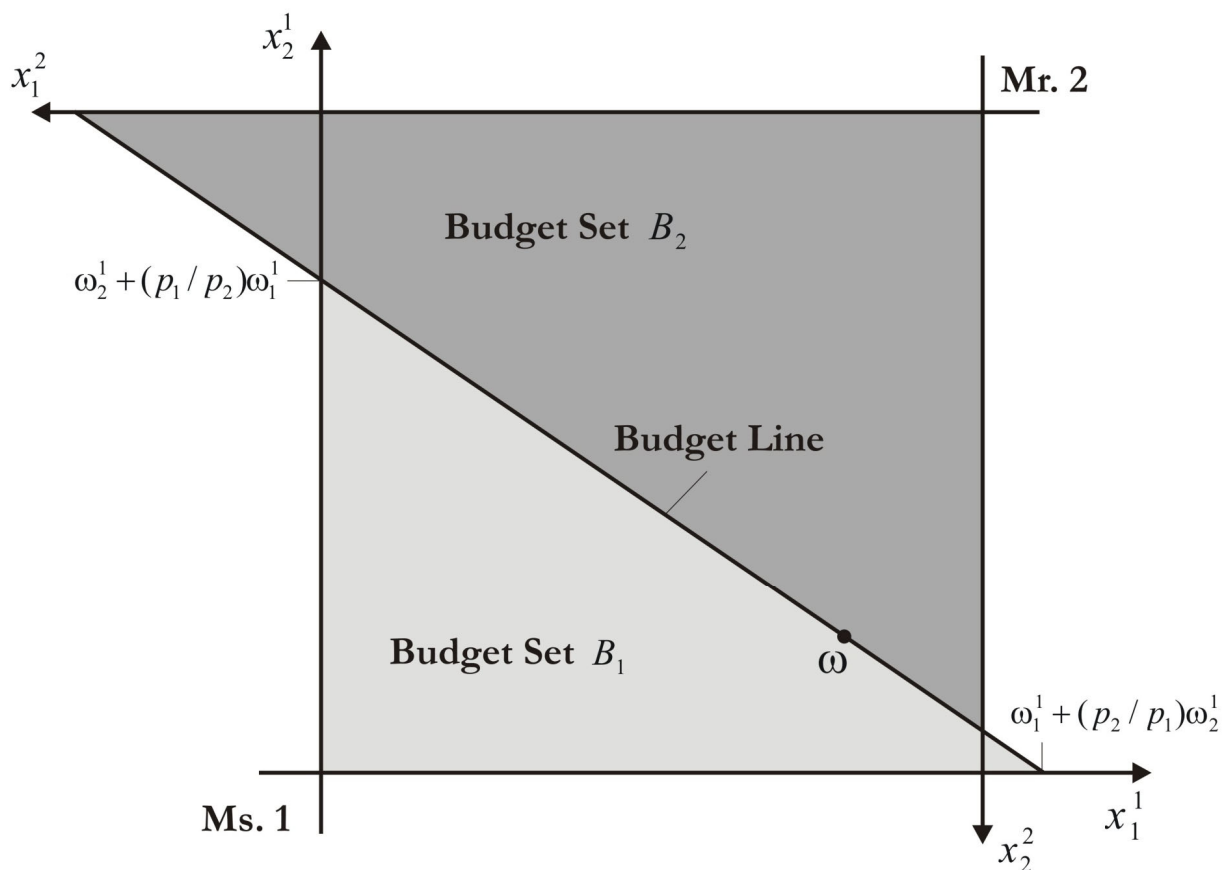
THERE MAY BE **GAINS FROM TRADE**



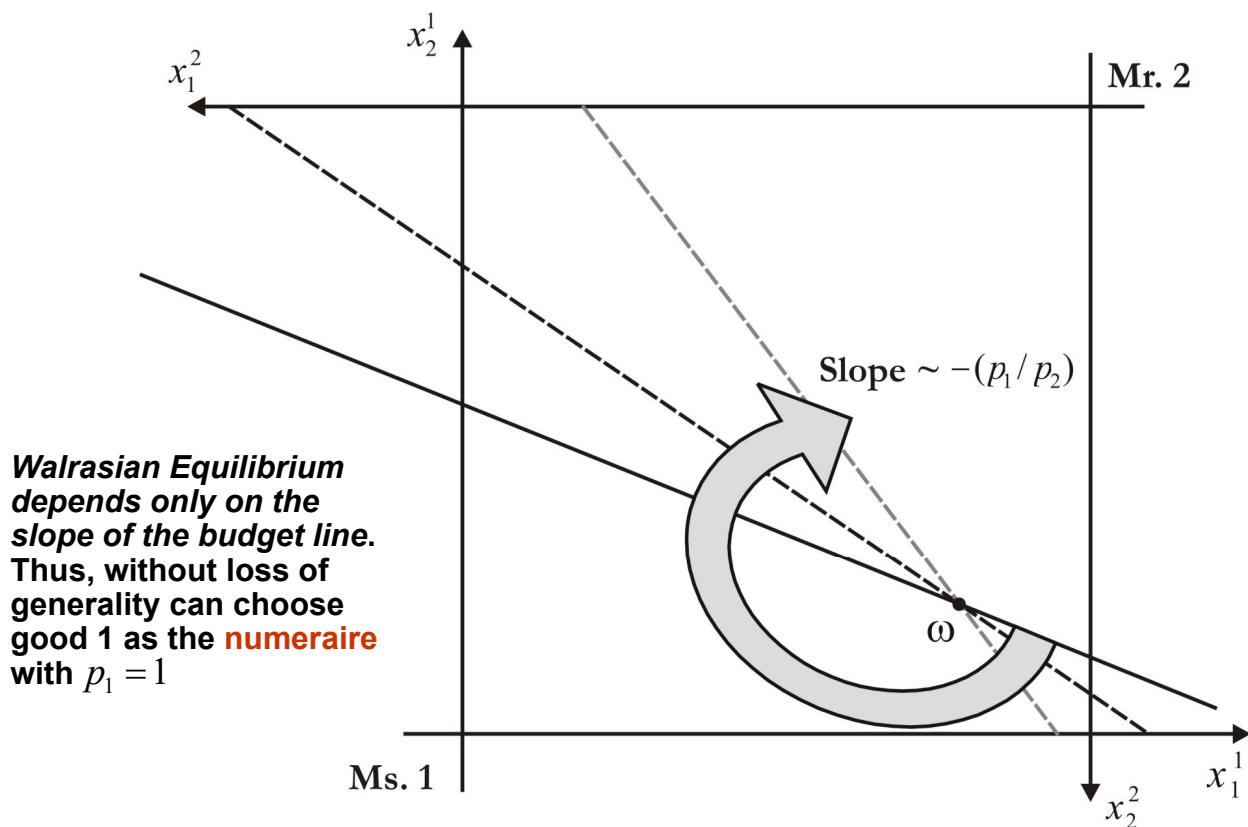
PARETO-OPTIMAL ALLOCATIONS AND CONTRACT CURVE



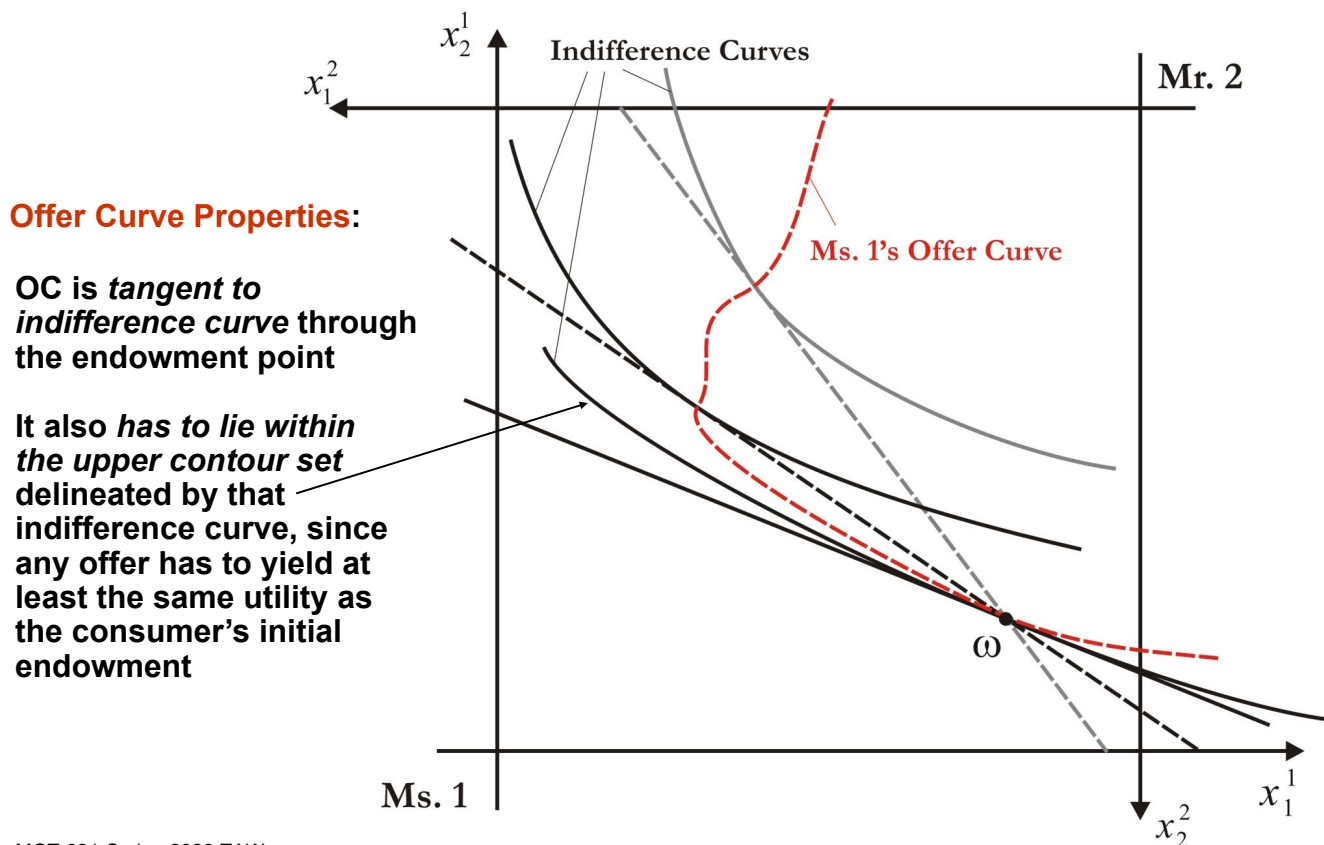
BUDGET LINE AND BUDGET SETS



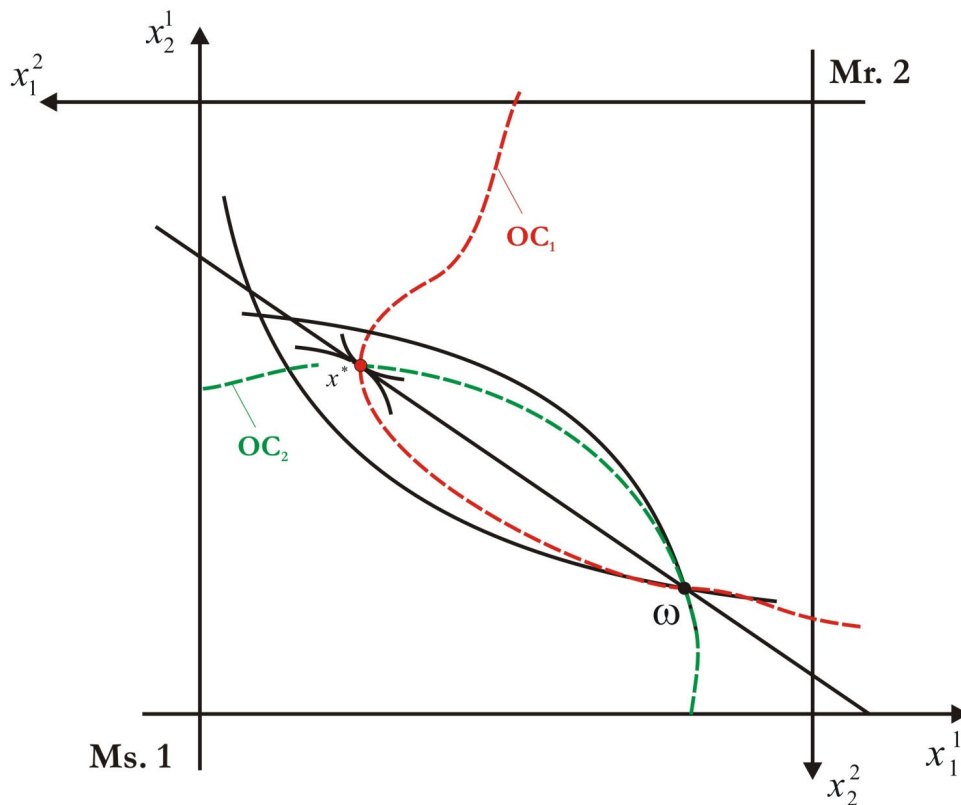
THE SLOPE OF THE BUDGET LINE IS $(-p_1/p_2)$



THE **OFFER CURVE** DESCRIBES THE OPTIMAL CONSUMPTION CHOICE AS A FUNCTION OF MARKET PRICES



IN A WALRASIAN EQUILIBRIUM THE OFFER CURVES INTERSECT



EXAMPLE: EXCHANGE WITH COBB-DOUGLAS UTILITIES Compute the Walrasian Equilibrium

Consider **two consumers**, 1 and 2, with **Cobb-Douglas utility functions**

$$u_c(x_1^c, x_2^c) = (x_1^c)^\alpha (x_2^c)^{1-\alpha}$$

where $\alpha \in (0,1)$ is a constant and $c \in \{1,2\}$. The **endowment vectors** are $\omega^1 = (1,2)$ and $\omega^2 = (2,1)$ respectively

Given a price vector $p = (p_1, p_2)$, consumer 1's **utility maximization problem** can be stated (after taking the logarithm) in the form

$$x^1(p) = \arg \max_{(x_1, x_2) \in B(p, p_1 + 2p_2)} \{ \alpha \log x_1 + (1-\alpha) \log x_2 \} = \underbrace{\left(\frac{\alpha(p_1 + 2p_2)}{p_1}, \frac{(1-\alpha)(p_1 + 2p_2)}{p_2} \right)}_{\text{Consumer 1's offer curve}}$$

Similarly, we find **consumer 2's offer curve**,

$$x^2(p) = \left(\frac{\alpha(2p_1 + p_2)}{p_1}, \frac{(1-\alpha)(2p_1 + p_2)}{p_2} \right)$$

EXCHANGE WITH COBB-DOUGLAS UTILITIES (cont'd)

Using the **Demand = Supply** condition for the Walrasian equilibrium we can clear the market for good 1,

$$x_1^1(p) + x_1^2(p) = \omega_1^1 + \omega_1^2$$

or equivalently,

$$\frac{\alpha(p_1+2p_2)}{p_1}+\frac{\alpha(2p_1+p_2)}{p_1}=3$$

which yields $\frac{p_1}{p_2} = \frac{\alpha}{1-\alpha}$ **and** $\hat{x}_1^1 = \hat{x}_2^1 = 2 - \alpha, \quad \hat{x}_1^2 = \hat{x}_2^2 = 1 + \alpha$

In other words, the price ratio in equilibrium is equal to the marginal rate of substitution between the two goods at the equilibrium allocation.⁽¹⁾

Note also that market clearing for good 1 implies market clearing for good 2 (why?)

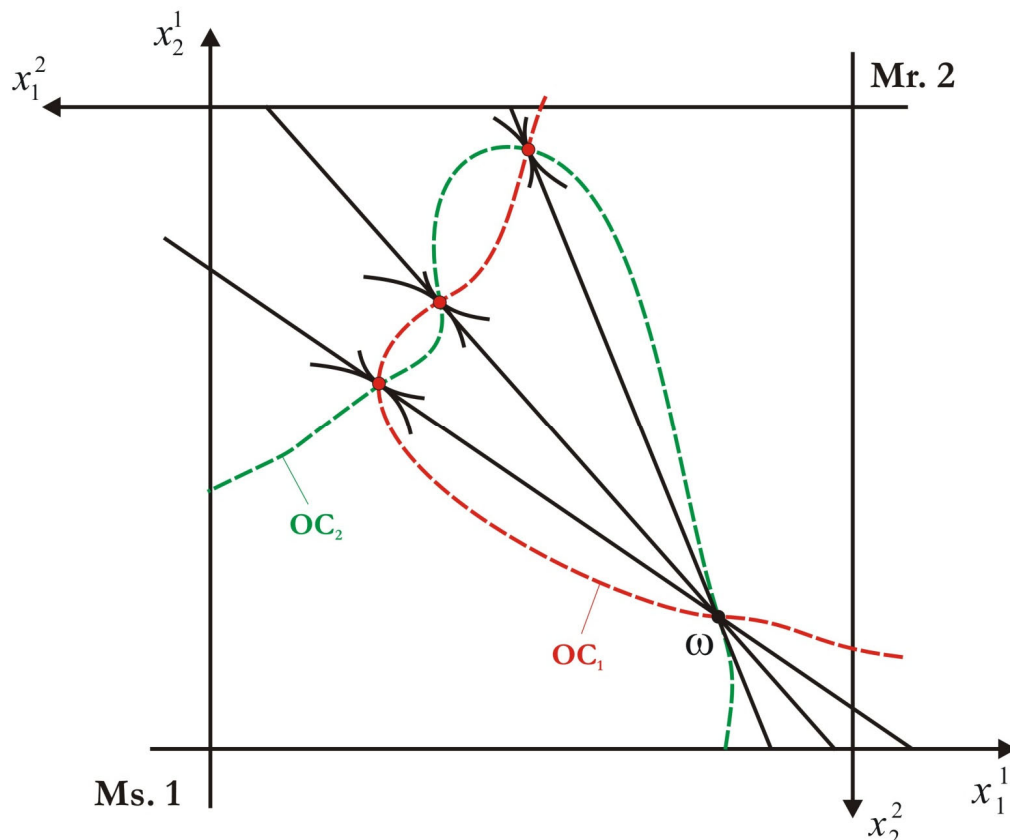
(1) The marginal rate of substitution for consumer 1 between goods 1 and 2 is $MRS_{1,2}(\hat{x}^1) = \frac{\frac{\partial u_1}{\partial x_1} \Big|_{(\hat{x}_1^1, \hat{x}_2^1)}}{\frac{\partial u_1}{\partial x_2} \Big|_{(\hat{x}_1^1, \hat{x}_2^1)}} = \frac{\alpha}{1-\alpha} \frac{\hat{x}_1^1}{\hat{x}_2^1} = \frac{\alpha}{1-\alpha} \frac{2-\alpha}{2-\alpha} = \frac{\alpha}{1-\alpha}$.

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THERE MAY BE DIFFERENT WALRASIAN EQUILIBRIA

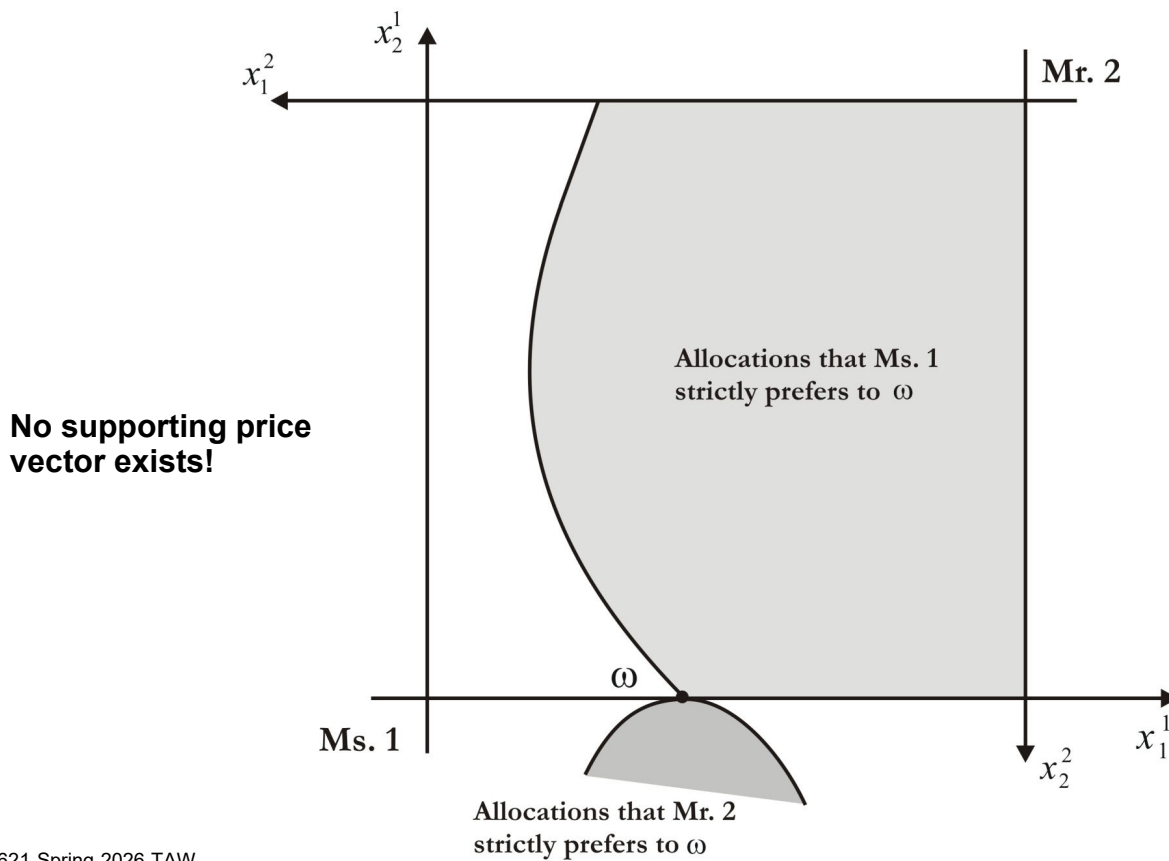
Nonuniqueness



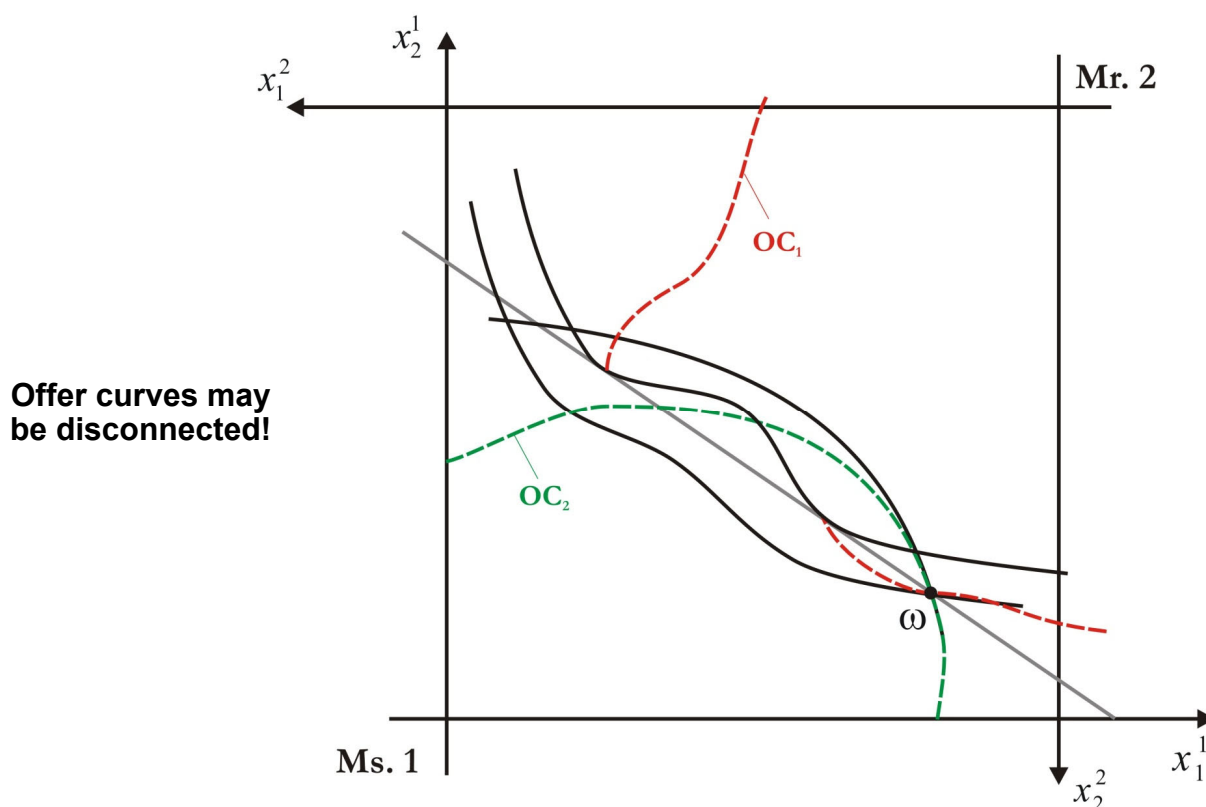
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A WALRASIAN EQUILIBRIUM MAY NOT EXIST Example: Preferences Not Strictly Monotone



A WALRASIAN EQUILIBRIUM MAY NOT EXIST (cont'd) Example: Nonconvex Preferences



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General Equilibrium: The Standard Model

Pure Exchange

Production Economies

Key Concepts to Remember

ROBINSON CRUSOE ECONOMY

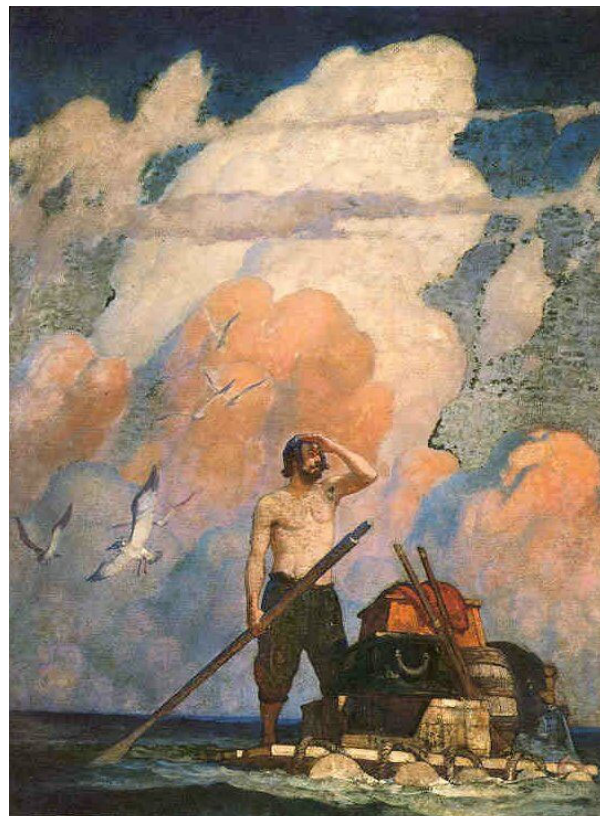
Robinson is alone on an island. He can *either* rest (i.e., consume leisure x_1) *or* use his own labor to pick yummy coconuts x_2

You can think of the firm “**Robinson Crusoe Enterprises**” producing coconuts using Robinson’s labor $z = \bar{L} - x_1$ as the only production input, i.e., the firm maximizes profits

$$\pi(w, p) = \max_{z \geq 0} \{pf(z) - wz\}$$

where $f(\cdot)$ is the firm’s *production function*,⁽¹⁾ w is the wage Robinson pays himself, p is the price of coconuts, and $\bar{L} > 0$ is a constant

Question: If Robinson maximizes his utility $u(x_1, x_2)$, *what is the Walrasian equilibrium of his private economy?*



(1) For simplicity assume that the production function is strictly concave, increasing, and continuously differentiable; we also assume that Robinson's utility is smooth. In addition it is sometimes useful to add the assumptions that $f'(0) = 0$ (to allow for the possibility of inaction), and the so-called Inada conditions $f'(0) = \infty$, $f'(\bar{L}) = 0$ (to guarantee strict interiority of the optimizer).

ROBINSON CRUSOE ECONOMY (cont'd)

Answer: Naturally, Robinson owns all of “Robinson Crusoe Enterprises,” whence we obtain his utility maximization problem,

$$x(w, p) = \arg \max_{(x_1, x_2) \in B((p, w), I)} u(x_1, x_2)$$

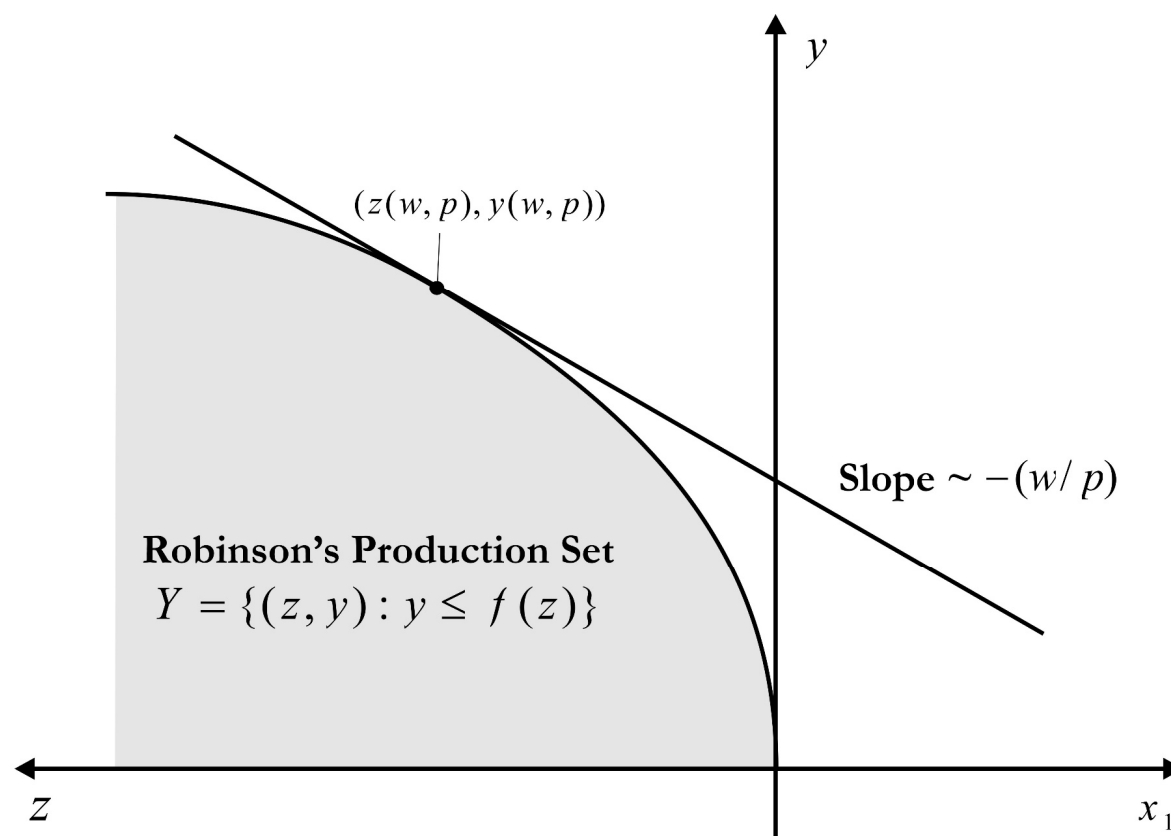
where $B((w, p), I) = \{(x_1, x_2) \in \mathfrak{R}_+^2 : px_2 \leq w(\bar{L} - x_1) + \pi(w, p)\}$

From the definition of a **Walrasian equilibrium**, we obtain the following conditions:

- **Profit maximization:** $f'(z^*(w, p)) = \frac{w}{p}$
- **Utility Maximization:** $\left[\frac{\partial u}{\partial x_1} - \left(\frac{w}{p} \right) \frac{\partial u}{\partial x_2} \right] (x_1^*, \frac{w}{p}(\bar{L} - x_1^*) + y^*(w, p) - \frac{w}{p}z^*(w, p)) = 0$
 where $y^*(w, p) = f(z^*(w, p))$ is the **equilibrium production quantity of coconuts**
- **Demand = Supply:** $x_1^* = \bar{L} - z^*$
 $x_2^* = y^*$

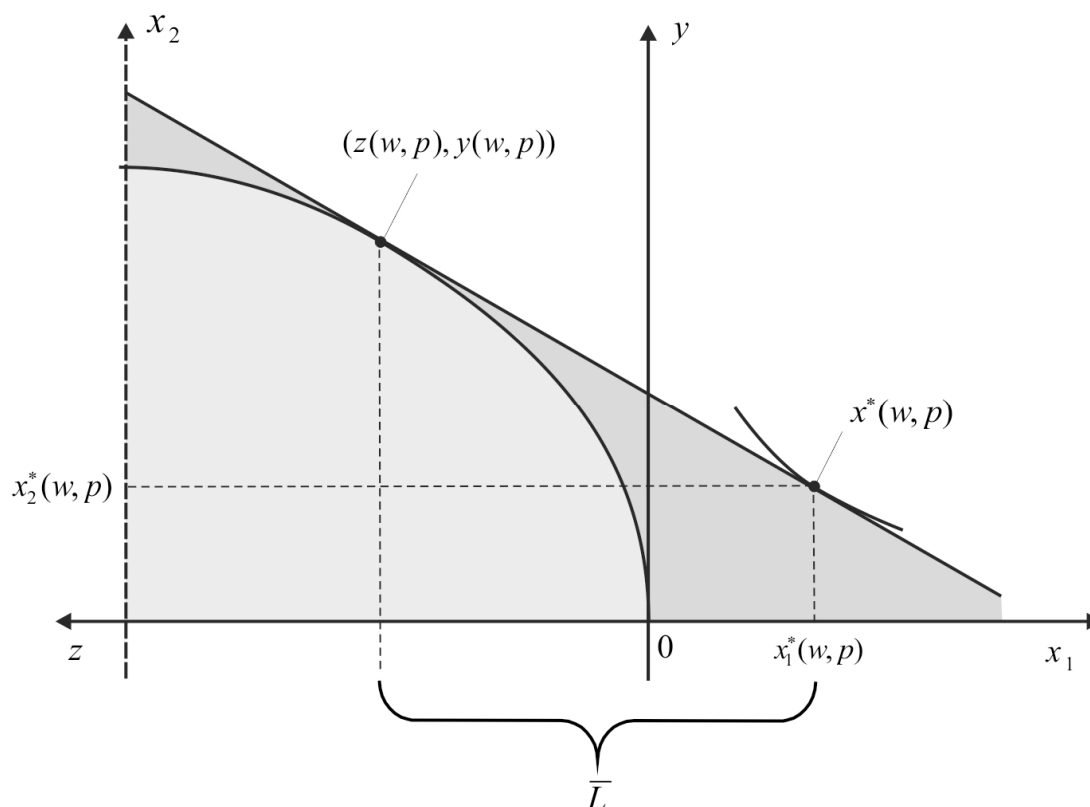
MGT-621-Spring-2026-TAW Note: Without loss of generality we can take coconuts as the numeraire good (Robinson pays himself in coconuts), i.e., $p = 1$. - 21 -

ROBINSON CRUSOE ECONOMY (cont'd) Production Problem



ROBINSON CRUSOE ECONOMY (cont'd)

Walrasian Equilibrium



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2X2 PRODUCTION ECONOMY

Consider now a simple **two-input two-output production economy**:

- **2 Outputs**, $j \in \{1, 2\}$: there are **2N firms** producing one output each ⁽¹⁾
- **2 Inputs**, capital K^j and labor L^j
- The **production function** of each firm for output j is given by

$$y^j = f^j(K^j, L^j)$$

where $f^j(0, 0) = 0$ (**possibility of inaction**) and f^j is strictly concave

- **N consumers** of each of two types, $c \in \{1, 2\}$, with increasing and strictly quasi-concave utility functions $u_c(x_1^c, x_2^c)$, where x_j^c is the amount a consumer of type c consumes of product j
- We assume for simplicity that **consumers do not want to consume either capital or labor**
- **Consumers start with zero endowments** in the production goods $j \in \{1, 2\}$ and with $\bar{K}_c$ units of capital and $\bar{L}_c$ units of labor. In addition, each consumer of type c owns the fraction θ_j^c of the outstanding shares in the firms producing output j

(1) More precisely, we assume that there are N firms producing output 1 and N firms producing output 2.

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2X2 PRODUCTION ECONOMY (cont'd)

Definition: A **symmetric allocation** is one in which *all consumers of type c receive the same consumption vector $x^c = (x_1^c, x_2^c)$ and all firms of type j produce the same output level y^j using the same input vector (K^j, L^j) .*

A **symmetric allocation** $((x^1, x^2), (y^1, K^1, L^1), (y^2, K^2, L^2))$ is **feasible**, if **demand = supply**, i.e.,

$$y^j = x_j^1 + x_j^2 \quad (1)$$

$$\bar{K}_c = K_c^1 + K_c^2 \quad (2)$$

$$K^j = K_1^j + K_2^j \quad (3)$$

$$\bar{L}_c = L_c^1 + L_c^2 \quad (4)$$

$$L^j = L_1^j + L_2^j \quad (5)$$

$$y^j = f^j(K^j, L^j) \quad (6)$$

Let F be the set of feasible allocations.

2X2 PRODUCTION ECONOMY (cont'd)

Definition: A **symmetric Walrasian equilibrium** is a specification of a price p_j for each output j , a price r of capital, a wage w for labor, a consumption vector $\hat{x}^c$ for each consumer type c , and a production vector $(\hat{y}^j, \hat{K}^j, \hat{L}^j)$ for each type of firm, such that the following three conditions are satisfied.

- **Utility maximization** $\hat{x}^c = \arg \max_{x \in B_c} \{u_c(x)\}$

$$B_c = \{(x_1^c, x_2^c) \in \mathfrak{R}_+^2 : p_1 x_1^c + p_2 x_2^c \leq r \bar{K}_c + w \bar{L}_c + \theta_1^c \pi_1 + \theta_2^c \pi_2\}$$

- **Profit maximization** $(\hat{y}^j, \hat{K}^j, \hat{L}^j) = \arg \max_{y^j = f^j(K, L)} \{p_j y^j - r K^j - w L^j\}$

- **Demand = supply**, i.e., $((\hat{x}^1, \hat{x}^2), (\hat{y}^1, \hat{K}^1, \hat{L}^1), (\hat{y}^2, \hat{K}^2, \hat{L}^2))$ is a **symmetric feasible allocation** for each $j \in \{1, 2\}$, where

$$\pi_j = p_j \hat{y}^j - r \hat{K}^j - w \hat{L}^j$$

is firm j 's equilibrium profit

2X2 PRODUCTION ECONOMY (cont'd)

First-order necessary optimality conditions hold in a symmetric WE:

- **Utility maximization**
$$\frac{\partial u_1(\hat{x}^1) / \partial x_1^1}{\partial u_1(\hat{x}^1) / \partial x_2^1} = \frac{\partial u_2(\hat{x}^2) / \partial x_1^2}{\partial u_2(\hat{x}^2) / \partial x_2^2} = \frac{p_1}{p_2} \quad (7)$$

- **Profit maximization**
$$p_1 \frac{\partial f^1(\hat{K}^1, \hat{L}^1)}{\partial K^1} = p_2 \frac{\partial f^2(\hat{K}^2, \hat{L}^2)}{\partial K^2} = r \quad (8)$$

$$p_1 \frac{\partial f^1(\hat{K}^1, \hat{L}^1)}{\partial L^1} = p_2 \frac{\partial f^2(\hat{K}^2, \hat{L}^2)}{\partial L^2} = w \quad (9)$$

From (7)—(9) we obtain

$$\frac{\partial u_c(\hat{x}^c) / \partial x_1^c}{\partial u_c(\hat{x}^c) / \partial x_2^c} = \frac{p_1}{p_2} = \frac{\partial f^2(\hat{K}^2, \hat{L}^2) / \partial K^2}{\partial f^1(\hat{K}^1, \hat{L}^1) / \partial K^1} = \frac{\partial f^2(\hat{K}^2, \hat{L}^2) / \partial L^2}{\partial f^1(\hat{K}^1, \hat{L}^1) / \partial L^1}$$

All marginal rates of substitution are determined by the equilibrium price ratio of the traded market commodities.

AGENDA

General Equilibrium: The Standard Model

Pure Exchange

Production Economies

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Edgeworth Box**
- **Pareto Optimality**
- **Budget Line**
- **Endowment**
- **Numeraire Good**
- **Offer Curve**
- **Walrasian Equilibrium (Competitive Equilibrium) (w/ or w/o transfers)**
- **Pareto Set**
- **Contract Curve/Core**
- **Pure Exchange / Production Economy**
- **Private Ownership Economy**
- **Price-Taking Behavior**
- **Walras' Law**

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8. (Optional) *General Equilibrium, Part II*

Autumn 2026

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College of Management of Technology**

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AGENDA

Some Preliminaries

Fundamental Welfare Theorems

Existence of a Competitive Equilibrium

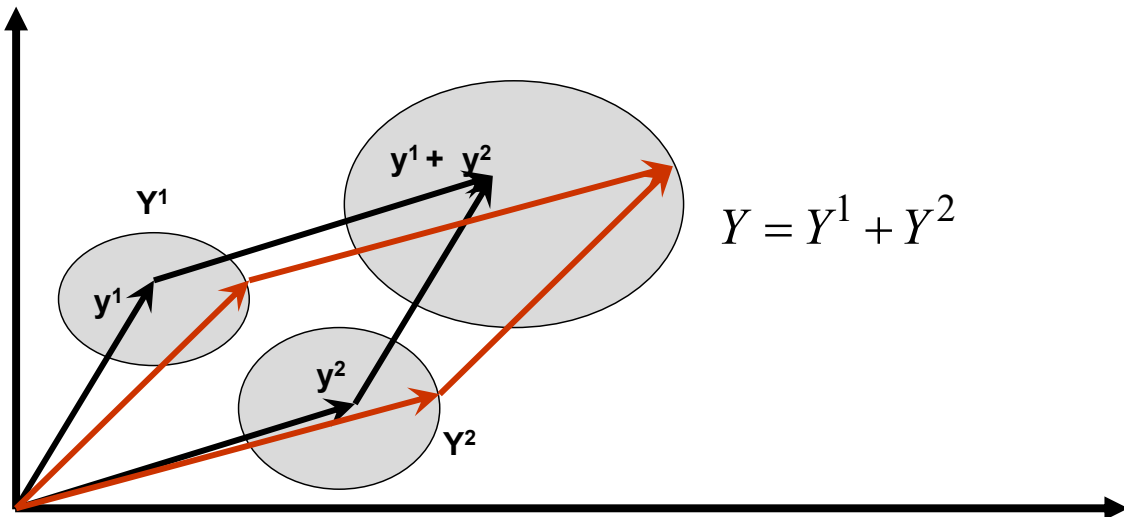
General Equilibrium vs. Partial Equilibrium

Key Concepts to Remember

CONCEPT OF SET SUMMATION

Set summation of set Y^1 and set Y^2 : $Y = Y^1 + Y^2 = \{y : y = y^1 + y^2, y^1 \in Y^1, y^2 \in Y^2\}$

Intuition. Choose any point, y^1 from set Y^1 and any point y^2 from set Y^2 ; the set Y consists of the set of all points $y^1 + y^2$.



FEASIBLE TOTAL OUTPUT IN THE ECONOMY

Initial Endowment of commodities by consumer c : ω^c

Net output by firm f : $y^f \in Y^f$

→ Total Supply:
$$\sum_{c=1}^C \omega^c + \sum_{f=1}^F y^f$$

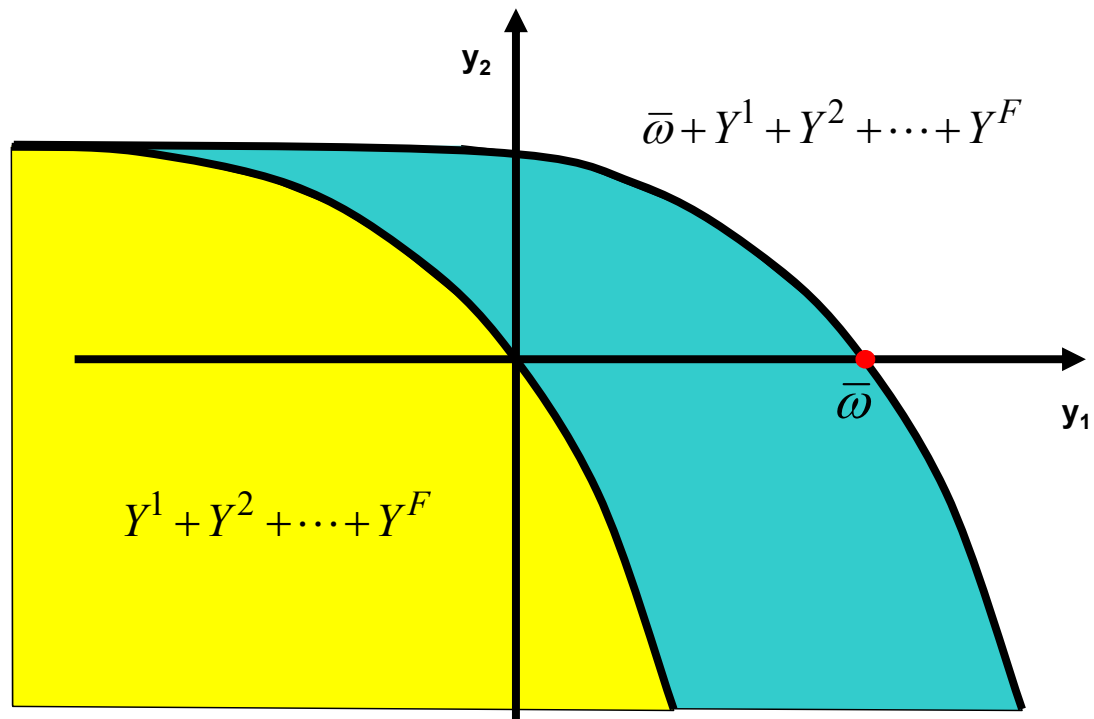
Define total Initial Endowments
$$\bar{\omega} = \sum_{c=1}^C \omega^c$$

Then, feasible set of total outputs is

$$Y = \bar{\omega} + Y^1 + Y^2 + \dots + Y^F$$

Remark. Note that this depends on the assumption that there are no externalities in production. If there are externalities, then the feasible set of total outputs is not set summation of individual production sets.

TOTAL FEASIBLE OUTPUT IN THE ECONOMY (Cont'd)

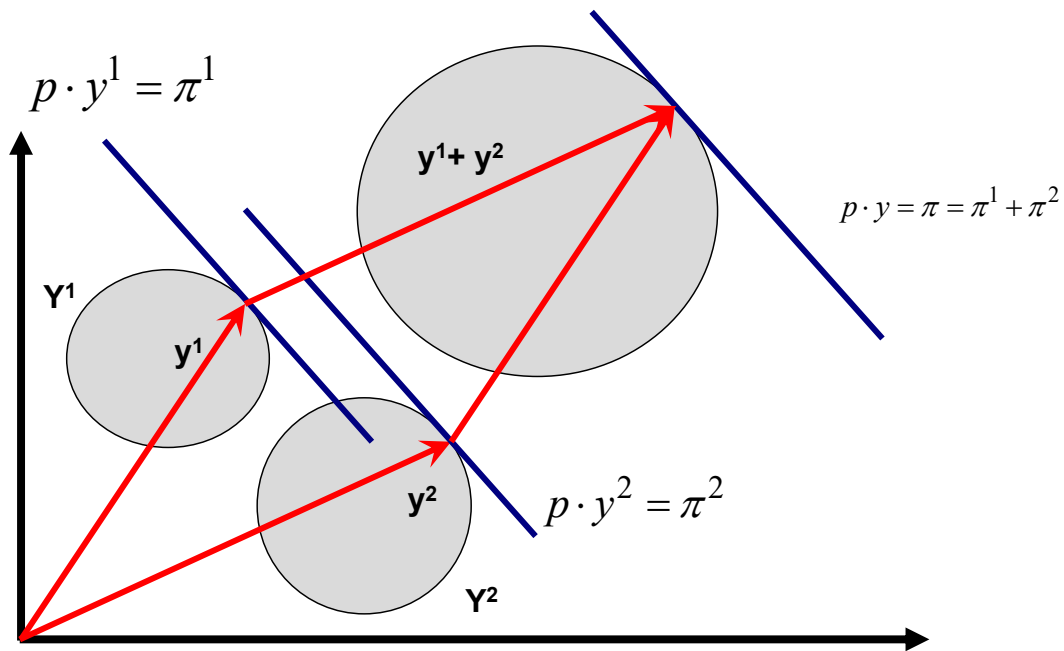


VALUING TOTAL OUTPUT AT MARKET PRICES

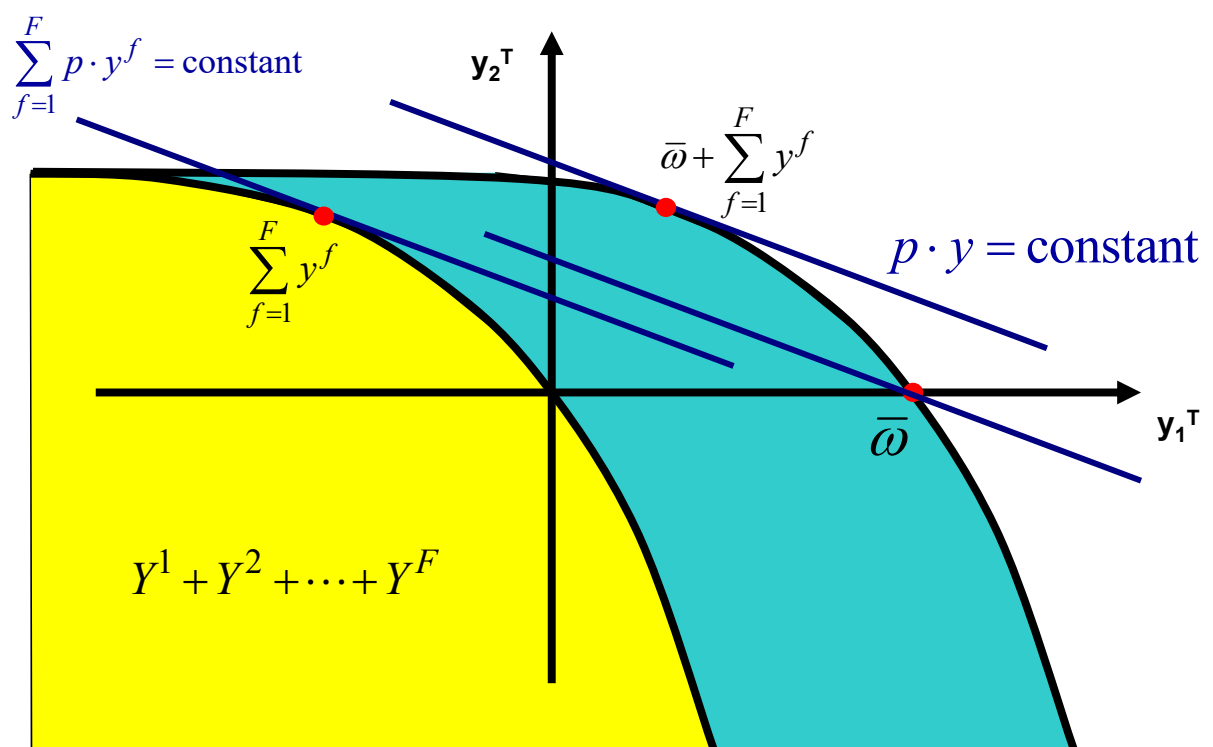
$$p \cdot y = \sum_{c=1}^C p \cdot \omega^c + \sum_{f=1}^F p \cdot y^f$$

Value of Total Output in Economy = Value of Initial Endowments + Sum of Firms' Profits

INDIVIDUAL MAXIMIZATION IMPLIES GLOBAL MAXIMIZATION



MAXIMUM VALUE OF TOTAL OUTPUT



CONSUMER CHOICE

Consumer maximizes utility, subject to budget constraint

$$u^{c*} = \max u^c(x^c)$$
$$\text{s.t. } p \cdot x^c \leq w^c$$

Equivalently, consumer minimizes expenditure for achieving a certain utility level

$$w^{c*} = \min p \cdot x^c$$
$$\text{s.t. } u^c(x^c) \geq U^{c*}$$

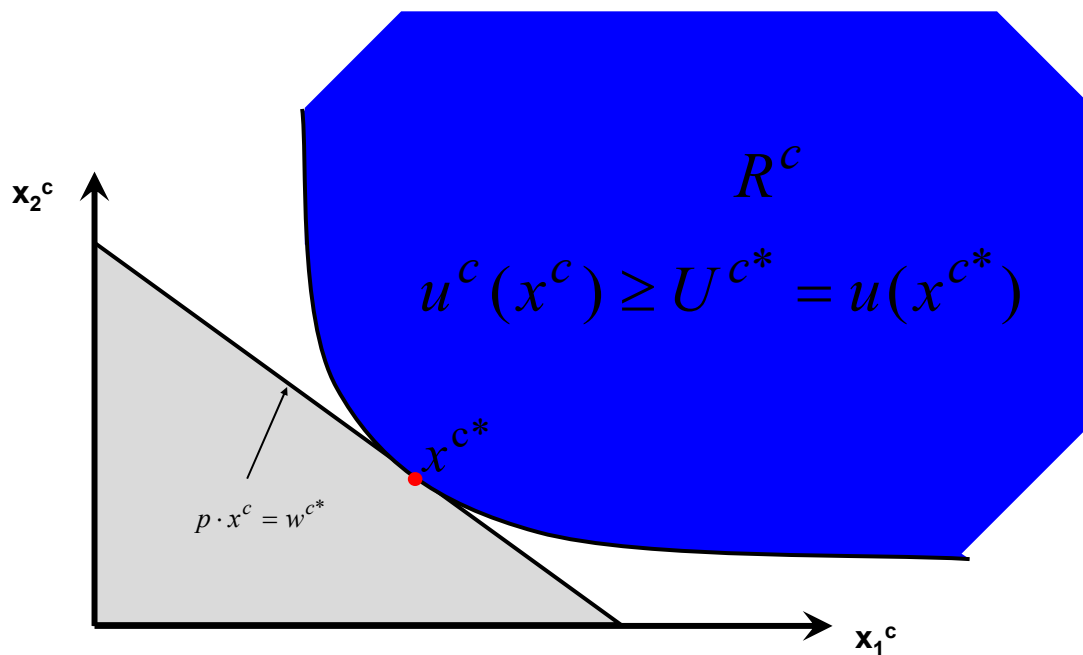
CONSUMER CHOICE (Cont'd)

Define set of consumption bundles weakly preferred to optimal choice as preference set for consumer c: R^c (upper contour set)

Then consumer **minimizes expenditure**, given x^c in R^c

$$\min_{x^c \in R^c} p \cdot x^c$$

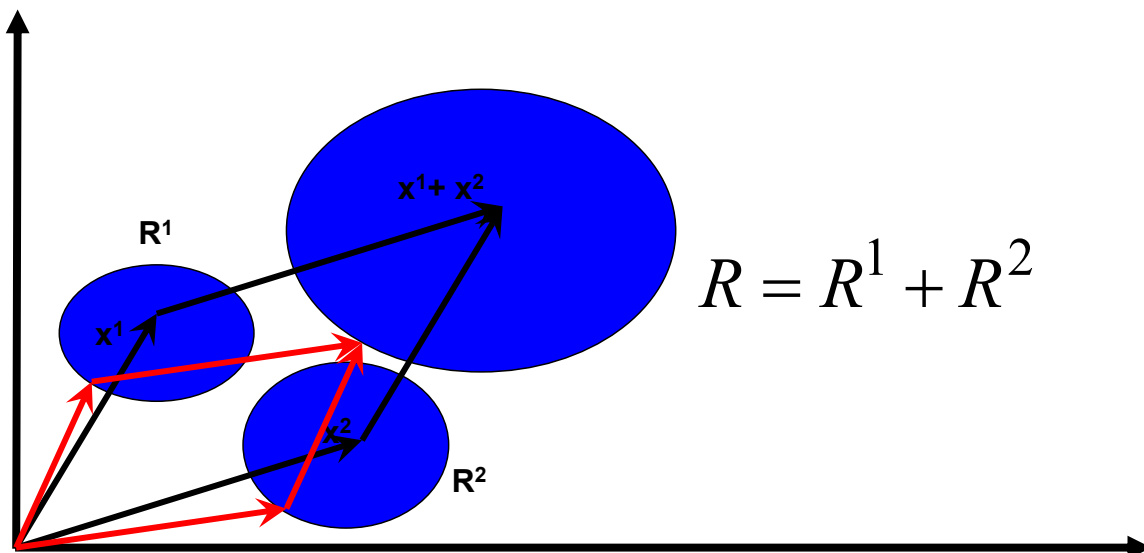
CONSUMER CHOICE (Cont'd)



SET SUMMATION OF INDIVIDUAL PREFERENCE SETS

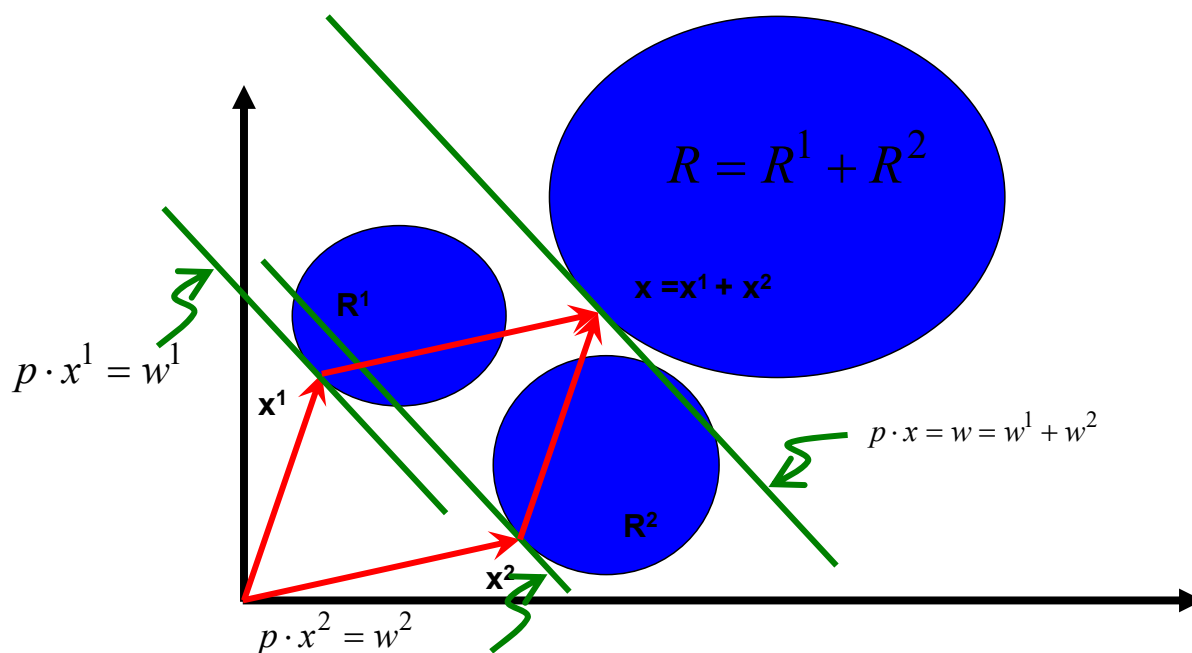
Set summation of individual preference sets is the set of total consumption bundles that allows each consumer to have utility at least as high as his/her U^c . R is the aggregate preference set.

Total consumption in interior of R could allow Pareto-superior allocations to consumers.



Remark. Note that this depends on the assumption that there are no externalities in consumption. If there are externalities, then the aggregate preference set is not set summation of individual preference sets.

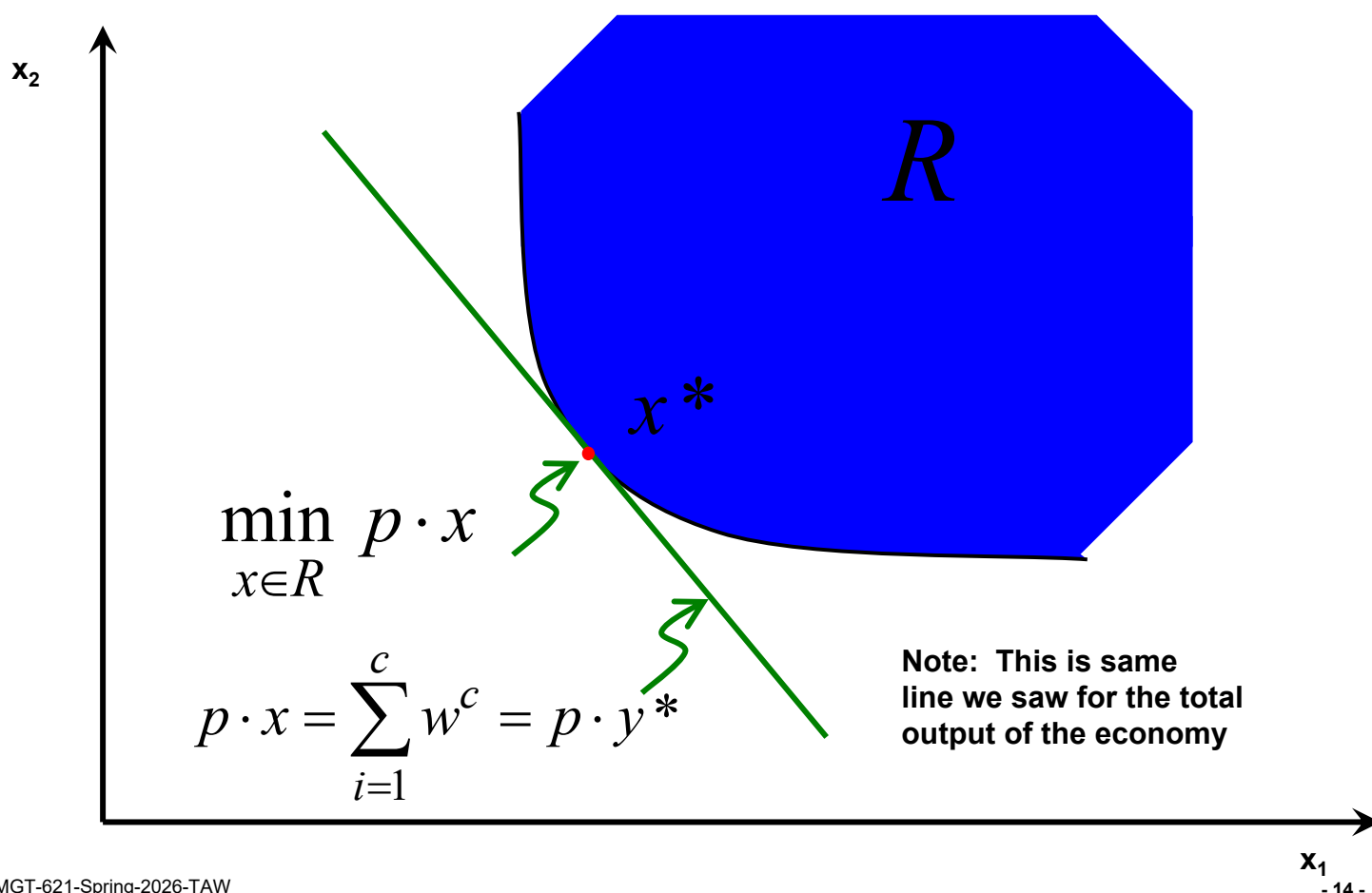
INDIVIDUAL MINIMIZATION IMPLIES GLOBAL MINIMIZATION



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MINIMIZATION OF TOTAL EXPENDITURE



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x_1
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AGENDA

Some Preliminaries

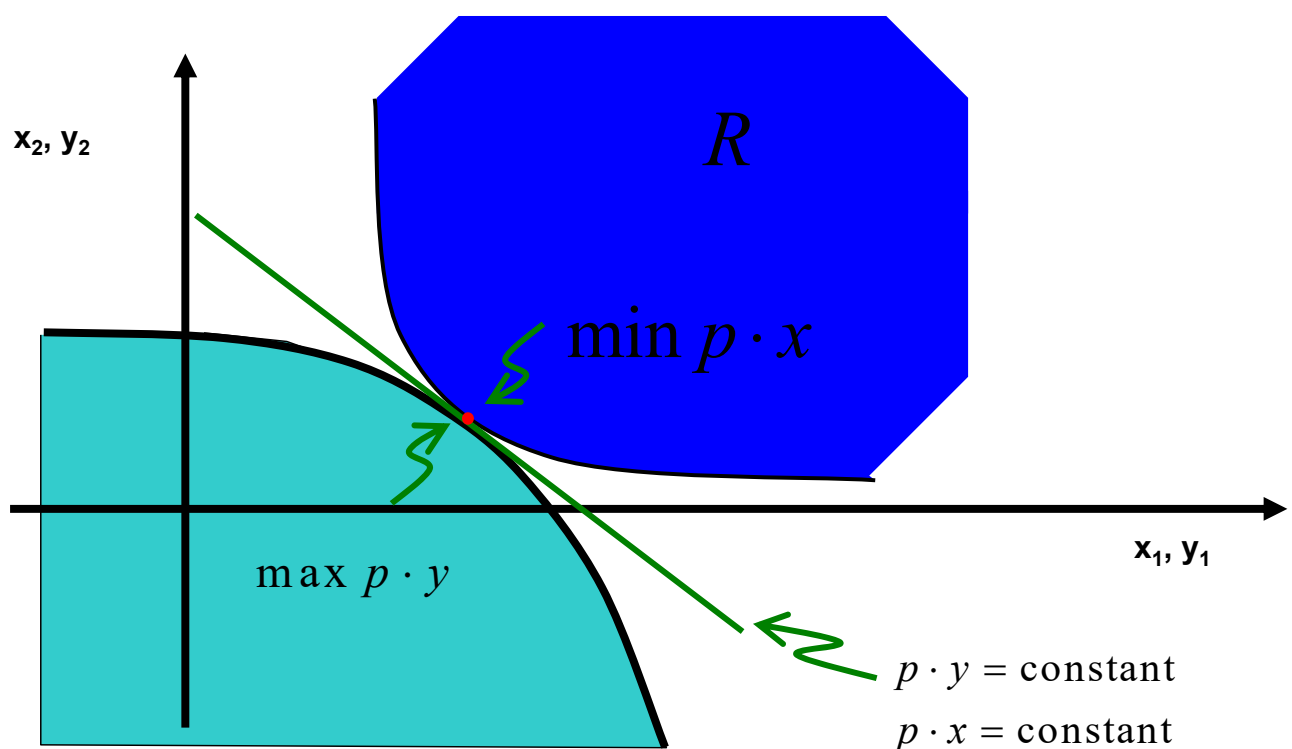
Fundamental Welfare Theorems

Existence of a Competitive Equilibrium

General Equilibrium vs. Partial Equilibrium

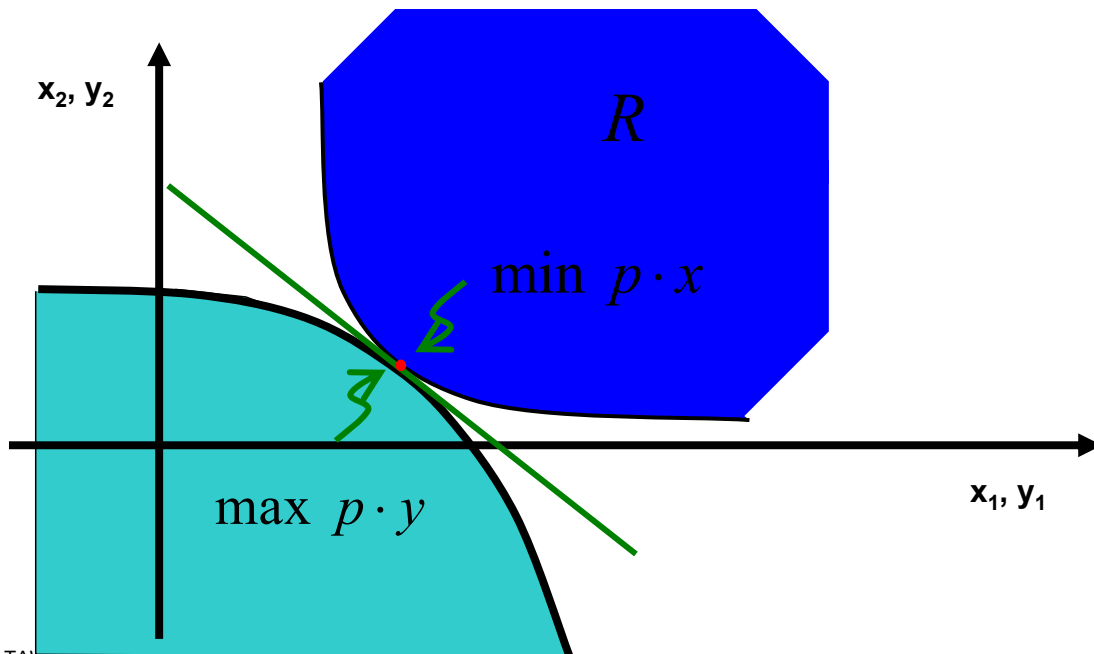
Key Concepts to Remember

COMPETITIVE EQUILIBRIUM MATCHES SUPPLY AND DEMAND



FIRST FUNDAMENTAL WELFARE THEOREM

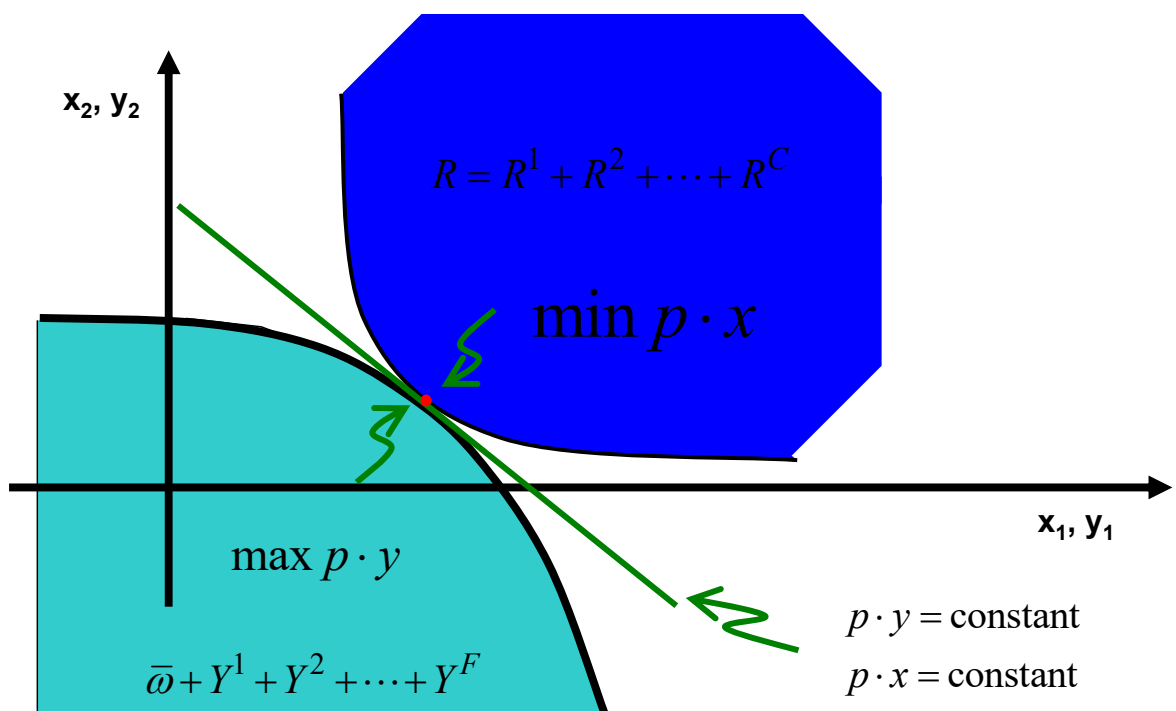
Competitive equilibrium implies set of economy-wide feasible outputs is separated from aggregate preference set, the set of points that allow Pareto-dominant allocation (neither set includes interior point of other set). Therefore, the competitive market equilibrium must be a Pareto-optimal allocation



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COMPETITIVE EQUILIBRIUM IS PARETO OPTIMAL



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FIRST FUNDAMENTAL WELFARE THEOREM

Definition: Assume that consumer c 's preferences are representable by a continuous utility function $u_c(\cdot)$. His preferences are **locally nonsatiated** if for any feasible consumption vector $x^c \in \mathfrak{R}_+^N$ and any $\varepsilon > 0$ there exists another feasible consumption vector $\hat{x}^c \in U_\varepsilon(x^c) = \{y \in \mathfrak{R}_+^N : \|y - x^c\| < \varepsilon\}$ such that $u_c(\hat{x}^c) > u_c(x^c)$.⁽¹⁾

Theorem (1st FWT): Assume that for all consumers $c \in \{1, \dots, C\}$ the utility function is locally nonsatiated. If $(p, (\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is a **Walrasian equilibrium**, then the allocation $((\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is **Pareto optimal**.

(1) Local nonsatiation is implied by strict monotonicity of consumer c 's utility function. The converse is not true since some of the components of the consumption vector may not be desirable, i.e., it may contain "bads" instead of "goods". (However, it is not possible that all consumption goods are "bads," since then at 0 would become a (global) satiation point.)

FIRST FUNDAMENTAL WELFARE THEOREM Proof

Proof: [by contradiction]

Suppose that $((x^1, \dots, x^C), (y^1, \dots, y^F))$ is a feasible allocation, such that for all $c \in \{1, \dots, C\}$:

$$u_c(x^c) \geq u_c(\hat{x}^c) \quad (1)$$

and for some c , say $c = c'$, we have a strict inequality. Then, necessarily (by **utility maximization**), it is

$$p \cdot x^{c'} > p \cdot \hat{x}^{c'} \quad (2)$$

and **local nonsatiation implies** that as a consequence of (1), for all $c \in \{1, \dots, C\}$:⁽¹⁾

$$p \cdot x^c \geq p \cdot \hat{x}^c$$

Hence, using (2),

$$p \cdot \left(\sum_{c=1}^C x^c \right) = \sum_{c=1}^C (p \cdot x^c) > \sum_{c=1}^C (p \cdot \hat{x}^c) = p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) \quad (3)$$

(1) Otherwise, any $\bar{x}^c \in \mathfrak{R}_+^N$ sufficiently close to x^c must satisfy $p \cdot \bar{x}^c < p \cdot \hat{x}^c$. But by local nonsatiation, there must exist at least one such $\bar{x}^c$ for which also $u_c(\bar{x}^c) > u_c(x^c)$. By transitivity this implies $u_c(\bar{x}^c) > u_c(\hat{x}^c)$, which contradicts the assumption that $\hat{x}^c$ solves the utility maximization problem as part of a Walrasian equilibrium. If some consumers were local satiated one may be able to transfer small amounts of money from consumers that are locally indifferent to a consumer that cares at the margin.

FIRST FUNDAMENTAL WELFARE THEOREM

Proof (cont'd)

Feasibility of the WE (i.e., demand = supply) **implies**

$$\sum_{c=1}^C x^c = \sum_{c=1}^C \omega^c + \sum_{f=1}^F y^f \quad (4)$$

and

$$\sum_{c=1}^C \hat{x}^c = \sum_{c=1}^C \omega^c + \sum_{f=1}^F \hat{y}^f \quad (5)$$

Combining (3)—(5) we obtain

$$p \cdot \left(\sum_{c=1}^C \omega^c + \sum_{f=1}^F y^f \right) > p \cdot \left(\sum_{c=1}^C \omega^c + \sum_{f=1}^F \hat{y}^f \right)$$

whence

$$p \cdot \left(\sum_{f=1}^F y^f \right) > p \cdot \left(\sum_{f=1}^F \hat{y}^f \right) \quad (6)$$

FIRST FUNDAMENTAL WELFARE THEOREM

Proof (cont'd)

Since the allocation $((x^1, \dots, x^C), (y^1, \dots, y^F))$ is by assumption feasible, we have that $y^f \in Y_f$ for all $f \in \{1, \dots, F\}$. **Profit maximization implies** that for all $f \in \{1, \dots, F\}$:

$$p \cdot \hat{y}^f \geq p \cdot y^f$$

But then it must be true that

$$p \cdot \left(\sum_{f=1}^F \hat{y}^f \right) \geq p \cdot \left(\sum_{f=1}^F y^f \right)$$

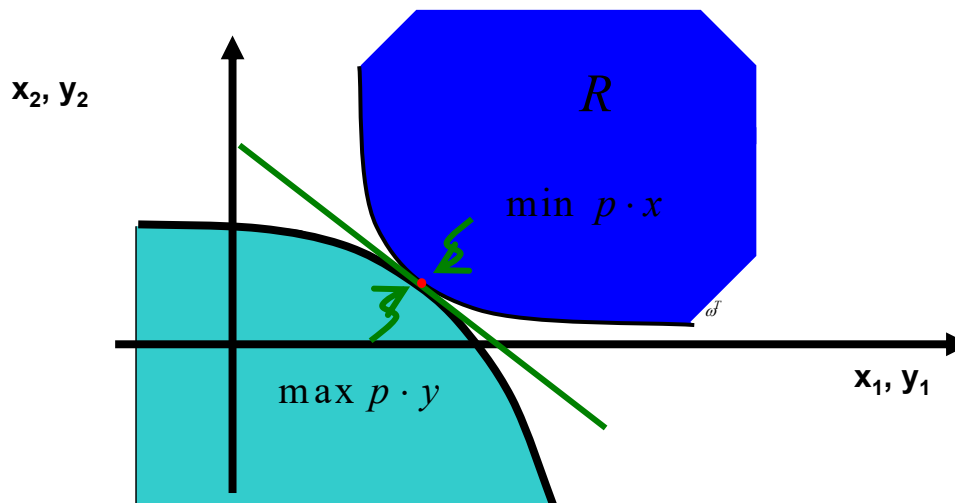
which contradicts (6). QED

SECOND FUNDAMENTAL WELFARE THEOREM

The common point is the **competitive equilibrium**, since

- (1) it minimizes expenditure,
- (2) it maximizes profits,
- (3) it has all supplies equal to all demands, and
- (4) it has all profits allocated to consumers.

However, wealth is not necessarily consistent with initial endowments. Thus, a **lump-sum wealth redistribution** is likely to be required.



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SECOND FUNDAMENTAL WELFARE THEOREM

Theorem (2nd FWT): Assume that for all consumers $c \in \{1, \dots, C\}$ the utility function is locally nonsatiated, continuous, and has convex upper contour sets. Let $\bar{\omega} \in \mathbb{R}_+^N$ be some vector of initial resources (endowments). (i) **If, starting from $\bar{\omega}$, the allocation $((\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is Pareto optimal, then there exists a price vector $p \in \mathbb{R}_+^N$ such that**

- for all $c \in \{1, \dots, C\}$: $u_c(x^c) \geq u_c(\hat{x}^c) \Rightarrow p \cdot x^c \geq p \cdot \hat{x}^c$
- for all $f \in \{1, \dots, F\}$: $y^f \in Y_f \Rightarrow p \cdot \hat{y}^f \geq p \cdot y^f$

(ii) **If, in addition, for all $c \in \{1, \dots, C\}$ there exists a vector $\bar{x}_c \in \mathbb{R}_+^N$ such that $p \cdot \hat{x}^c > p \cdot \bar{x}_c$, then there is a division of initial resources $\bar{\omega}, (\omega^1, \dots, \omega^C)$, and of firm ownership shares, $(\theta^1, \dots, \theta^C)$, such that $(p, (\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is a Walrasian equilibrium relative to $(\omega^1, \dots, \omega^C)$ and $(\theta^1, \dots, \theta^C)$.**

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SEPARATING HYPERPLANE THEOREM

Definition: A plane $P = \{x \in X : f(x) = 1\}$ separates two sets $A, B \subset X$, if

$$x \in A \Rightarrow f(x) \leq 1$$

$$x \in B \Rightarrow f(x) \geq 1$$

Hahn-Banach Theorem: Let A and B be two disjoint nonempty convex sets in a vector space X . If A has an inner point, then there exists a plane P separating A and B .⁽¹⁾

Separating Hyperplane Theorem: Let $A, B \subset \mathbb{R}^N$ be two disjoint nonempty convex sets. Then there exists a nonzero vector $p \in \mathbb{R}^N$ and a scalar $\alpha \in \mathbb{R}$ such that

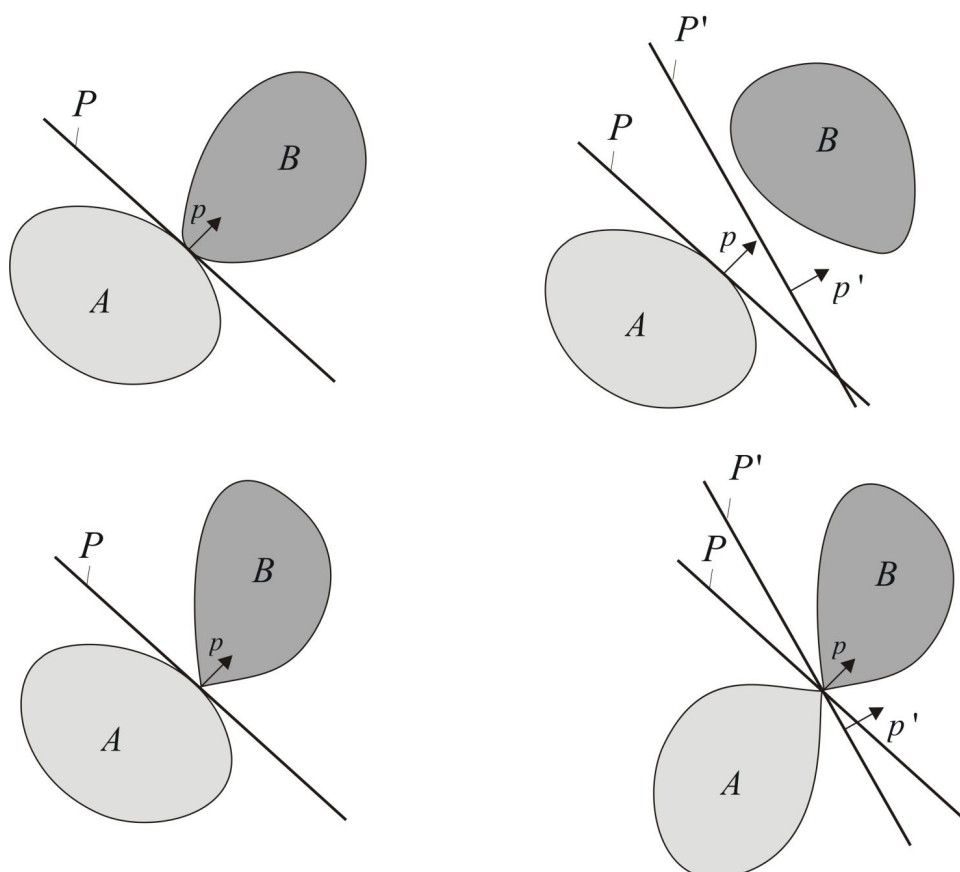
$$p \cdot x \leq \alpha \leq p \cdot y$$

for any $(x, y) \in A \times B$.⁽²⁾

(1) For a proof of this theorem, see e.g., Berge, C. (1963), "Topological Spaces," Oliver & Boyd, Edinburgh and London, UK, pp. 154—157. Reprinted by Dover Publications in 1997.

(2) In other words, it is possible to select a linear form f in the Hahn-Banach theorem. For a proof of that theorem, see MWG, p. 948.

SEPARATING HYPERPLANE THEOREM Geometric Interpretation



SECOND FUNDAMENTAL WELFARE THEOREM Proof

Proof: [proceeds in 7 steps]

Step 1: Apply the Separating Hyperplane Theorem

For all consumers $c \in \{1, \dots, C\}$, the set of preferred allocations (upper contour set),

$$V^c(\hat{x}^c) = \{x^c \in \mathfrak{R}_+^N : u_c(x^c) > u_c(\hat{x}^c)\}$$

is convex. As a result, $V = \sum_{c=1}^C V^c(\hat{x}^c)$ is convex. Similarly, convexity of the production set Y_f for all $f \in \{1, \dots, F\}$ implies that

$$Y = \sum_{f=1}^F Y_f + \{\bar{\omega}\}$$

is convex. By assumption we know that the allocation $((\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is Pareto optimal, i.e.,

$$\left(\sum_{c=1}^C V^c(\hat{x}^c) \right) \cap \left(\sum_{f=1}^F Y_f + \{\bar{\omega}\} \right) = V \cap Y = \emptyset$$

In other words, *there is nothing that the economy can produce that makes everybody better off.*

SECOND FUNDAMENTAL WELFARE THEOREM Proof (cont'd)

The separating hyperplane theorem implies that for any $(x, y) \in V \times Y$ there exists a vector p and a scalar α , such that $p \cdot y \leq \alpha \leq p \cdot x$

Step 2: Show that $p \cdot \left(\sum_{f=1}^F \hat{y}^f + \bar{\omega} \right) = p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) = \alpha$

Since $((\hat{x}^1, \dots, \hat{x}^C), (\hat{y}^1, \dots, \hat{y}^F))$ is feasible, we have $\sum_{c=1}^C \hat{x}^c = \sum_{f=1}^F \hat{y}^f + \bar{\omega} \in Y$, so that by Step 1:

$$\alpha \geq p \cdot \left(\sum_{f=1}^F \hat{y}^f + \bar{\omega} \right) = p \cdot \left(\sum_{c=1}^C \hat{x}^c \right)$$

Now, for each $c \in \{1, \dots, C\}$ and $n \geq 1$, let

$$\hat{x}^c(n) = \hat{x}^c + \left(\frac{1}{n}, \dots, \frac{1}{n} \right)$$

By local nonsatiation⁽¹⁾ it is $\hat{x}^c(n) \in V^c(\hat{x}^c)$ and thus $\sum_{c=1}^C \hat{x}^c(n) \in V$. Hence by Step 1:

$$p \cdot \left(\sum_{c=1}^C \hat{x}^c(n) \right) \geq \alpha$$

(1) Actually we are using monotonicity here. For a justification see Step 3, where the same construction is used.

SECOND FUNDAMENTAL WELFARE THEOREM Proof (cont'd)

Taking the limit for $n \rightarrow \infty$ gives thus $\alpha \leq \lim_{n \rightarrow \infty} p \cdot \left(\sum_{c=1}^C \hat{x}^c(n) \right) = p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) = p \cdot \left(\sum_{f=1}^F \hat{y}^f + \bar{\omega} \right)$

Step 3: Show that $x \in \bar{V} \Rightarrow p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) \leq p \cdot x$, where $\bar{V} = \sum_{c=1}^C \underbrace{\{x^c \in \mathcal{R}_+^N : u_c(x^c) \geq u_c(\hat{x}^c)\}}_{\bar{V}^c(\hat{x}^c)}$

For simplicity, let us assume here that all the commodities are desirable, so that local nonsatiation is equivalent to monotonicity of the consumers' utility functions. For any $x^c \in \bar{V}$ let

$$x^c(n) = x^c + \left(\frac{1}{n}, \dots, \frac{1}{n} \right)$$

so that by monotonicity, $x^c(n) \in V^c(\hat{x}^c)$ and $\sum_{c=1}^C x^c(n) \in V$

Hence, by Step 1, $p \cdot \left(\sum_{c=1}^C x^c(n) \right) \geq \alpha$, so that after taking the limit for $n \rightarrow \infty$ we obtain

$$p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) = \alpha \leq \lim_{n \rightarrow \infty} p \cdot \left(\sum_{c=1}^C x^c(n) \right) = p \cdot \left(\sum_{c=1}^C x^c \right) = p \cdot x$$

SECOND FUNDAMENTAL WELFARE THEOREM Proof (cont'd)

Step 4: $y \in Y \Rightarrow p \cdot \left(\sum_{f=1}^F \hat{y}^f + \bar{\omega} \right) \geq p \cdot y$ and thus⁽¹⁾ $y^f \in Y_f \Rightarrow p \cdot \hat{y}^f \geq p \cdot y^f$

Step 5: $x \in \bar{V} \Rightarrow p \cdot \left(\sum_{c=1}^C \hat{x}^c \right) \leq p \cdot x$ and thus⁽¹⁾ $x^c \in \bar{V}^c(\hat{x}^c) \Rightarrow p \cdot \hat{x}^c \leq p \cdot x^c$

Step 6: Show that: if for all consumers $c \in \{1, \dots, C\}$ there exists a vector $\bar{x}^c \in \mathcal{R}_+^N$ such that $p \cdot \bar{x}^c < p \cdot \hat{x}^c$, then $u_c(x^c) > u_c(\hat{x}^c) \Rightarrow p \cdot \hat{x}^c < p \cdot x^c$.

By Step 5, $p \cdot \bar{x}^c < p \cdot \hat{x}^c$ implies that $u_c(\hat{x}^c) > u_c(\bar{x}^c)$ and (since $u_c(x^c) > u_c(\hat{x}^c)$) also $p \cdot \hat{x}^c \leq p \cdot x^c$. Thus, $p \cdot \bar{x}^c < p \cdot (\beta x^c + (1-\beta)\bar{x}^c) < p \cdot x^c$ for any $\beta \in (0,1)$.

By the continuity of $u_c(\cdot)$ there is a $\beta \in (0,1)$ such that $u_c(\hat{x}^c) = u_c(\beta x^c + (1-\beta)\bar{x}^c)$.

But $\beta x^c + (1-\beta)\bar{x}^c \in \bar{V}^c(\hat{x}^c)$, so that by Step 5 $p \cdot \hat{x}^c \leq p \cdot (\beta x^c + (1-\beta)\bar{x}^c)$, and thus $p \cdot \hat{x}^c \leq p \cdot (\beta x^c + (1-\beta)\bar{x}^c) < p \cdot x^c$ as claimed.

(1) Just consider the inequality for each consumer/producer individually by setting all other components to $\hat{x}^c$ or $\hat{y}^f$, so that they cancel out.

SECOND FUNDAMENTAL WELFARE THEOREM

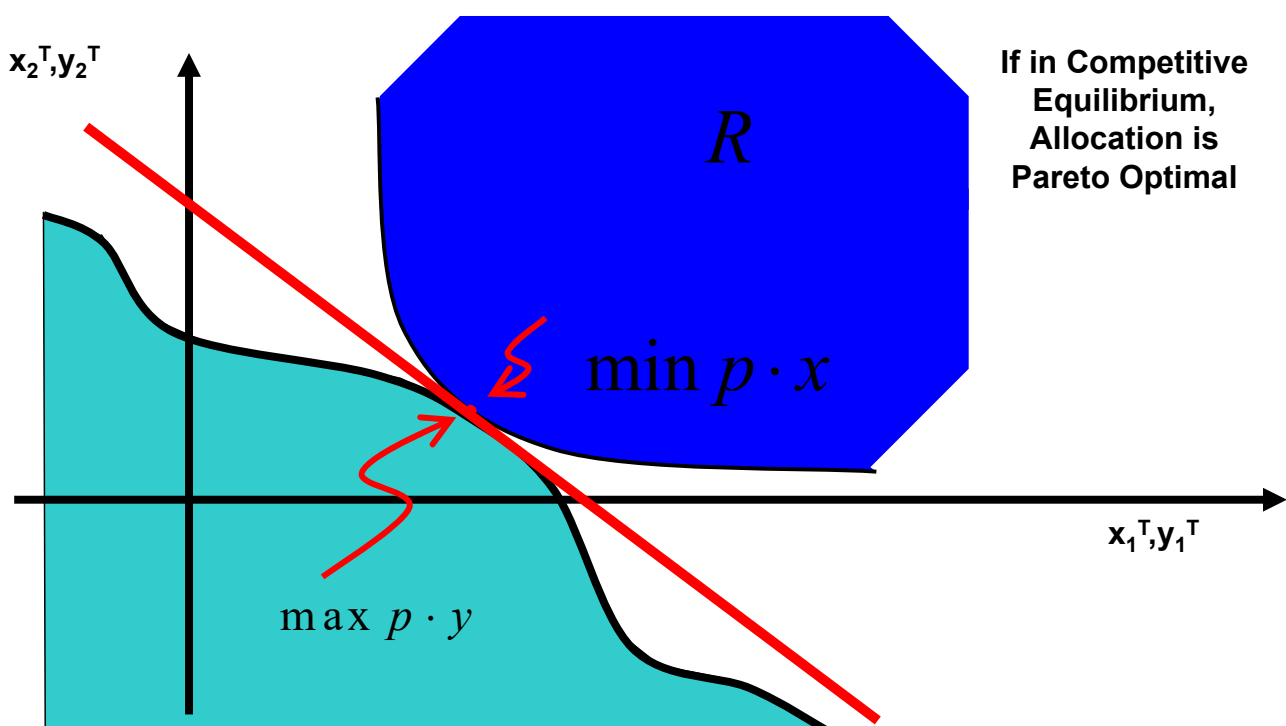
Proof (cont'd)

Step 7: It is now **enough to choose a division** of the initial endowment $\omega, (\omega^1, \dots, \omega^C)$, and of firm ownership shares, $(\theta^1, \dots, \theta^C)$, **such that**

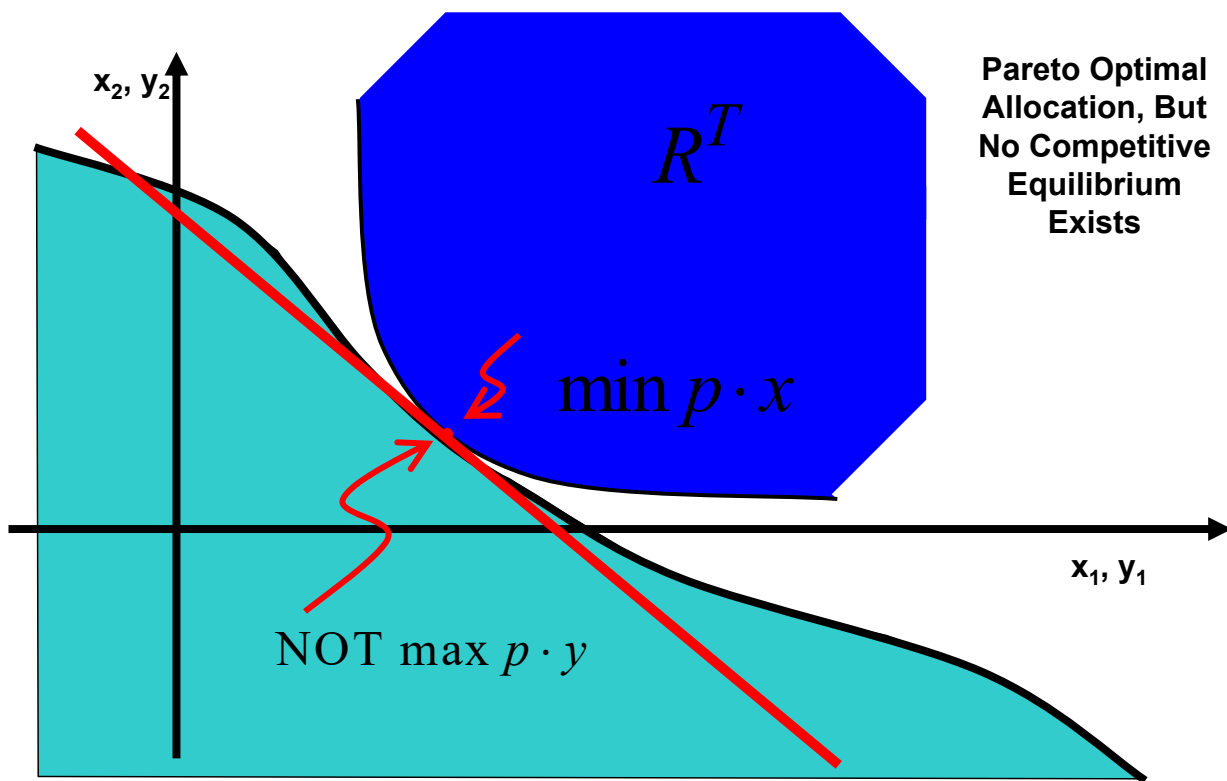
$$p \cdot \hat{x}^c = p \cdot \omega^c + \sum_{f=1}^F \theta_f^c (p \cdot \hat{y}^f)$$

which completes our proof. QED

NONCONVEXITY (1st FWC)



NONCONVEXITY (2nd FWC)



AGENDA

Some Preliminaries

Fundamental Welfare Theorems

Existence of a Competitive Equilibrium

General Equilibrium vs. Partial Equilibrium

Key Concepts to Remember

HOMOGENEOUS FUNCTIONS

Definition: A function $g : \mathbb{R}_+^N \rightarrow \mathbb{R}$ is **homogeneous of degree** k , if for any $\lambda > 0$ and $x \in \mathbb{R}_+^N$:

$$g(\lambda x) = \lambda^k g(x)$$

Examples:

- The **supply function** $y^f(p) = \arg \max_{y \in Y_f} p \cdot y$

is homogeneous of **degree zero**. Indeed, for any $\lambda > 0$ we have that

$$y^f(p) = \arg \max_{y \in Y_f} p \cdot y = \arg \max_{y \in Y_f} \{(\lambda p) \cdot y\}$$

- The **profit function** $\pi^f(p) = \max_{y \in Y_f} p \cdot y$ is also **homogeneous of degree one**, since

$$\lambda \pi^f(p) = \lambda \max_{y \in Y_f} p \cdot y = \max_{y \in Y_f} \{(\lambda p) \cdot y\} = \pi^f(\lambda p)$$

EXCESS DEMAND

Definition: The **excess demand function** for consumer $c \in \{1, \dots, C\}$ is

$$\begin{aligned} z^c(p) &= x^c(p, I^c) - \omega^c \\ &= x^c(p, p \cdot \omega^c + \sum_{f=1}^F \theta_f^c \pi^f(p)) - \omega^c \end{aligned}$$

Summing up over all consumers and subtracting the firms' production, the function

$$z(p) = \sum_{c=1}^C z^c(p) - \sum_{f=1}^F y^f(p)$$

denotes **excess market demand** (also referred to **aggregate excess demand function**).

Exercise: Show that the excess demand function and the excess market demand are homogeneous of degree zero.

WALRAS' LAW

Proposition (Walras' Law): For any price vector p the value of excess market demand is zero, i.e.,

$$p \cdot z(p) = 0$$

Proof: Consumer c 's budget constraint implies that

$$p \cdot z^c(p) = p \cdot x^c(p, p \cdot \omega^c + \sum_{f=1}^F \theta_f^c \pi^f(p)) - p \cdot \omega^c = \sum_{f=1}^F \theta_f^c \pi^f(p)$$

Adding up over all consumers and subtracting the firms' production yields

$$\begin{aligned} p \cdot z(p) &= p \cdot \left[\sum_{c=1}^C z^c(p) - \sum_{f=1}^F y^f(p) \right] = \sum_{f=1}^F \sum_{c=1}^C \theta_f^c \pi^f(p) - \sum_{f=1}^F p \cdot y^f(p) \\ &= \sum_{f=1}^F \pi^f(p) - \sum_{f=1}^F p \cdot y^f(p) = \sum_{f=1}^F p \cdot y^f(p) - \sum_{f=1}^F p \cdot y^f(p) = 0 \end{aligned}$$

QED

EXISTENCE OF A WALRASIAN EQUILIBRIUM

Proposition: Assume that the **supply function** $y^f(p)$ and the (finite) demand function

$$x^c(p, p \cdot \omega^c + \sum_{f=1}^F \theta_f^c \pi^f(p)) < \infty$$

exist for all $c \in \{1, \dots, C\}$, $f \in \{1, \dots, F\}$, and all $p \in \Delta = \{\hat{p} \in [0, 1] : \sum_{i=1}^N \hat{p}_i = 1\}$
Suppose further that

- The **production sets** Y_f are **closed, bounded, and strictly convex**
- The **utility functions** $u_c(\cdot)$ are **continuous, locally nonsatiated, and with strictly convex upper contour sets** $V^c(\cdot)$

Then there exists a price vector $p^* \in \Delta$ **such that excess market demand is zero, i.e.,** $z(p^*) = 0$ **(this price supports a WE)**

EXISTENCE OF A WALRASIAN EQUILIBRIUM

Proof

Proof: The supply function $y^f(p)$ and the (finite) demand function

$$x^c(p, p \cdot \omega^c + \sum_{f=1}^F \theta_f^c \pi^f(p))$$

exist for all $c \in \{1, \dots, C\}$, $f \in \{1, \dots, F\}$, are unique as a consequence of the imposed convexity/concavity assumptions, and are continuous⁽¹⁾ in $p \in \Delta$. Hence the market excess demand function $z(p) = (z_1(p), \dots, z_n(p))$ is unique and continuous on Δ .

Let us now define

$$z_i^+(p) = \max\{z_i(p), 0\}$$

and the corresponding vector

$$z^+(p) = (z_1^+(p), \dots, z_n^+(p))$$

Then the mapping $h: \Delta \rightarrow \Delta$ with

$$h(p) = \frac{p + z^+(p)}{\sum_{i=1}^n (p_i + z_i^+(p))}$$

is well-defined and continuous on Δ .

(1) This can be concluded e.g., from Berge's maximum theorem (cf. an earlier lecture).

EXISTENCE OF A WALRASIAN EQUILIBRIUM

Proof (cont'd)

Brower's fixed-point theorem implies that the mapping h possesses a fixed point p^* in Δ , i.e.,

$$h(p^*) = \frac{p^* + z^+(p^*)}{\sum_{i=1}^n (p_i^* + z_i^+(p^*))} = p^*$$

Using Walras' Law we find

$$0 = p^* z(p^*) = h(p^*) z(p^*) = \frac{\overbrace{p^* z(p^*)}^{=0} + z^+(p^*) z(p^*)}{\sum_{i=1}^n (p_i^* + z_i^+(p^*))} \stackrel{\sum_{i=1}^n p_i^* = 1}{=} \frac{z^+(p^*) z(p^*)}{1 + \sum_{i=1}^n z_i^+(p^*)}$$

and therefore

$$0 = z^+(p^*) z(p^*) = \sum_{i=1}^n z_i(p^*) \max\{0, z_i(p^*)\}$$

which implies that

$$z_i(p^*) \leq 0$$

(1) This can be concluded e.g., from Berge's maximum theorem (cf. an earlier lecture).

EXISTENCE OF A WALRASIAN EQUILIBRIUM

Proof (Cont'd)

In addition, since $z^+(p^*) = p^* \sum_{i=1}^N z_i^+(p^*)$, good i can be in excess supply, i.e., $z(p^*) < 0$, only if it is worthless, that is to say only if $p_i^* = 0$.

In particular, $0 = p^* \cdot z(p^*) = p_1 z_1(p^*) + \cdots + p_N z_N(p^*)$ implies that

$$z_i(p^*) < 0 \Rightarrow p_i^* = 0$$

One can also show that the fixed point p^* needs to occur in the interior of the simplex Δ , (cf. MWG, p. 586) so that the excess demand must vanish in equilibrium,

$$z(p^*) = 0$$

QED

AGENDA

Some Preliminaries

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General Equilibrium vs. Partial Equilibrium

Key Concepts to Remember

GENERAL VS. PARTIAL EQUILIBRIUM ANALYSIS

To see how General Equilibrium Theory can yield predictions that are radically different from Partial Equilibrium Theory, consider the following example.

Example: Tax Incidence

Consider an economy with N cities (where N is a large number).

- In each city there is a single price-taking firm that produces a single consumption good using the increasing, strictly concave production function $f(\cdot)$
- There are M identical workers. Each worker is free to move between cities to be paid the highest wage.
- Each worker derives utility from the single consumption good that is available. Without loss of generality the price of the consumption good can be normalized to 1.

Question: If a tax on labor is levied in city 1, who bears the cost (firms or workers)?

GENERAL VS. PARTIAL EQUILIBRIUM (cont'd)

Analysis I (Partial Equilibrium) – Consider only City 1

- Before the tax is introduced, given that workers can move freely, wages must be equal in each city, i.e.,

$$w_1 = \dots = w_N = \bar{w} = f'(M/N)$$

which yields each firm's equilibrium profit, $\bar{\pi} = f(M/N) - \bar{w}(M/N)$

- The supply of workers in city 1 must be completely elastic, and thus the equilibrium wage after the tax $t \geq 0$ is introduced must still be equal to $\bar{w}$
- Hence, we find that in city 1, output drops to

$$\pi_1 = f(L_1) - (\bar{w} + t)L_1 < \bar{\pi}$$

where the labor used in city 1, L_1 , is such that $f'(L_1) = \bar{w} + t$

- As a result, since $L_1 < M/N$, some labor moves away from city 1, but all the tax is borne by producers!

GENERAL VS. PARTIAL EQUILIBRIUM (cont'd)

Analysis II (General Equilibrium)

- *Before the tax is introduced*, we obtain the same analysis as before.
- Let $w(t) = w_1 = \dots = w_N$ be the **common equilibrium wage** in all cities *after the tax* $t \geq 0$ is introduced
- **Demand = Supply** yields $(N-1)L(t) + L_1(t) = M$, where $L(t)$ is the equilibrium labor demand in cities 2,...,N, and $L_1(t)$ is the equilibrium labor demand in city 1
- **Profit maximization** yields $f'(L_1(t)) = w(t) + t$ and $f'(L(t)) = w(t)$
- Using the boundary condition for $t = 0$, when $L(0) = L_1(0) = M / N$, we find by differentiating the optimality conditions and evaluating at $t = 0$:

$$f''(L_1(0))L'_1(0) = -f''(M/N)(N-1)L'(0) = w'(0) + 1$$

$$f''(M/N)L'(0) = w'(0)$$

so that $w'(0) = -1/N$. In other words, the **wage rate in all cities declines** with an imposition of a tax on labor

GENERAL VS. PARTIAL EQUILIBRIUM (cont'd)

- Let us now **consider the change of firm profits**, which again can be done by differentiating profits with respect to t and evaluating at $t=0$:⁽¹⁾

$$(N-1)\bar{\pi}'(\bar{w})w'(0) + \bar{\pi}'(\bar{w})(w'(0) + 1) = \bar{\pi}'(\bar{w})\left(-\frac{N-1}{N} + \frac{N-1}{N}\right) = 0$$

In other words, **aggregate profits are very little affected (and in the limit unaffected) by a (small) tax.**

- We therefore find that (at least for small taxes) **virtually all of the tax in city 1 is incurred by the workers**, which is the *opposite* conclusion of what we obtained using partial equilibrium analysis!

(1) A more detailed proof of this statement can be given as follows. Let $S(t) = (f(z_1) - (w+t)z_1) + (N-1)(f(z) - wz) = f(z_1) + (N-1)f(z) - w(z_1 + (N-1)z) - tz_1$ be the sum of the firms' profits at the tax rate t . Note that (supply = demand) implies that $z_1 + (N-1)z = M$, so that $S(t) = f(z_1) + (N-1)f(z) - wM - tz_1$. We obtained earlier that $w'(0) = -1/N$, and $z_1(0) = z(0) = M/N$. We can therefore differentiate $S(t)$ with respect to t and evaluate at $t = 0$:

$$S'(0) = f'(M/N)(z'_1(0) + (N-1)z'(0)) - w'(0)M - z_1(0) = f'(M/N)\frac{d}{dt}\bigg|_{t=0} (z_1(t) + (N-1)z(t)) - (-(M/N) + (M/N)) = f'(M/N)(M)' - 0 = 0$$

The significance of this result is that small deviations from zero in the tax level have an arbitrarily small impact on aggregate profit, whereas the impact on the workers' utilities has a strictly positive slope. In other words, in the limit a very small positive tax is borne entirely by the workers.

AGENDA

Some Preliminaries

Fundamental Welfare Theorems

Existence of a Competitive Equilibrium

General Equilibrium vs. Partial Equilibrium

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Set Summation**
- **Walrasian Equilibrium (Competitive Equilibrium) (w/ or w/o transfers)**
- **Fundamental Welfare Theorems**
- **Separating Hyperplane Theorem**
- **Walras' Law**
- **General Equilibrium vs. Partial Equilibrium**

MGT 621 – MICROECONOMICS

Thomas A. Weber

9. Markets and Intermediaries

Autumn 2026

**École Polytechnique Fédérale de Lausanne
College of Management of Technology**

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AGENDA

What is an intermediary?

Intermediary's Value Proposition

Key Concepts to Remember

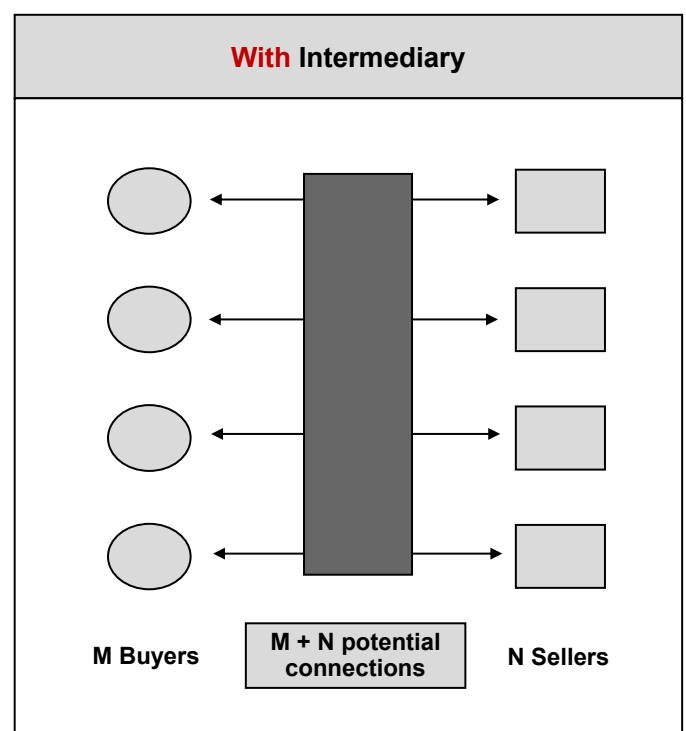
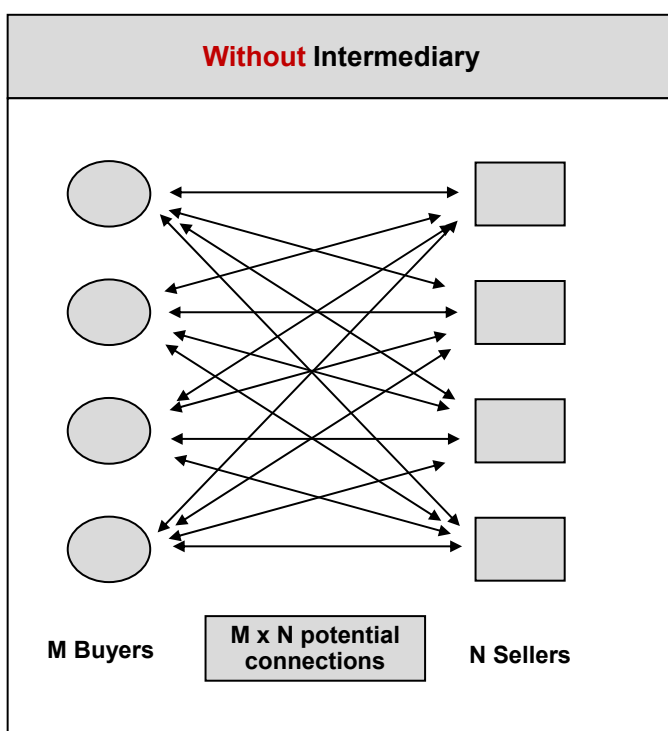
WHAT IS AN INTERMEDIARY

Definition. An **intermediary** offers intermediation services between two trading parties by acting as a *conduit for goods and services* offered by a supplier to a consumer. Typically the intermediary offers an *added value to the transaction* that is not available in a direct exchange between the two trading parties.



"I was hoping we could avoid the middleman."

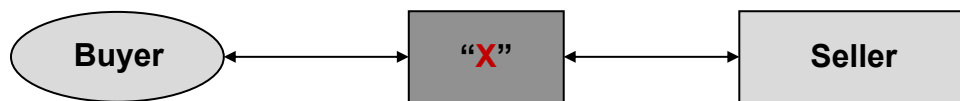
MARKETS CAN BE **WITH OR WITHOUT** INTERMEDIARIES



THERE ARE MANY DIFFERENT NAMES FOR THE SAME THING **Intermediary** ...

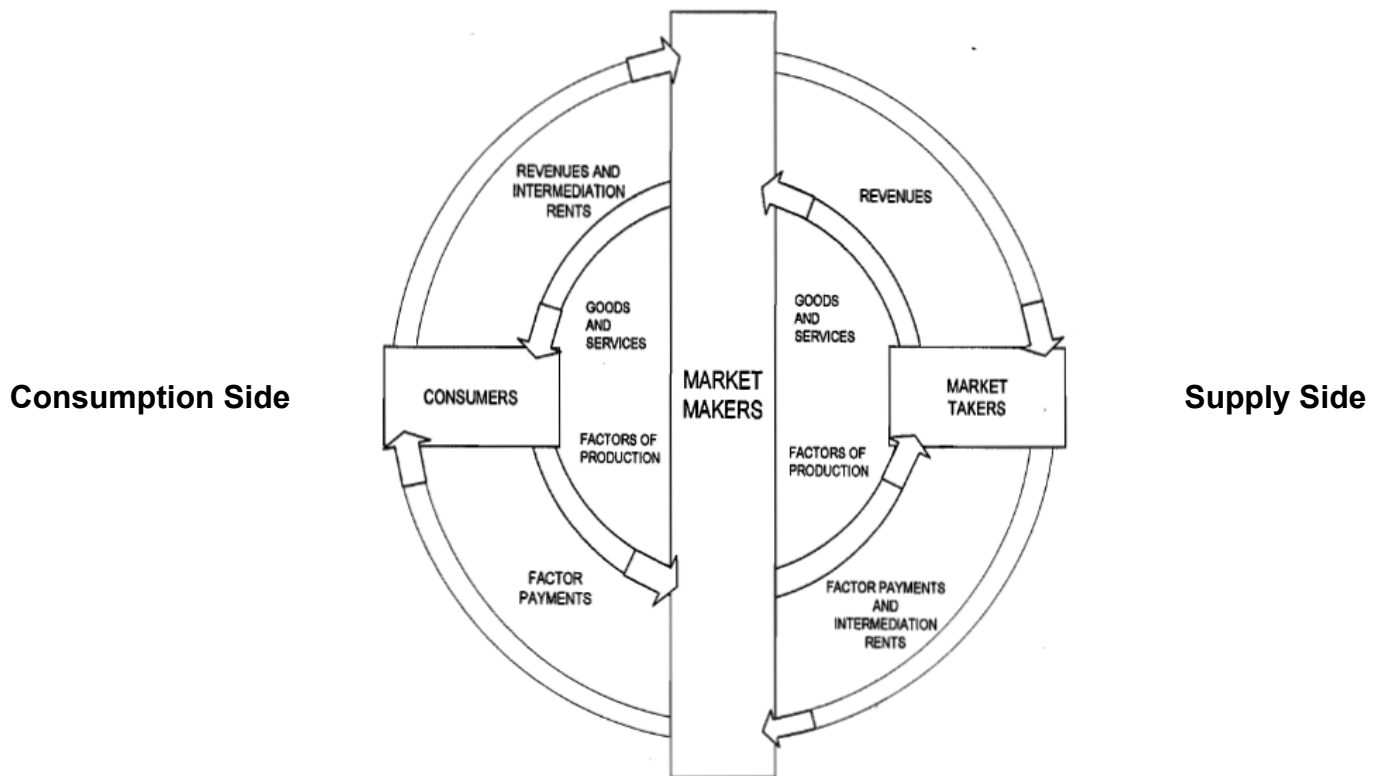
... also known as:

Portal, Middleman, Electronic Broker, Infomediary, Information Aggregator, Vortex, Butterfly Market, Net Market Maker, Hub, Metamediary, Reintermediary, Distributor, Importer, Exporter, Wholesaler, Retailer, Shipper, Information Provider, Trading Company, Dealer, Market Maker, ...



THERE ARE **FOUR MAIN ROLES** OF MARKET INTERMEDIARIES ... in **Physical** and **Electronic** Markets

INTERMEDIARIES ARE **MARKET MAKERS** ... and Create **Two-Sided Markets** ...



Source: Spulber (1999)

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EXAMPLES OF INTERMEDIARIES

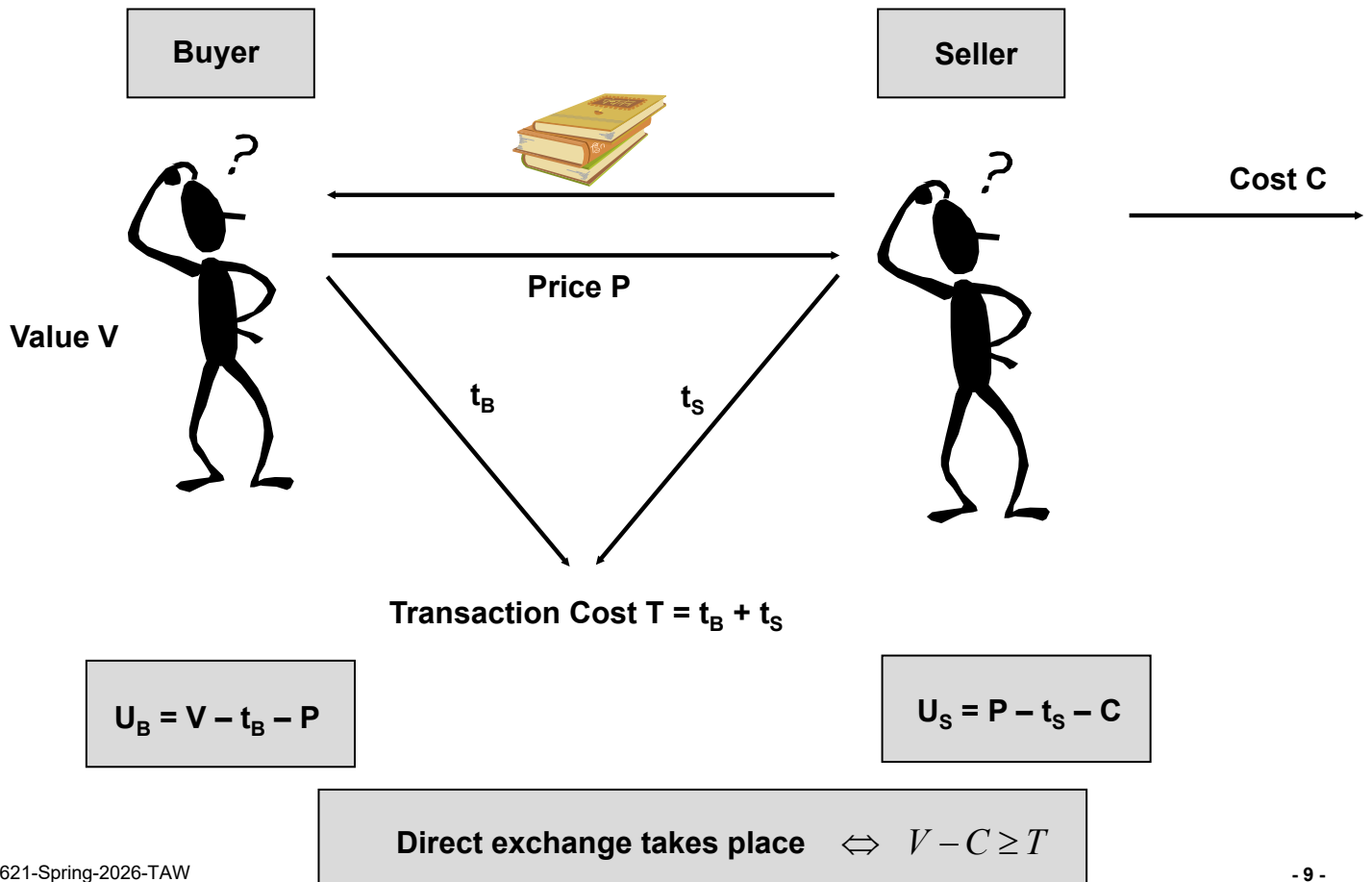
There are plenty ...

- 
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DIRECT EXCHANGE



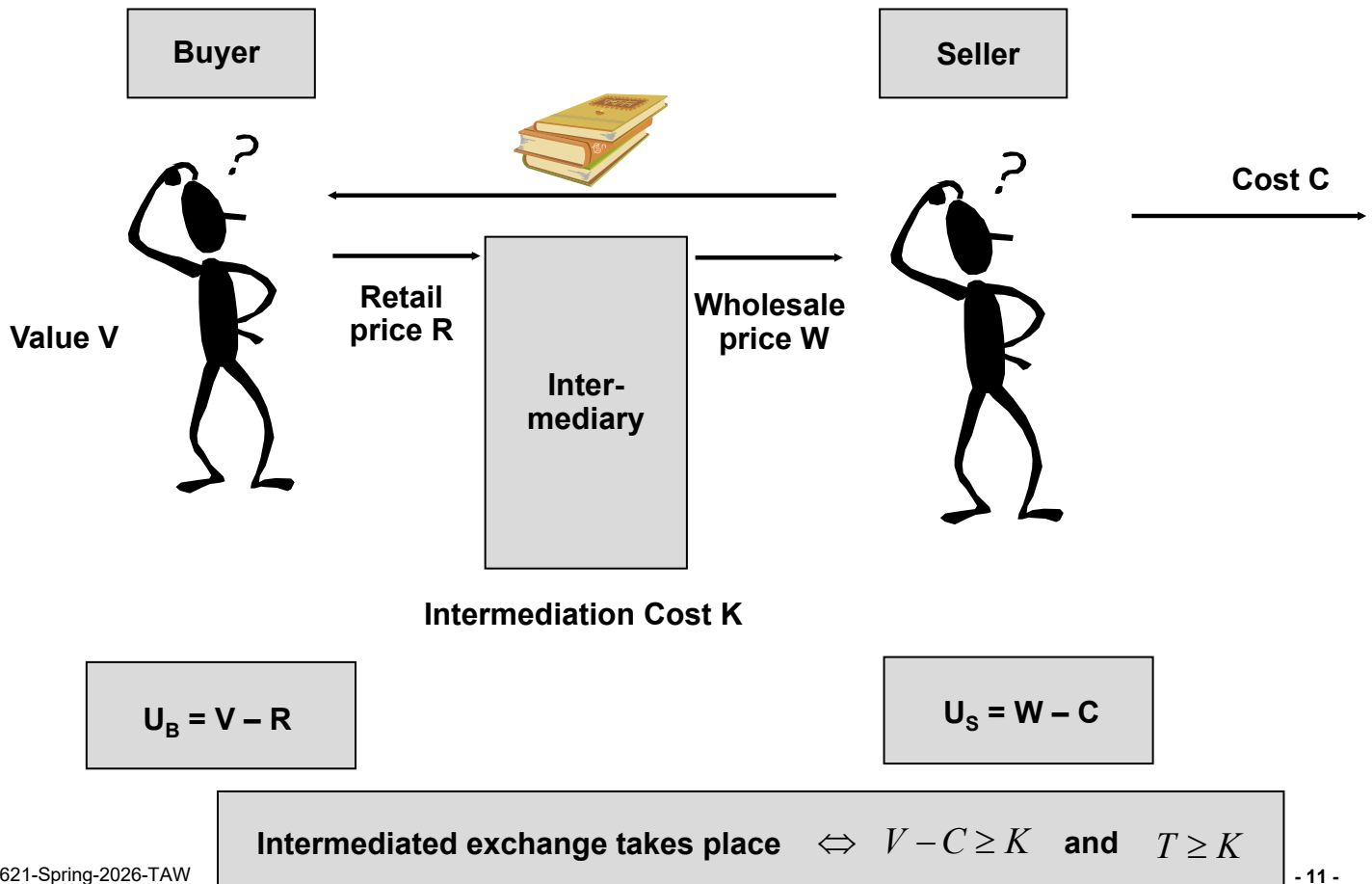
AGENDA

What is an intermediary?

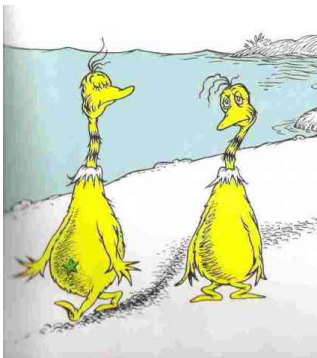
Intermediary's Value Proposition

Key Concepts to Remember

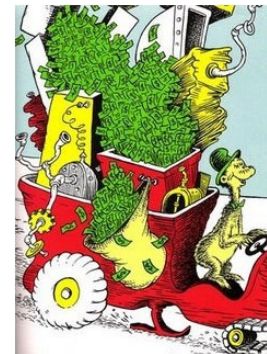
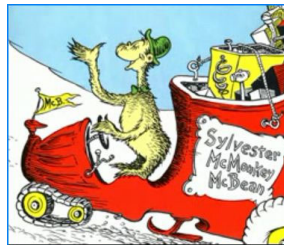
INTERMEDIATED EXCHANGE



TRANSACTION COST Example: The **Sneetches**



"Now, the Star-Belly Sneetches had bellies with stars, The Plain-Belly Sneetches had none upon thars. The stars weren't so big; they were really quite small. You would think such a thing wouldn't matter at all. But because they had stars, all the Star-bellied Sneetches would brag, "We're the best kind of Sneetch on the beaches."



REASONS FOR LOWER TRANSACTION COSTS

- Intermediary trades **larger volume** → **Economies of scale**
- **Commitment** power → **Intermediary can guarantee prices**
- **Longevity** of Intermediary → **Reputation** → **Trust**
- **Information aggregation** → **Intermediary knows more**
- **Inventory** → Intermediary can achieve **immediacy** by keeping an inventory

MONOPOLISTIC INTERMEDIARY

The **profit-maximizing** prices (R,W) satisfy

$$V - R = (V - C - T)/2 = W - C.$$

The intermediary's "**markup**" is therefore

$$R - W = T.$$

Hence, the intermediary is **viable** if and only if

$$K \leq T.$$

PRICE COMPETITION BETWEEN INTERMEDIARIES

“Bertrand Price Competition”

• Assume two intermediaries have **identical intermediation cost K** . Then in a simultaneous-move price-setting game their gains are dissipated fully, since undercutting the opponent is a dominant strategy, as long as payoffs are positive.

• Hence, at the unique Nash equilibrium, both intermediaries charge (R, W) such that

$$V - R = (V - C - K)/2 = W - C \quad \text{and} \quad R - W = K$$

• The **intermediaries' equilibrium payoffs are zero**, while the seller's and the buyer's payoffs are $(V - C - K)/2$, respectively.

Intermediaries can enable social gains by lowering transaction cost.

TRANSACTION COST DECREASE: **IMMEDIACY**

• Assume that the buyer and seller are equally sensitive to the time value of money and have a **common per-period discount factor $\delta \in (0, 1)$** . When conducting a direct exchange, the gains from trade, $V - C$, are realized in the following period. The surplus to be divided between the trading parties becomes

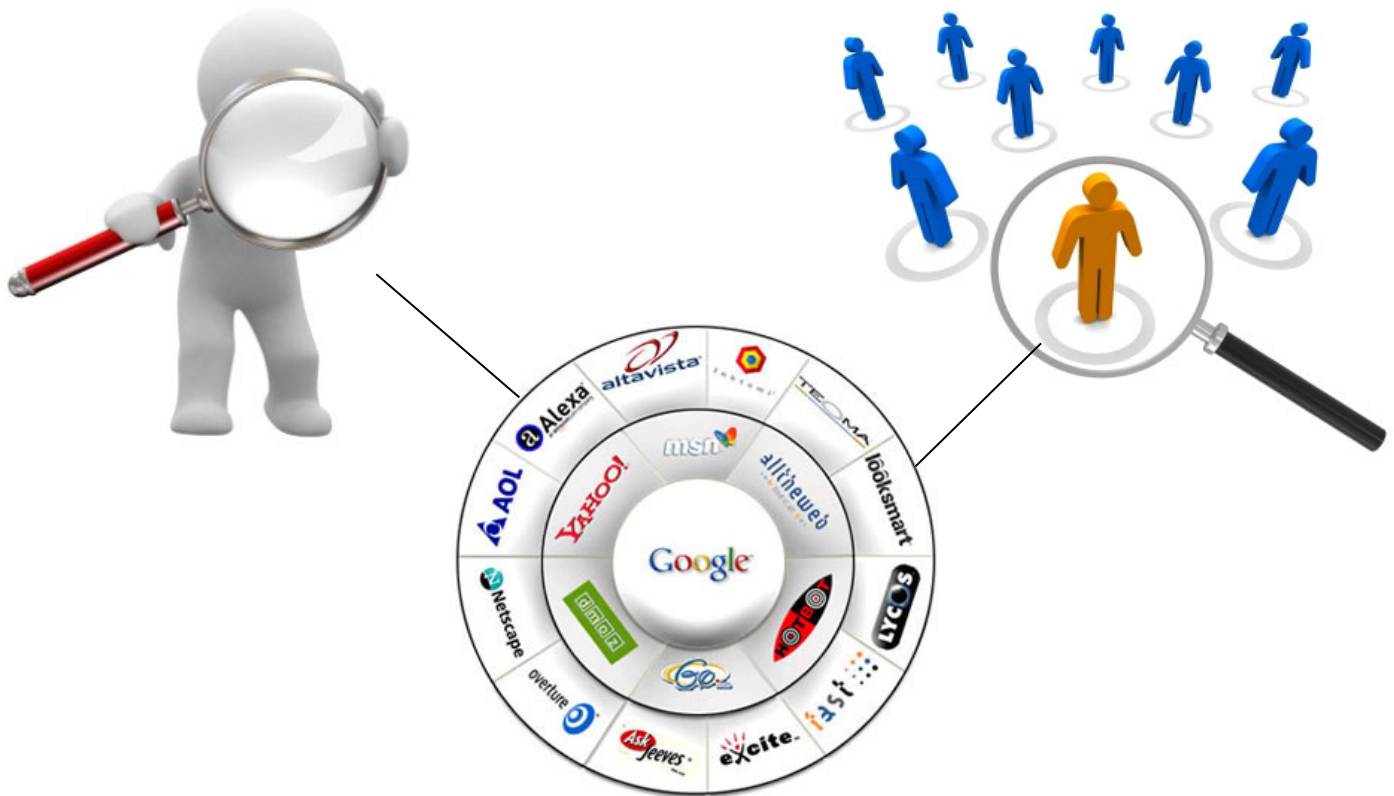
$$S = \delta (V - C).$$

• An intermediary, **e.g., by keeping an inventory** of the items to be traded, can **provide immediacy** of the exchange.

• It is **viable** if and only if the intermediation cost K is such that

$$K \leq (1 - \delta)(V - C).$$

SEARCH → MATCHING



TRANSACTION COST DECREASE: ELIMINATION OF SEARCH

• Assume that the **buyer and seller** have a probability $\beta \in (0,1)$ of meeting in a **direct exchange**. Then **with probability $(1 - \beta)$ no exchange takes place**, resulting in a transaction cost

$$T = (1 - \beta)(V - C).$$

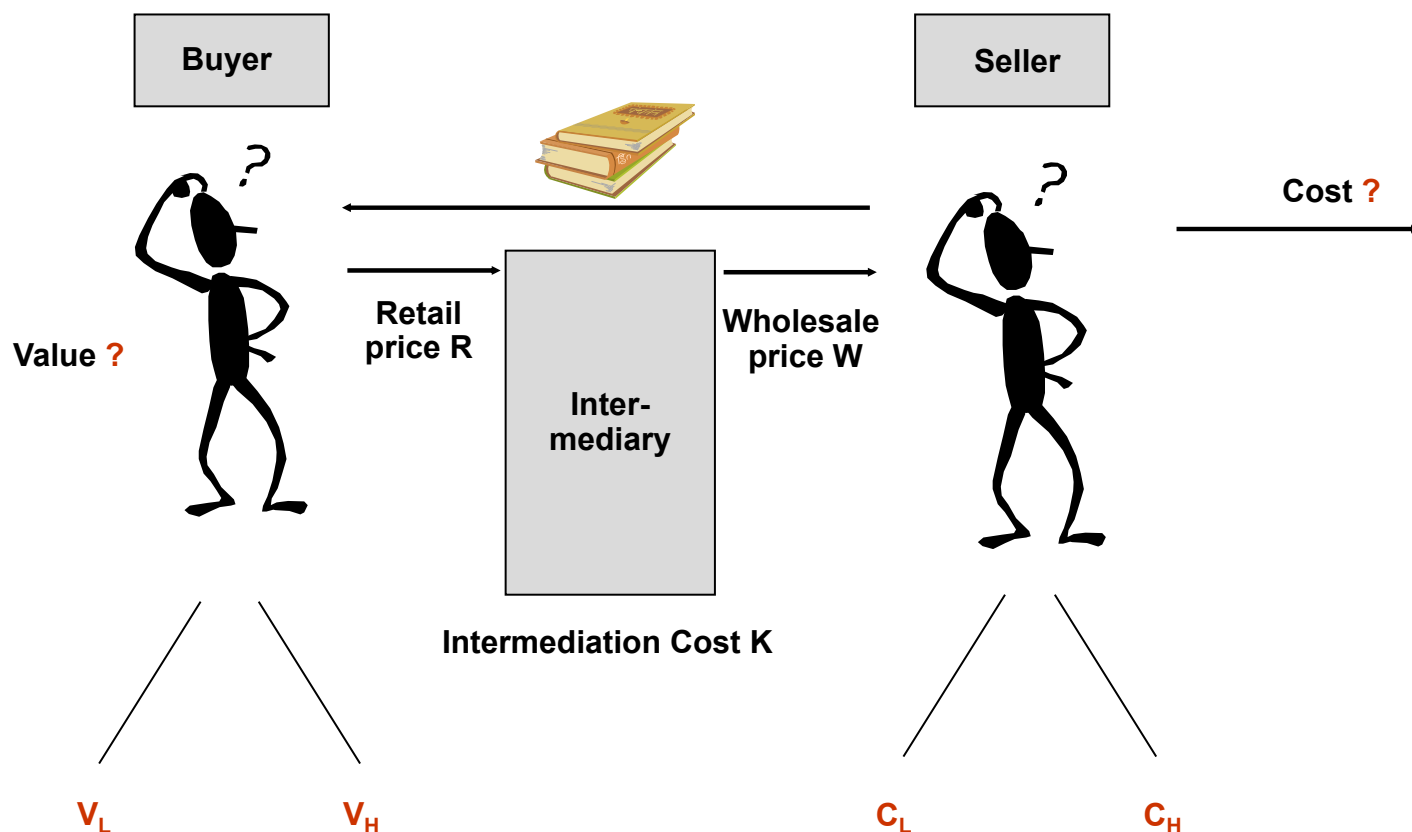
• Hence, an well-known intermediary that provides a trading platform can be **viable** if the intermediation cost K is such that

$$K \leq (1 - \beta)(V - C).$$

• The **lower the probability of matching between buyers and sellers**, the **higher the likelihood that an intermediary emerges**.

BILATERAL ASYMMETRIC INFORMATION

Type of the Counterparty Generally Unknown



INTERMEDIARY CAN ENABLE TRADE WHEN MARKETS FAIL

•Consider a **buyer whose value for an item is either high (V_H) or low (V_L)**, with equal probability (where $V_H > V_L > 0$), so that **in expectation**

$$V = (V_H + V_L)/2.$$

•Suppose further that a **seller has either high opportunity cost (C_H) or low opportunity cost (C_L)**, with equal probabilities (where $C_H > C_L > 0$), so that **in expectation**

$$C = (C_H + C_L)/2.$$

•Assume that $V_H > C_H > V_L > C_L$. After meeting and learning each other's type they decide to transact or not. Thus, with probability 1/4 there is no trade. The **expected gains** from direct transaction are therefore

- Buyer L: $(V_L - C_L)/4$
- Buyer H: $(V_H - C)/2$
- Seller L: $(V - C_L)/2$
- Seller H: $(V_H - C_H)/4$

Total ex-ante expected surplus:

$$V_H - C_L - (C_H - V_L)/2 \quad (< V_H - C_L)$$

ENABLE TRADE WHEN MARKETS FAIL (Cont'd)

•If a monopolist intermediary offers prices $R = V_H - (V_H - C)/2$ and $W = C_L + (V - C_L)/2$, then **buyer H's** and **seller L's** expected surplus from using the intermediary are equal to the expected surplus from direct exchange, since

$$V_H - R = V_H - V_H + (V_H - C)/2 = (V_H - C)/2$$

and

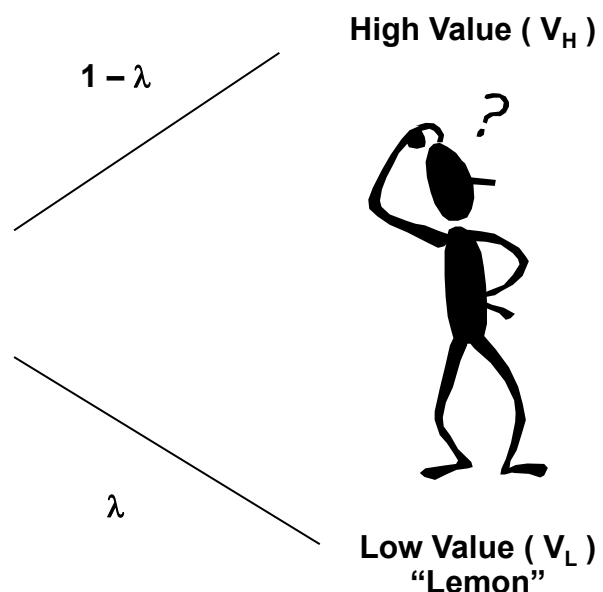
$$W - C_L = C_L + (V - C_L)/2 - C_L = (V - C_L)/2.$$

•However, **buyer L's** and **seller H's** surplus are **negative**, preventing them from using the intermediary. Hence, they will be inactive in equilibrium, while the intermediary is viable if its intermediation cost K is such that

$$R - W = V_H - (V_H - C)/2 - C_L - (V - C_L)/2 = (V_H - V_L + C_H - C_L)/4 \geq K.$$

•Thus, an **intermediary may produce a separating equilibrium** in a market that has a positive probability of failing, provided intermediation costs are low enough.

ADVERSE SELECTION Example: Used Car



With **probability λ** , a given used car is a **“lemon.”**

Question. Under what conditions on the parameters (λ, V_L, V_H) and on the seller's cost does the market fail?

INTERMEDIARY MAY ALLEVIATE ADVERSE SELECTION

•Consider a **seller whose product is of either high (H) or low quality (L)**. The seller's opportunity cost increases with the quality of the good supplied, $C_H > C_L > 0$.

•The **buyer's willingness to pay is increasing in the product quality**, $V_H > V_L > 0$.

•Let $\lambda \in (0,1)$ be the probability that the good is a "lemon", i.e., is of low quality, such that the following "**lemons condition**" is satisfied:

$$V = \lambda V_L + (1 - \lambda) V_H < C_H.$$

•Hence, **if the buyer cannot distinguish between the two product qualities in equilibrium, the high-quality seller will leave the market**, as the buyer's willingness to pay does not cover the cost of providing the good.

•Therefore, the **lemons condition implies that only lemons are directly exchanged in the market**. The payoff for the remaining buyer and seller type (L,L) is $(V_L - C_L)/2$. Trade occurs with probability λ .

ALLEVIATE ADVERSE SELECTION (Cont'd)

•**Assume that a trusted intermediary is able to observe the quality** of the seller's product at a cost K and then to communicate that information to the buyer.

$$V = \lambda V_L + (1 - \lambda) V_H < C_H.$$

•**The intermediary can then make the prices (R,W) contingent on the observed quality (L or H)**. The optimal intermediation prices are such that buyer's and sellers are just as well off as under direct exchange, so that

$$(R_H, W_H) = (V_H, C_H)$$

and

$$(R_L, W_L): \quad W_L - C_L = (V_L - C_L)/2 \quad \text{and} \quad V_L - R_L = \lambda (V_L - C_L)/2.$$

The intermediary is **viable** if $(1 - \lambda)(R_H - W_H) + \lambda(V_L - (1 + \lambda)(R_L - W_L)/2 - C_L) \geq K$, i.e., ultimately if $(1 - \lambda)(V_H - C_H) + \lambda(V_L - (1 + \lambda)(V_L - C_L)/2 - C_L) \geq K$.

INTERMEDIARIES CAN MITIGATE MORAL HAZARD

• Suppose that a buyer can enhance the default value V_L obtained from a certain good or service to V_H by making a relationship-specific investment I . This investment is non-contractable.

• The seller's cost is $C < V_L$, and the surplus is evenly divided such that both parties obtain $(V_i - C)/2$ for $i = L, H$.

• Assume that required investment is "substantial", i.e., $V_H - V_L > I > (V_H - V_L)/2$.

• Then the buyer will not find it worthwhile to make the relationship-specific investment, since

$$(V_H - C)/2 - I < (V_L - C)/2.$$

MITIGATE MORAL HAZARD (Cont'd)

An intermediary can set prices (R, W) such that

$$V_H - I - R = (V_L - C)/2 = W - C.$$

The intermediary is viable if

$$R - W = V_H - V_L - I \geq K.$$

INTERMEDIARIES AND TRANSACTION COST

Summary

- **Provide Immediacy**
- **Reduce Search & Matching Cost**
- **Enable Trade when Markets Fail (Bilateral Asymmetric Information)**
- **Alleviate Adverse Selection**
- **Mitigate Moral Hazard**

... and reduce “coordination problems”

CONCLUSION

Public-Policy Implications with respect to Intermediaries

- Should the government **encourage the entrance** of intermediaries?
- Should the government **act as** an intermediary?
- Should the government **encourage the competition** of intermediaries?

AGENDA

What is an intermediary?

Intermediary's Value Proposition

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Intermediary / Market Maker / Two-Sided Market**
- **Transaction Cost**
- **Intermediation Cost**
- **Immediacy**
- **Search Cost**
- **Market Failure**
- **Adverse Selection**
- **Moral Hazard**