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FIRST MOVER AND SECOND MOVER ADVANTAGES*

BY ESTHER GAL-OR[1]

1. INTRODUCTION

The objective of the present note is to demonstrate that when two identical players move sequentially in a game, the player that moves first (Stackleberg leader) earns lower (higher) profits than the player that moves second (Follower) if the reaction functions of the players are upwards (downwards) sloping, respectively.

2. THE MODEL

We assume two identical players where the strategy choice of player i is denoted by s_i and the payoff function of player i is denoted by $\pi^i(s_i, s_j)$ $i, j=1, 2$ $j \neq i$, where π^i is twice continuously differentiable, $\pi^i_{11}<0$ and π^i_2 and π^i_{12} never vanish. (Subscripts denote partial derivatives.) The first argument of the payoff function of a player corresponds to his own strategy choice and the second argument corresponds to his rival's strategy choice. The assumptions made about the payoff function imply that the payoff of player i is a strictly concave function of s_i and both π^i and π^i_1 are strictly monotonic functions of the strategy choice of player $j \neq i$. We assume that the order of moves does not affect payoffs conditional on the strategies, i.e. the payoff of player i from taking action 'a' when player j takes action 'b' is the same whether 'a' was taken before or after 'b'. Since the players are identical $\pi^i(v, w)=\pi^j(v, w)$ $i, j=1, 2$ for every v and w. Without any loss of generality we denote by "player 1" the player that moves first (the Stackleberg player) and by "player 2" the player that moves second (the follower).

The strategy of player 1 is a number $s_1 \in [\underline{s}, \bar{s}]$ and the strategy of player 2 is a decision rule $s_2(\cdot)$ where $s_2: [\underline{s}, \bar{s}] \rightarrow [\underline{s}, \bar{s}]$. In Definition 1, we specify the properties of the Nash equilibrium with sequential move[2].

Definition 1. *Nash Equilibrium with Sequential Move. The pair* (s_1^*, s_2^*) *corresponds to a Nash equilibrium with sequential move if*:
(a) $s_2^* \equiv g(s_1^*) = \operatorname*{argmax}_s \pi^2(s, s_1^*)$; (b) $s_1^* = \operatorname*{argmax}_s \pi^1(s, g(s))$.
The function $g(\cdot)$ that is defined by (a) can be interpreted as the reaction function of player 2.

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[1] I wish to acknowledge the helpful comments of two anonymous referees.
[2] We consider only subgame perfect equilibria.

If an interior equilibrium satisfying Definition 1 exists, then the following first order necessary conditions hold

$$(1)\qquad \begin{aligned} &G^1(s_1^*, s_2^*) \equiv \pi_1^1(s_1^*, s_2^*) - \pi_2^1(s_1^*, s_2^*)\pi_{12}^2(s_2^*, s_1^*)/\pi_{11}^2(s_2^*, s_1^*) = 0 \\ &G^2(s_1^*, s_2^*) \equiv \pi_1^2(s_2^*, s_1^*) = 0 \end{aligned}$$

where subscripts denote partial derivatives. The second order condition is

$$(2)\qquad \begin{aligned} &G_1^1(s_1^*, s_2^*) - G_2^1(s_1^*, s_2^*)\pi_{12}^2(s_2^*, s_1^*)/\pi_{11}^2(s_2^*, s_1^*) < 0 \\ &G_2^2(s_1^*, s_2^*) < 0. \end{aligned}$$

If inequality (2) holds globally, the equilibrium is unique. Inspection of the expressions for G_i^i and G_j^i in ineq. (2) shows that existence and uniqueness of equilibrium can be a problem in sequential move games. Even if *a priori* we restrict ourselves to concave profit functions ineq. (2) does not necessarily follow.[3] Notice that sign of the slope of the reaction function of player 2 is determined by the sign of π_{12}^2, namely the cross partial derivative of the payoff function. If it is positive, the reaction function $g(\cdot)$ is upwards sloping, and if it is negative, the reaction function $g(\cdot)$ is downwards sloping. We first restrict consideration to the case that $\pi_{12}^2>0$, namely the strategy choice of the follower is positively related to the strategy choice of the Stackleberg leader.

LEMMA 1. (i) *When* $\pi_{12}^2(s_2^*, s_1^*)>0$ *and* $\pi_2^1(s_1^*, s_2^*)>0$, *then* $s_1^*>s_2^*$.

(ii) *When* $\pi_{12}^2(s_2^*, s_1^*)>0$ *and* $\pi_2^1(s_1^*, s_2^*)<0$, *then* $s_1^*<s_2^*$.

PROOF. (i) Suppose $s_1^* \leqslant s_2^*$. Then $\pi_1^1(s_1^*, s_2^*) \geqslant \pi_1^1(s_2^*, s_2^*) \geqslant \pi_1^1(s_2^*, s_1^*) = \pi_1^2(s_2^*, s_1^*) = 0$. The first inequality follows since $\pi_{11}^1<0$; the second inequality follows since $\pi_{12}^2>0$; the third equality follows since the players are identical; and the fourth follows from equation (1). Also, from equation (1),

$$\pi_1^1(s_1^*, s_2^*) - \frac{\pi_2^1(s_1^*, s_2^*)\pi_{12}^2(s_2^*, s_1^*)}{\pi_{11}^2(s_2^*, s_1^*)} = 0.$$

Since $\pi_1^1 \geqslant 0$, $\pi_{12}^2>0$ and $\pi_{11}^2<0$, it follows that $\pi_2^1(s_1^*, s_2^*) \leqslant 0$. But, the last is a contradiction to the presumption that $\pi_2^1(s_1^*, s_2^*)>0$. Hence, $s_1^*>s_2^*$.

(ii) Suppose $s_1^* \geqslant s_2^*$. Then $\pi_1^1(s_1^*, s_2^*) \leqslant \pi_1^1(s_2^*, s_2^*) \leqslant \pi_1^1(s_2^*, s_1^*) = \pi_1^2(s_2^*, s_1^*) = 0$. From the second part of equation (1) it follows that $\pi_2^1(s_1^*, s_2^*) \geqslant 0$, which is a contradiction to the presumption of part (ii). Hence, $s_1^*<s_2^*$. Q.E.D.

Proposition 1 follows from Lemma 1.

PROPOSITION 1. *When the reaction function of the follower is upwards sloping* ($\pi_{12}^i>0$), *then* $\pi^2(s_2^*, s_1^*)>\pi^1(s_1^*, s_2^*)$.

[3] It is easy to show that restricting the function $g(\cdot)$ in Definition 1 to be convex or concave is also not sufficient. However, if third order derivatives of the profit function vanish and if own effects dominate cross effects ($\pi_{11}^i| > (|\pi_{12}^i|$) ineq. (2) holds.

PROOF. $\pi^2(s_2^*, s_1^*) \geqslant \pi^2(s_1^*, s_1^*) > \pi^2(s_1^*, s_2^*) = \pi^1(s_1^*, s_2^*)$.

The first inequality follows since s_2^* maximizes $\pi^2(\cdot, s_1^*)$ for a given choice of s_1^*. The second inequality follows since from Lemma 1 $s_1^* > s_2^*$ when $\pi_2^2 > 0$, and $s_1^* < s_2^*$ when $\pi_2^2 < 0$. The last equality follows by the identity of firms. Q.E.D.

According to Proposition 1, there are inherent second mover advantages when the strategy choice of players are positively related. This result is reversed when the strategies are negatively related.

LEMMA 2. (i) *When* $\pi_{12}^2(s_2^*, s_1^*) < 0$ *and* $\pi_2^1(s_1^*, s_2^*) > 0$, *then* $g(s_2^*) > s_1^*$;

(ii) *when* $\pi_{12}^2(s_2^*, s_1^*) < 0$ *and* $\pi_2^1(s_1^*, s_2^*) < 0$, *then* $g(s_2^*) < s_1^*$,

where $g(\cdot)$ is the reaction function defined in Definition 1.

PROOF. (i) Suppose $g(s_2^*) \leqslant s_1^*$. Then, $\pi_1^1(s_1^*, s_2^*) \leqslant \pi_1^1(g(s_2^*), s_2^*) = \pi_1^2(g(s_2^*), s_2^*) = 0$. From the second part of equation (1) it follows that $\pi_2^1(s_1^*, s_2^*) \leqslant 0$ contradicting the presumption that $\pi_2^1(s_1^*, s_2^*) > 0$. Hence, $g(s_2^*) > s_1^*$.

(ii) Suppose $g(s_2^*) \geqslant s_1^*$. Then $\pi_1^1(s_1^*, s_2^*) \geqslant \pi_1^1(g(s_2^*), s_2^*) = \pi_1^2(g(s_2^*), s_2^*) = 0$. Since $\pi_1^1 \geqslant 0$ and $\pi_{12}^1 \leqslant 0$ it follows from the second part of equation (1) that $\pi_2^1(s_1^*, s_2^*) \geqslant 0$. The last is a contradiction to the presumption that $\pi_2^1(s_1^*, s_2^*) < 0$. Hence, $g(s_2^*) < s_1^*$. Q.E.D.

Proposition 2 is a direct implication of Lemma 2.

PROPOSITION 2. *When the reaction function of the follower is downwards sloping*, $(\pi_{12}^i < 0)$ *then* $\pi^1(s_1^*, s_2^*) > \pi^2(s_2^*, s_1^*)$.

PROOF. $\pi^1(s_1^*, s_2^*) = \pi^1(s_1^*, g(s_1^*)) \geqslant \pi^1(s_2^*, g(s_2^*)) > \pi^1(s_2^*, s_1^*) = \pi^2(s_2^*, s_1^*)$. The first equality and the second inequality follow from Definition 1, the third inequality follows from Lemma 2 ($g(s_2^*) > s_1^*$ when $\pi_2^1 > 0$ and $g(s_2^*) < s_1^*$ when $\pi_2^1 < 0$) and the last equality is implied by the identity of firms. Q.E.D.

According to Proposition 2 there are inherent first mover advantages when the strategies of the players are negatively related. This result is different from the one obtained in Proposition 1 when the second mover was earning higher profits.

The intuition for the results can be explained as follows. Downwards sloping reaction functions refer to markets in which the leader can make a preemptive move; upwards sloping reaction functions refer to followers copying or undercutting the leader. An example of the former is when an incumbent firm invests in excess capacity (Spence [1979], Dixit [1980]). Examples of the latter are (i) when an entrant undercuts the price of the incumbent as in the contestable market literature (Baumol [1982]) or (ii) when the follower in the development stage invests more than the leader and is consequently more likely to collect a patent in a research and development game (Reinganum [1983]).

The above analysis suggests that firms are unwilling to commit first when reaction functions are upwards sloping and are willing to commit first when reaction functions are downwards sloping. This result can be used to consider a new game with time of commitment the strategic choice. For example, the firms may have the option of commiting themselves at $t=1$ or $t=2$, with payoffs determined after $t=2$. The derivation of the equilibria of such a game requires the additional computation of payoffs, when both players commit at the same time.[4]

3. AN EXAMPLE

We assume two firms, each producing a differentiated product at no cost. The demand is linear, namely

$$p_i = a - bq_i - cq_j \quad a,\, b,\, c > 0;\; b \geqslant c;\; j \neq i \tag{3}$$

where p_i is the price and q_i the amount produced of product i. Since $c>0$, the two products are substitutes and since $b \geqslant c$, "cross effects" are dominated by "own effects". We will consider two different games: in the first, the players choose prices as strategies and in the second, they choose output levels as strategies.

It is fairly easy to demonstrate that the reaction function of player 2 with prices as strategies is

$$p_2 = \begin{cases} [cp_1 + a(b-c)]/2b & \text{if} \quad 0 \leqslant p_1 < a[1-bc/(2b^2-c^2)] \\ [bp_1 - a(b-c)]/c & \text{if} \quad a[1-bc/(2b^2-c^2)] \leqslant p_1 < a[1-c/2b] \\ \dfrac{a}{2} & \text{if} \quad a[1-c/2b] \leqslant p_1\,. \end{cases}$$

In the first region, $q_1>0$ and in the last two regions, firm 2 chooses a price that drives firm 1 out of the market, namely $q_1=0$. Solving for the equilibrium with sequential move yields:

$$p_1^* = a(b-c)(2b+c)/2(2b^2-c^2) > p_2^* = a(b-c)(2bc+4b^2-c^2)/4b(2b^2-c^2)$$

$$\pi_1^* = a^2(b-c)(2b+c)^2/8b(b+c)(2b^2-c^2) < \pi_2^*$$
$$= a^2(b-c)(2bc+4b^2-c^2)^2/16b(b+c)(2b^2-c^2)^2.$$

Hence, the follower undercuts the price of the leader and earns higher profits. Nevertheless, he does not drive firm 1 out of the market, since p_1^* falls in the first region of firm 2's reaction function.

With output levels as strageties, the reaction function of player 2 is

$$q_2 = (a - cq_1)/2b.$$

The equilibrium with sequential move is

$$q_1^* = a(2b-c)/2(2b^2-c^2) > q_2^* = a[4b^2-2bc-c^2]/4b(2b^2-c^2)$$

$$\pi_1^* = a^2(2b-c)^2/8b(2b^2-c^2) > \pi_2^* = a^2[4b^2-c^2-2bc]^2/16b(2b^2-c^2)^2.$$

University of Pittsburgh, U. S. A.

[4] This generalization was suggested by a referee.

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