École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-621 Microeconomics (Prof. Weber)

Final Exam

Autumn 2021

Monday, October 4, 2021

This is an *open-book exam*. You will have **180 minutes** to complete it. Use the point totals listed below to allocate your time appropriately. Please write all answers and supporting work in your answer booklet.

You must return the question sheet with the answer booklets.

The distribution of points for the four questions in this exam is as follows:

- Problem 1: 35 points
- Problem 2: 25 points
- Problem 3: 35 points
- Problem 4: 25 points

Total: 120 points

Good luck!

I hereby certify that this exam is based solely on my own efforts, and is otherwise also in accordance with the *EPFL Honor Code* as put forth in the course syllabus.

Name: ....

Signature: ....

Problem 1. (35 Points) Claude considers buying shares of a company. He believes that three months from now the stock will be worth either 90 or 120, with equal probability. Claude's utility for money is of the form $u(x) = 1 - \exp(-\rho x)$ for all $x \in \mathbb{R}$, where $\rho > 0$ is his constant-absolute-risk-aversion (CARA) parameter. Claude's wealth of $y \geq 500$ is entirely available for stock purchases or for saving.[1] If Claude does not use all his wealth to purchase shares, he keeps the balance in a savings account at zero percent interest. For simplicity, we assume that there is no discounting between different time periods.

(i) Determine Claude's willingness to pay p_b for one share. Does it depend on his wealth y? If yes, how. If no, why not?

(ii) How many shares would Claude buy at the price $p_0 = 100$?

(iii) Based on your result in part (ii), if Claude bought z shares at the price $p_0 = 100$, at what price per share p_s would he be willing to sell them again? Explain the difference. In particular, why is there a "wealth effect" even though CARA-investors are commonly thought of as free of such effects?

(iv) Instead of buying the stock, Claude considers buying a call option with a strike price of $s = 105$ that matures in three months. Determine Claude's willingness to pay p_c for one call option.[2]

(v) Consider a change in the stock's payoff distribution such that the stock is valued in three months at either 70 or at 140, with equal probability. Compare the new payoff distribution with the old one in terms of money lotteries and determine which one stochastically dominates the other (in which dominance order?). Find Claude's willingness to pay $\hat{p}_b$ for one new share, and compute his willingness to pay $\hat{p}_c$ for one call option with strike price $s = 105$. Compare and contrast your results to your answers in parts (i) and (iv).

[1] Assume that it is impossible for Claude to borrow any additional funds.

[2] A (European) *call option* at the strike price s entitles the holder to buy (at its maturity date) a certain underlying stock at the price s. Clearly, if the stock price at maturity is below s, then the call option becomes worthless.

Problem 2. (25 Points) A firm's feasible production vectors $y = (-z, q)$ are in its production possibilities set

$$Y = \left\{(-z, q) : (z, q) \in \mathbb{R}^2_+, (q \leq \min\{1, z\}) \text{ or } (q \leq 2 \leq z - 1)\right\},$$

where $z \geq 0$ denotes the firm's input and $q \geq 0$ its output.

(i) Draw the production possibilities set.

(ii) Which of the following statements about the firm's production possibilities set are correct? Explain.

1. Y is closed.
2. Y exhibits the no-free-lunch property.
3. Y allows for the possibility of inaction.
4. Y allows for free disposal.
5. The firm's production is irreversible.
6. Y exhibits nonincreasing/nondecreasing/constant returns to scale.
7. Y is additive.
8. Y is convex.

(iii) Given a unit price $w > 0$ for the input, determine the firm's cost function $C(q; w)$, and provide a practical example of a firm that might have a cost function of this type.

(iv) As a function of $q > 0$ is the firm's average cost $AC(q)$ increasing or decreasing? Explain.

(v) Given any price vector $(w, p) \in \mathbb{R}^2_{++}$, determine the firm's set of profit-maximizing feasible production vectors $y^*(w, p) \in Y$. Plot y^* as a function of w/p. [Hint: When is the firm indifferent between several feasible production vectors?]

Problem 3. (35 Points) Consider 2 firms, A and B, which sell differentiated widgets on a common market. The quantities and prices of their products are denoted by q_A, q_B and p_A, p_B respectively. The demand for firm A's widgets is given by

$$q_A = 24 - 2p_A + p_B,$$

and the demand for firm B's widgets is

$$q_B = 24 - 2p_B + p_A.$$

Assume that **each firm** $j \in \{A, B\}$ **chooses only its price** $p_j \geq 0$.

Part I. Suppose that both firms move *simultaneously* at time $t = 0$.

(i) Determine the firms' respective payoffs $\pi_j(p_j, p_{-j})$.

(ii) Plot the firms' best-response correspondences.

(iii) Are the firms' actions strategic substitutes or strategic complements? Why?

(iv) Is the game supermodular? Why or why not? [Hint: the game is supermodular if the agents' payoff functions exhibit increasing differences.]

(v) Determine the Nash equilibrium strategies and the associated equilibrium payoffs.

Part II. Assume that firm A gets to choose its price p_A (at $t = 0$) *before* firm B can choose its price p_B (at $t = 1$).

(vi) Determine the unique subgame-perfect Nash equilibrium of this game including firm payoffs. Discuss why your finding may be unusual and what is driving this result. [Hint: "subgame perfection" means that the equilibrium can be found via backward induction.]

Part III. Assume that at time $t = -1$ both firms can spend money on R&D and the firm that spends most gets to move first.[3]

(vii) Describe what happens at $t = -1$ in a subgame-perfect Nash equilibrium. How much money do both firms expect to spend on R&D respectively?

[3] If both firms happen to spend the same amount, then each firm gets to move first with 50 percent probability.

Problem 4. (25 Points) Consider two consumers $i \in \{1, 2\}$, who are endowed with a certain labor capacity $\bar{L}_i = 1$ each. In the private-ownership economy there is one firm, which is initially (at $t = 0$) completely owned by consumer 2, and produces at time $t = 1$ a quantity

$$y_s = f(L_{1s}, L_{2s}) = \min\{\lambda L_{1s}, L_{2s}\}$$

of a desirable consumption good, where L_{is} is the labor provided by consumer i in state s (see below), and $\lambda = 1/2$ is a constant. Both consumers initially do not possess any amount of the consumption good. Consumer i's utility is

$$u_{is}(x_{is}, L_{is}) = \alpha_{is} \log(x_{is}) + \log(1 - L_{is}),$$

where $\alpha_{is} = 1 - (1 - i)s$, $s \in \{0, 1\}$ is a state of nature, and x_{is} is the amount of the good consumed by consumer $i \in \{1, 2\}$ in state s. Assume that the firm pays the same wage w_s for any unit of labor in state s so that the firm's profits can be written as

$$\pi_s(L_{1s}, L_{2s}) = p_s f(L_{1s}, L_{2s}) - w_s(L_{1s} + L_{2s}),$$

where p_s is the market price of the consumption good in state s, which we take to be the *numeraire* (i.e., $p_s = 1$).

(i) Determine the state-contingent Walrasian equilibrium, $(p_s, w_s, (\hat{L}_{1s}, \hat{L}_{2s}), (\hat{x}_{1s}, \hat{x}_{2s}))$, for $s \in \{0, 1\}$. [Hint: read part (ii) to get an idea about the results before starting your computations.]

(ii) What are the firms' state-contingent Walrasian equilibrium profits, $\hat{\pi}_s$? Explain why profits and wages vary with s, even though output and labor input do in equilibrium not vary with s.

(iii) In equilibrium, which state does consumer 1 prefer, and which state does consumer 2 prefer?

(iv) Based on your answer in (iii), if the consumers could trade in firm shares (at $t = 0$) *before* the state of nature $s \in \{0, 1\}$ realizes (at $t = 1$), would they do so? Explain. [No computations required.]