École Polytechnique Fédérale de Lausanne
College of Management of Technology

MGT-621 Microeconomics (Prof. Weber)

Final Exam

Autumn 2020

Monday, October 5, 2020

This is an *open-book exam*. You will have **180 minutes** to complete it. Use the point totals listed below to allocate your time appropriately. Please write all answers and supporting work in your answer booklet.

You must return the question sheet with the answer booklets.

The distribution of points for the four questions in this exam is as follows:

- Problem 1: 30 points
- Problem 2: 20 points
- Problem 3: 25 points
- Problem 4: 25 points

Total: 100 points

Good luck!

I hereby certify that this exam is based solely on my own efforts, and is otherwise also in accordance with the *EPFL Honor Code* as put forth in the course syllabus.

Name: ....

Signature: ....

Problem 1. (30 Points) Laura likes to consume two goods, 1 and 2. She has a positive income of w, and her utility is given by the function

$$u(x) = x_1 \exp(x_2),$$

for any $x = (x_1, x_2)$ in the commodity space $\mathcal{X} = \mathbb{R}^2_+$. The current price vector is $p = (p_1, 1) \gg 0$.

(i) Draw the iso-utility curves for $u(x) \in \{1, 2, 3\}$. Be sure to label each axis of your graph and label any point where any one of the three iso-utility curves crosses an axis.

(ii) Compute Laura's marginal rate of substitution $\text{MRS}^L_{12}(x)$ between goods 1 and 2 at the consumption bundle x (not necessarily an optimal consumption point for her) and interpret your result. Could Laura's friend Jane have a different marginal rate of substitution between the two goods at the same consumption bundle x?

(iii) Assume now that Laura and Jane are consuming at their respective optimal bundles x^{L*} and x^{J*} in $\mathcal{X}$ and that each of these bundles contains positive amounts of both goods. Given that Jane's iso-utility curves are smooth, what can one say about $\text{MRS}^L_{12}(x^{L*})$ and $\text{MRS}^J_{12}(x^{J*})$?

(iv) Under what conditions on w and p_1 will Laura choose to consume only good 1? What happens to your answer in part (iii) in a situation where exactly one of the two agents (Laura) finds it optimal to consume only one good? Explain.

(v) Find Laura's indirect utility $v(p, w)$. [Hint: Recall that at the current price vector $p = (p_1, p_2)$ good 2 costs $p_2 = 1$, so your answer just depends on p_1 and w.]

(vi) Calculate Laura's compensating variation $C(w)$ and equivalent variation $E(w)$ for a change in commodity prices from $p = (1, 1)$ to $\hat{p} = (2, 1)$.

Problem 2. (20 Points) Charlie has a budget of $w = \$30$ to buy ground coffee and cocoa powder for his birthday party. Usually he goes to Mr. Wonka's store, which is a short walk from his home. Based on Charlie's past experience, Mr. Wonka always has a very good cocoa powder, but the quality $q \in \{0, 1\}$ of his coffee is always random, with a $100r\%$ chance of high quality ($q = 1$) and a $100(1-r)\%$ chance of low quality ($q = 0$).[1] The prices for cocoa powder and ground coffee are $p_1 = \$1/\text{ounce}$ and $p_2 = \$1.50/\text{ounce}$, respectively. Charlie's utility for having x_1 ounces of cocoa powder and x_2 ounces of ground coffee of quality q is $u(x; q) = x_1 x_2 + 10x_2^q$, where $x = (x_1, x_2) \geq 0$.

(i) Find the consumption bundle $x(p, w)$ that maximizes Charlie's expected utility $\text{EU}(x) = E\,[u(x; q)|\, r]$, subject to his budget constraint, where $p = (p_1, p_2)$.

(ii) How do Charlie's demand for cocoa powder and expected utility change in response to the change in the distribution of q? Explain your answers.

(iii) There is another store, Poots, within driving distance from Charlie's home, which features exactly the same prices as Wonka's store. But Charlie has been told that the new store's coffee quality is consistently high. Find Charlie's willingness to pay for renting a car to shop at Poots when $w = \$30$ and $r = 0.6$.

(iv) Repeat part (i) for $u(x; q) = x_1 + (1 + q)x_2$ and $u(x; q) = (x_1 + (1 + q)x_2)^2$ when $w = \$30$ and $r = 0.45$. Do these utility functions have the same ordinal characteristics for Charlie's preferences under uncertainty?

[1] The quality is revealed only after tasting a brewed cup of coffee.

Problem 3. (25 Points) Consider a single-output m-input firm with production function $F : \mathbb{R}_+^m \to \mathbb{R}_+$. Let $w = (w_1, \ldots, w_m) \gg 0$ be the price vector for the production input $z = (z_1, \ldots, z_m) \geq 0$, and let $q > 0$ denote the quantity of output produced by the firm.

(i) Provide a general expression for the firm's cost function $C(q; w)$.

(ii) Show that if the production set $\mathcal{Y} \triangleq \{y = (-z, q) \in \mathbb{R}^{m+1} : q \leq F(z)\}$ has nondecreasing returns to scale, then the 'average cost function' $AC(q; w) = C(q; w)/q$ is nonincreasing in q.

(iii) Let $m = 2$ and $F(z) = z_1^{1/4} z_2^{1/4}$. For a given price vector (w, p), where $p > 0$ is the (fixed) market price for the output, derive the firm's optimal profit $\pi(w, p)$ and its optimal (generalized) output vector $y^* = (-z^*, q^*)$.

(iv) Does the production function $F(\cdot)$ in part (iii) have nonincreasing returns to scale? Explain.

(v) Which of the following two state changes (A or B) would the firm in part (iii) prefer? Explain.

(A) Doubling of the market price for the output, i.e., a transition from (w, p) to $(w, 2p)$.

(B) Fifty percent price drop for the production inputs, i.e., a transition from (w, p) to $(w/2, p)$.

Problem 4. (25 Points) Consider an economy in which two EPFL students, Justin and Lina, consume two goods, tickets to ballet shows (B) and tickets to football games (F). Justin, by participating in a lottery, has won an initial allocation of tickets, $\omega^J = (\omega_B^J, \omega_F^J) = (5, 1)$; his utility from consuming any bundle $x^J = (x_B^J, x_F^J)$ of tickets is given by

$$u^J(x^J) = x_B^J + x_F^J.$$

Lina has a good friend who just gave her a bundle of $\omega^L = (\omega_B^L, \omega_F^L) = (1, 4)$ tickets for free; her utility from consuming a bundle $x^L = (x_B^L, x_F^L)$ of tickets is

$$u^L(x^L) = \log(x_B^L) + \log(x_F^L).$$

Justin and Lina are the only ones in the market for tickets (all others have sold out very fast). They both log on to `epfl.market` and trade tickets.[2] Without loss of generality, we set the price of the tickets to ballet shows to one (numeraire), i.e., $p_B = 1$. Both students are trying to maximize their respective utility functions.

(i) What are the equilibrium price p_F and consumption bundles x^J and x^L, if Justin and Lina act *competitively*? Draw the Pareto set (i.e., the "contract curve"), core, and equilibrium allocation in an Edgeworth box.

Justin is now in charge of `epfl.market`. To take advantage of his management position, Justin introduces a new trade system, which proceeds as follows.

Step 1. Justin sets the price p_F for football tickets.
Step 2. Given p_F, Lina chooses her optimal consumption bundle, $x^L(p_F) = (x_B^L(p_F), x_F^L(p_F))$.
Step 3. The actual trade takes place, where Lina receives $z^L(p_F) = (z_B^L(p_F), z_F^L(p_F))$
$= (x_B^L(p_F) - \omega_B^L, x_F^L(p_F) - \omega_F^L)$ from Justin.

(ii) Determine Lina's optimal consumption bundle $x^L(p_F)$ and trading quantities $z^L(p_F)$ given the price p_F.

(iii) What price p_F will Justin choose as a monopolist? Determine the equilibrium trading quantities, z^L, and allocations, x^J and x^L.

(iv) Describe the monopoly equilibrium allocation in the Edgeworth box from part (i). Specifically, draw Lina's budget set and the indifference curve at the monopoly equilibrium for both Justin and Lina. Is the resulting allocation of the two goods Pareto-optimal?

(v) How does the competitive price compare to the monopoly price? Is there more or less trade in a *competitive* equilibrium than under monopoly?

[2] Assume for simplicity that there are no indivisibilities.