MGT 528 – OPERATIONS: ECONOMICS & STRATEGY

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8. Cooperation & Relational Contracts

Autumn 2022

École Polytechnique Fédérale de Lausanne
College of Management of Technology

AGENDA

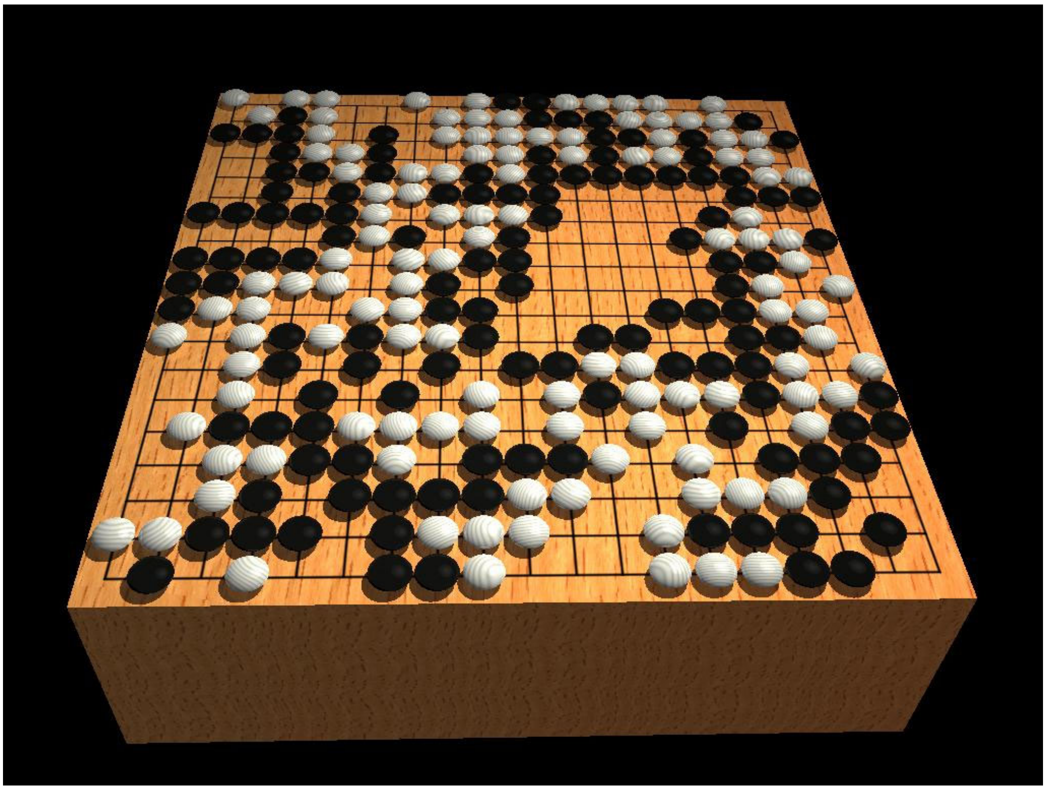
Dynamic Games: Coordination & Cooperation

Relational Contracts

Key Concepts to Remember

GAME THEORY

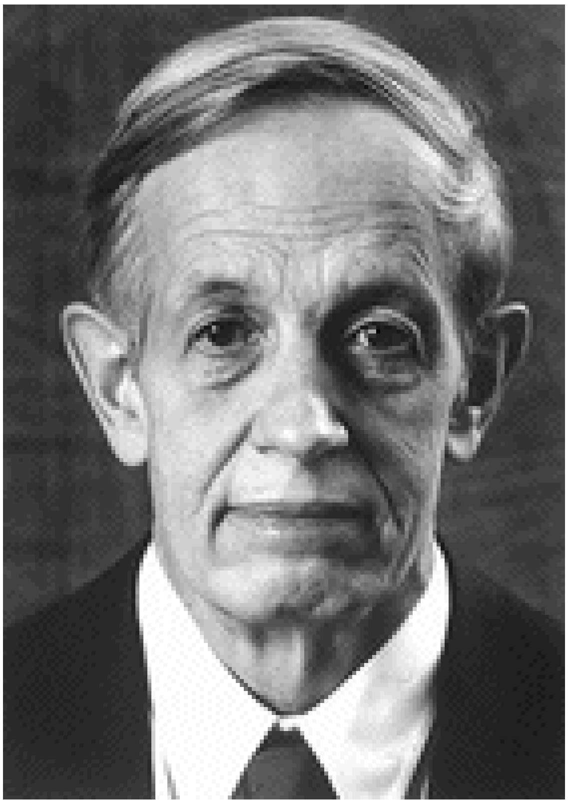
GAME THEORY



JOHN VON NEUMANN
(1903 – 1957)

Oskar Morgenstern
(1902 – 1976)

JOHN FORBES NASH
(1928 – 2015)



GAME THEORY

Game Theory is the *analysis of strategic interactions among agents*.

A *strategic interaction* is a situation in which each agent, when selecting his or her most preferred action, takes into account the likely decisions of the other agents.

Example: War

> ***"In war the will is directed at an animate object that reacts."***
>
> **- Carl von Clausewitz, *On War***

The objective of game theory is to provide predictions about the behavior of agents (players) in strategic interactions. The more precise these predictions are, the higher their "predictive power."

(1) Cf. von Clausewitz, C. (1976) *On War*, Princeton University Press, Princeton, NJ. Clausewitz lived from 1780 to 1831; for more details about his life and work, see http://www.clausewitz.com/. The first systematic academic treatment of game theory is von Neumann, J., Morgenstern (1944) *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

(NORMAL-FORM) STATIC GAME OF COMPLETE INFORMATION

Building Blocks

- **Players**, $i \in N = \{1,...,n\}$
- **Action Sets (Strategy Spaces),** A_i**, with elements** $a_i \in A_i$
- **Individual Payoffs,** $u_i(a)$**, where** $a = (a_i, a_{-i})$ **is a strategy profile,**

  a_i **is player i's action, and** a_{-i} **are all other players' actions**

Definition: A *Normal-Form Game* is a collection of players, action sets, and payoffs.

PRISONER'S DILEMMA
Example

Two suspects, 1 and 2, are being interrogated separately about a crime

- If **both confess**, each is sentenced to five years in prison
- If **both deny** their involvement, each is sentenced to one year in prison
- If **just one confesses**, he is released but the other one is sentenced to ten years in prison

Assume that each player's payoffs are proportional to the length of time of his prison sentence.

Formulate this game in normal form.

PRISONER'S DILEMMA (Cont'd)
Example

Normal-Form Representation

- Players, $i \in N = \{1,2\}$
- Action Sets, $A_i = \{Deny, Confess\}$
- Individual Payoffs, $u(a_1, a_2)$, defined by **"payoff matrix"**

Payoff Matrix[(1)]

		Player 2	
		Confess	Deny
Player 1	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

PRISONER'S DILEMMA (Cont'd)
Example

Find Prediction about Outcome of this Game

Player 2

		Confess	Deny
Player 1	Confess	(-5,-5)	(0,-10)
	Deny	(-10,0)	(-1,-1)

- Consider player 1's "best response" when fixing player 2's strategy
- Consider player 2's "best response" when fixing player 1's strategy

Hence, each player has a dominant strategy: no matter what the other player does, it is optimal (i.e., payoff-maximizing) for player i to select $a_i = Confess$.

Note also that the outcome is inefficient (i.e., does not maximize social surplus).

FUNDAMENTAL ASSUMPTIONS

Question: What assumptions are necessary to arrive at predictions about outcomes of normal-form games?

Assumption 1: All players are rational, i.e., they maximize (expected) payoffs.

Assumption 2: The players' payoff functions and action sets are common knowledge, i.e.,(1)

- Each player knows the rules of the game
- Each player knows that each player knows the rules
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- ...

Assumptions 1 and 2 imply a unique prediction in the Prisoner's Dilemma game; we will maintain these assumptions throughout this course

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(1) For a formal definition of common knowledge, see Osborne, M.J., Rubinstein, A. (1994) *A Course in Game Theory*, MIT Press, Cambridge, MA, pp. 73—75.

UNDERSTANDING RATIONALITY

Consider the following normal-form game (for which we just provide the payoff matrix):

		Player 2	
		L	R
Player 1	U	(4,4)	(-1000,3.9)
	D	(3.9,3.9)	(4,3.8)

Player 2 has a strictly dominant strategy; his dominated strategy can thus be eliminated. This leads to a unique prediction of the outcome (U,L) in this game.

Note though that player 1 has to be absolutely sure of the rationality of player 2!

PURE-STRATEGY NASH EQUILIBRIUM

Definition: For any normal-form game $\Gamma_N = \left\{N, \{A_i\}, \{u_i(\cdot)\}\right\}$ a pure-strategy Nash equilibrium is a strategy profile $a^* = (a_i^*, a_{-i}^*)$, such that for every $i \in N$:

$$\boxed{u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad \forall a_i \in A_i, i \in N}$$

Intuition:

In a Nash equilibrium no player can improve his payoffs by deviating unilaterally.

BATTLE OF THE SEXES

Consider the following game that Ann and Bert play all the time (at this point, we only look at a one-shot version of it).

- **Ann** would like to go out with Bert but would prefer to go **dancing (D)** rather than to the movies
- **Bert** would like to go out with Ann but would prefer to go to the **movies (M)** rather than dancing

Payoff Matrix:

		Bert	
		D	M
Ann	D	(2,1)	(0,0)
	M	(0,0)	(1,2)

BATTLE OF THE SEXES (cont’d)

The game has **three** Nash equilibria.

		Bert	
		D (q)	M (1-q)
Ann	D (p)	(2,1)	(0,0)
	M (1-p)	(0,0)	(1,2)

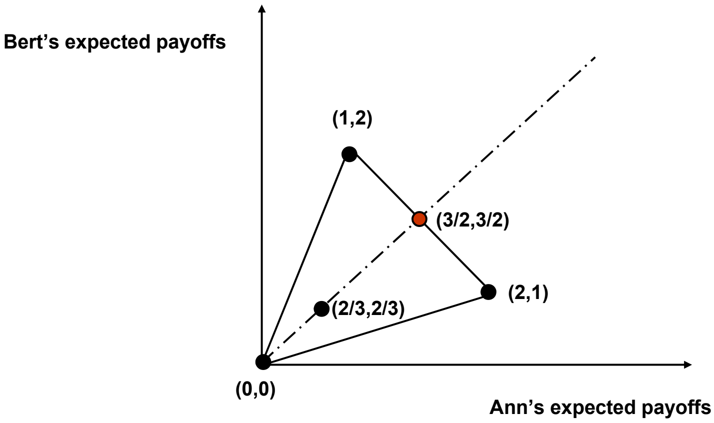
- **Two pure-strategy Nash equilibria: [D,D] and [M,M]**
- **One “mixed-strategy” Nash equilibrium: [p*,q*] = (2/3,1/3)** with expected payoffs (2/3,2/3)

BATTLE OF THE SEXES (cont'd)

Question: Can preplay communication help?

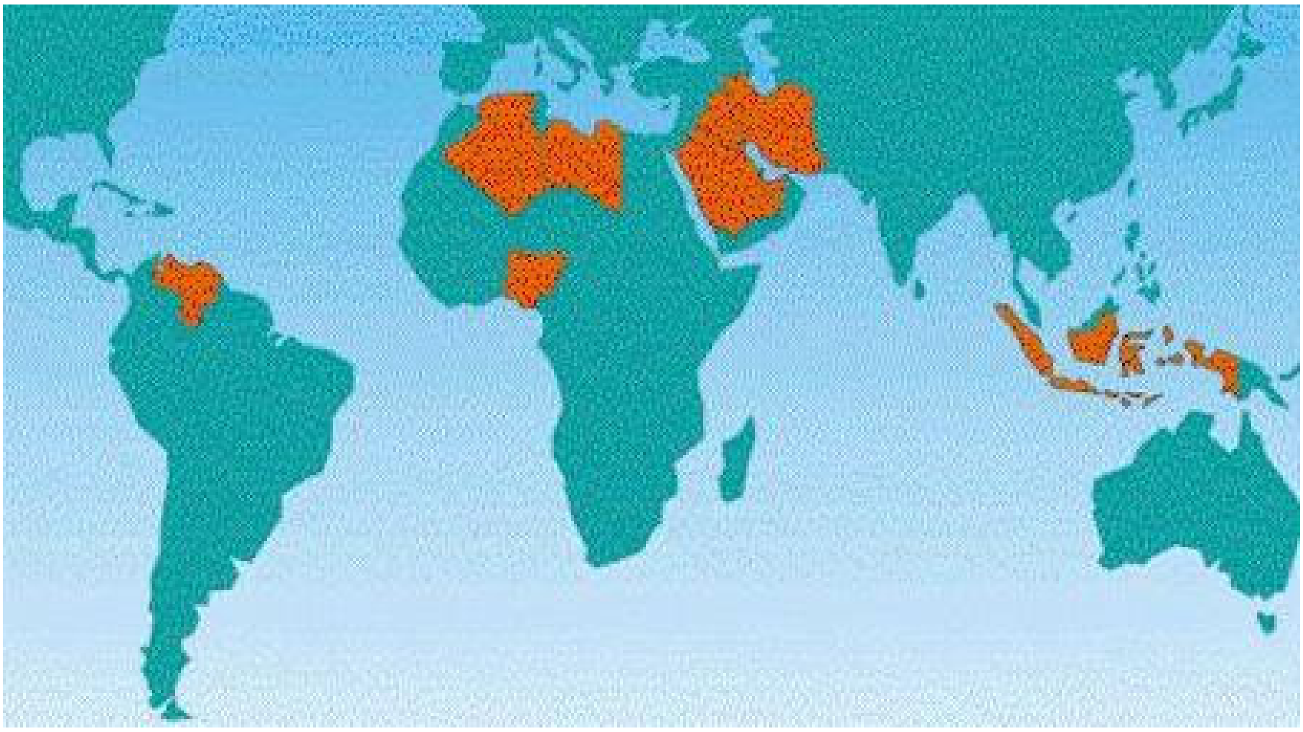
Suppose Ann and Bert flip a coin after having agreed on the following: if head shows, then they go dancing, otherwise they go to the movies

Using this external randomization, they are thus able to improve their expected payoffs (3/2,3/2), higher than the mixed-strategy expected payoffs of (2/3,2/3).



More generally, this leads to the notion of "correlated equilibria" which is a special case of Bayes-Nash equilibria (cf. games of imperfect information)

INDUSTRY ANALYSIS
Example



CHOOSING QUANTITIES: COURNOT DUOPOLY

Consider two firms, 1 and 2, choosing their production outputs q_1 and q_2 simultaneously. Each firm has a unit production cost of c (with 0 < c < 1).

- **The market (inverse) demand is given by** $p(q_1, q_2) = 1 - (q_1 + q_2)$

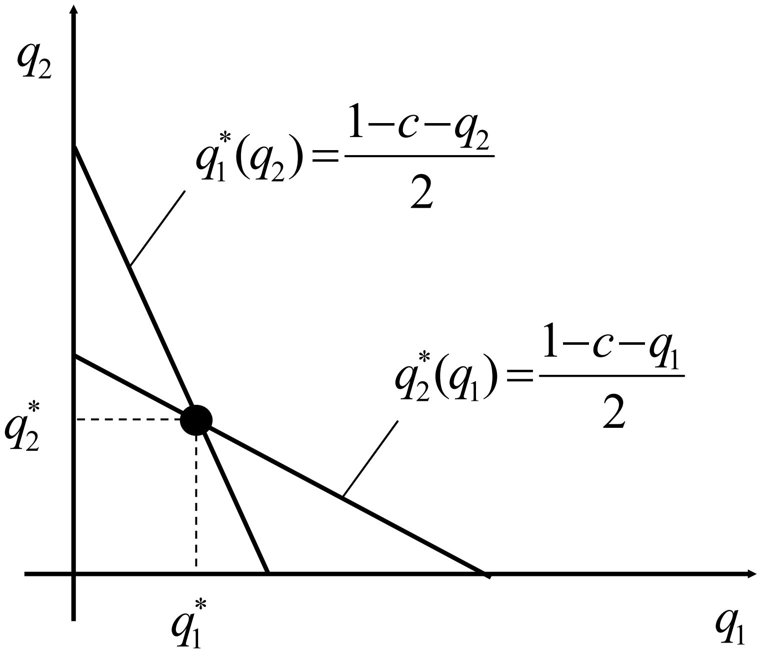
Question. Determine a Nash equilibrium of this game.

Solution.

Firm i's profit is $\Pi_i(q_1, q_2) = \big(p(q_1, q_2) - c\big)q_i = \big(1 - c - q_1 - q_2\big)q_i$

- **Its optimality condition is** $\dfrac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = 1 - c - 2q_i - q_j \overset{!}{=} 0$
- **Its best-response to q_j is therefore** $q_i^*(q_j) = \dfrac{1 - c - q_j}{2}$
- **Symmetry implies that at the Nash equilibrium** $q_i^* = \dfrac{1 - c - q_i^*}{2}$

COURNOT DUOPOLY (Cont'd)

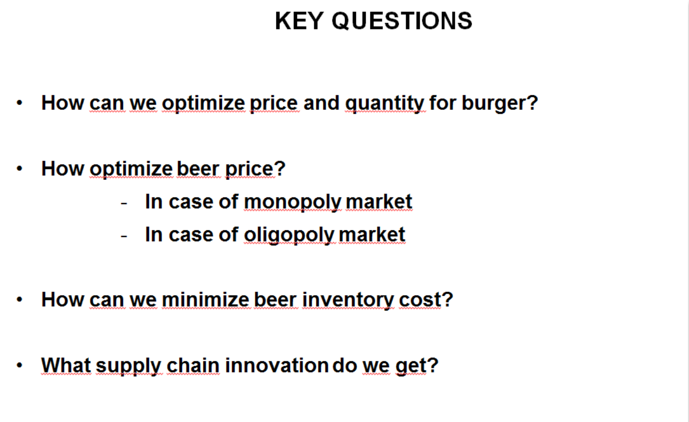


Unique Nash Equilibrium: $\boxed{q_1^* = q_2^* = \dfrac{1-c}{3}}$

STUDENT PROJECT (2014): THE GREAT ESCAPE

What is The Great Escape?

- The Great Escape is an old famous bar created in 2009
- It's a great place to have a beer, or burger, and watch sport game
- He is well situated: near Riponne
- But there are a lot competitors around

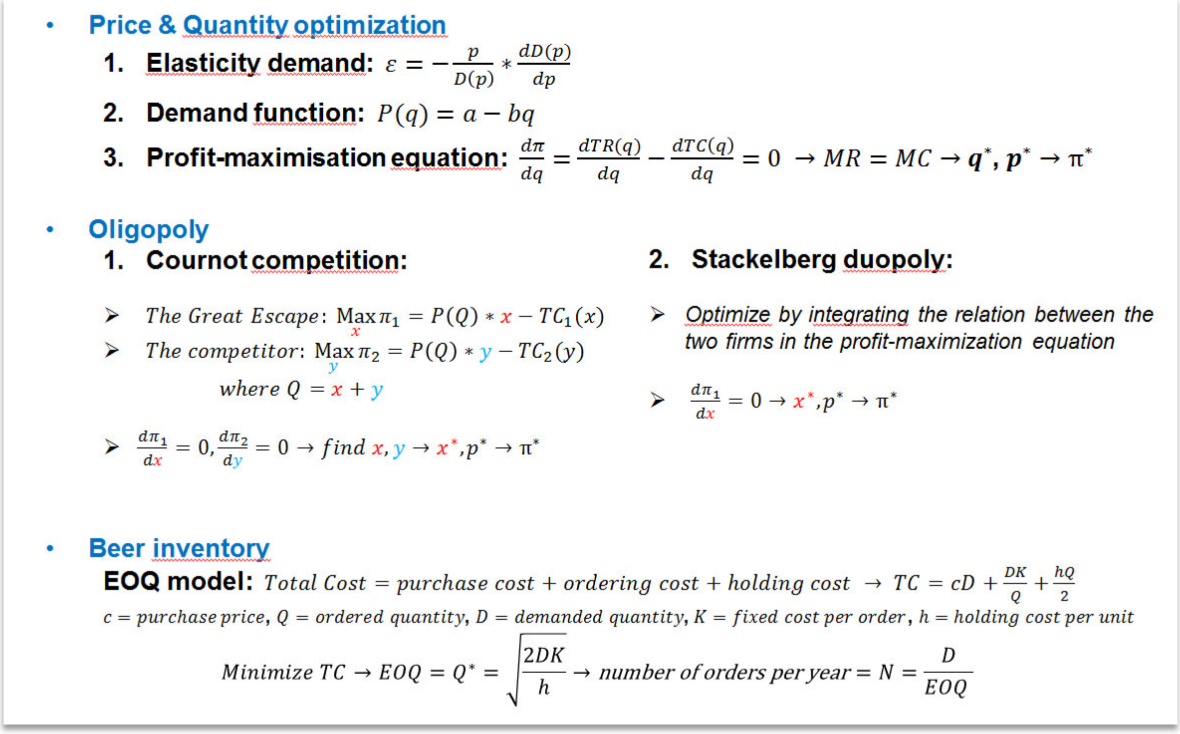


KEY QUESTIONS

- How can we optimize price and quantity for burger?
- How optimize beer price?
  - In case of monopoly market
  - In case of oligopoly market
- How can we minimize beer inventory cost?
- What supply chain innovation do we get?

Source: Degouy, L., Leynaud-Kieffer, L., Matz, A. (2014) "The Great Escape: Generating Value by Optimizing Price and Inventory Cost," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

THE GREAT ESCAPE (Cont'd)



- Price & Quantity optimization
  1. **Elasticity demand:** $\varepsilon = -\frac{p}{D(p)} * \frac{dD(p)}{dp}$
  2. **Demand function:** $P(q) = a - bq$
  3. **Profit-maximisation equation:** $\frac{d\pi}{dq} = \frac{dTR(q)}{dq} - \frac{dTC(q)}{dq} = 0 \rightarrow MR = MC \rightarrow \boldsymbol{q^*, p^*} \rightarrow \pi^*$

- Oligopoly
  1. **Cournot competition:**
     - $The\ Great\ Escape\colon \underset{x}{\text{Max}}\,\pi_1 = P(Q) * x - TC_1(x)$
     - $The\ competitor\colon \underset{y}{\text{Max}}\,\pi_2 = P(Q) * y - TC_2(y)$

       $where\ Q = x + y$
     - $\frac{d\pi_1}{dx} = 0, \frac{d\pi_2}{dy} = 0 \rightarrow find\ x, y \rightarrow x^*, p^* \rightarrow \pi^*$
  2. **Stackelberg duopoly:**
     - *Optimize by integrating the relation between the two firms in the profit-maximization equation*
     - $\frac{d\pi_1}{dx} = 0 \rightarrow x^*, p^* \rightarrow \pi^*$

- Beer inventory

  EOQ model: $Total\ Cost = purchase\ cost + ordering\ cost + holding\ cost\ \rightarrow TC = cD + \frac{DK}{Q} + \frac{hQ}{2}$

  $c = purchase\ price,\ Q = ordered\ quantity,\ D = demanded\ quantity,\ K = fixed\ cost\ per\ order,\ h = holding\ cost\ per\ unit$

  $$Minimize\ TC \rightarrow EOQ = Q^* = \sqrt{\frac{2DK}{h}} \rightarrow number\ of\ orders\ per\ year = N = \frac{D}{EOQ}$$

Source: Degouy, L., Leynaud-Kieffer, L., Matz, A. (2014) "The Great Escape: Generating Value by Optimizing Price and Inventory Cost," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

THE GREAT ESCAPE (Cont'd)

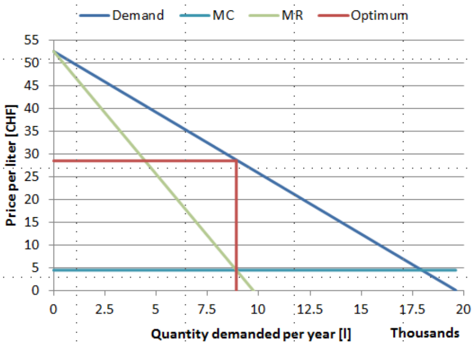
1. Pricing problem about burger

- **Actual:** $q = \mathbf{10'368}\ bugers/year, p = \mathbf{16}\ CHF/burger, \pi = \mathbf{25,576}\ CHF$
- **Optimal:** $q^* = \mathbf{8236}\ bugers/year,\ p^* = \mathbf{19.85}\ CHF/burger, \pi^* = \mathbf{32,127}\ CHF$
- **Increase profit of 20%**

2. Beer problem - Monopoly

Type of beer	Price per liter	Quantity per year	MC	Optimal quantity q*	Optimal price P(q*)
Heineken (lager)	13	7000	$4.05\ CHF/l$	$4505\ l$	$P_l(q^*) = 24.8\ CHF/l$
Hoegarden (wheat)	16	3500	$4.45\ CHF/l$	$2260\ l$	$P_w(q^*) = 30.2\ CHF/l$
London Pride (amber)	17	3500	$5.1\ CHF/l$	$2248\ l$	$P_a(q^*) = 32.3\ CHF/l$
Global	15	14,000	$4.41\ CHF/l$	$8905\ l$	$P(q^*) = 28.45\ CHF/l$

- **Actual:** $\pi = \pi_l + \pi_w + \pi_a = \mathbf{61,725}\ CHF$
- **Optimal:** $\pi = \mathbf{129{,}819}\ CHF$



3. Beer problem - Duopoly

- **Cournot competition**
  - $Price = \mathbf{28.45}\ CHF/l$
  - $\pi = \mathbf{59{,}764}\ CHF$
- **Stackelberg duopoly**
  - $Price = \mathbf{22.45}\ CHF/l$
  - $\pi = \mathbf{77{,}613}\ CHF$

Source: Degouy, L., Leynaud-Kieffer, L., Matz, A. (2014) "The Great Escape: Generating Value by Optimizing Price and Inventory Cost," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

THE GREAT ESCAPE (Cont'd)

4. Beer inventory – EOQ model

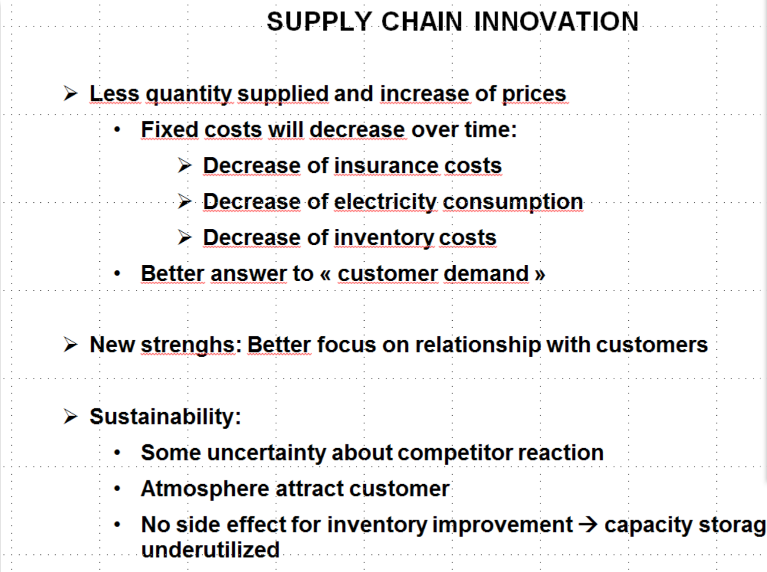
- $EOQ\ (global) = Q^* = \sqrt{\frac{2*14{,}000*324}{1.1}} = \mathbf{2871}\ \boldsymbol{l} \rightarrow N \cong \mathbf{5}\ orders$
- $EOQ\ (lager) = Q_l^* = \sqrt{\frac{2*7000*324}{1.1}} = \mathbf{2030}\ \boldsymbol{l} \rightarrow N \cong \mathbf{4}\ orders$
- $EOQ\ (wheat) = EOQ\ (amber) = Q_w^* = Q_a^* = \sqrt{\frac{2*3500*324}{1.1}} = \mathbf{1435}\ \boldsymbol{l} \rightarrow N \cong \mathbf{3}\ orders$

	D	Q*	TC
Global	14000	2871	**64934**
Lagger	7000	2030	29839
Wheat	3500	1435	16628
Amber	3500	1435	18902
Total			**65369**
Lagger	7000	2030	30584
Wheat	3500	1435	17155
Amber	3500	1435	19429
Total			**67168**

We suggest ordering the optimal quantity calculated above to minimize total inventory holding costs and ordering costs in both cases (one or three providers). Therefore, the fact to order the beers as a set is the best way.

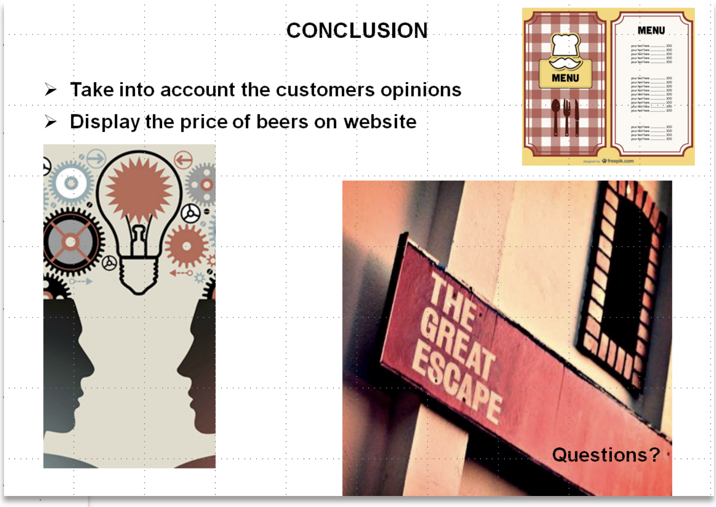
Source: Degouy, L., Leynaud-Kieffer, L., Matz, A. (2014) "The Great Escape: Generating Value by Optimizing Price and Inventory Cost," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

THE GREAT ESCAPE (Cont'd)



SUPPLY CHAIN INNOVATION

- Less quantity supplied and increase of prices
  - Fixed costs will decrease over time:
    - Decrease of insurance costs
    - Decrease of electricity consumption
    - Decrease of inventory costs
  - Better answer to « customer demand »
- New strenghs: Better focus on relationship with customers
- Sustainability:
  - Some uncertainty about competitor reaction
  - Atmosphere attract customer
  - No side effect for inventory improvement → capacity storage underutilized



CONCLUSION

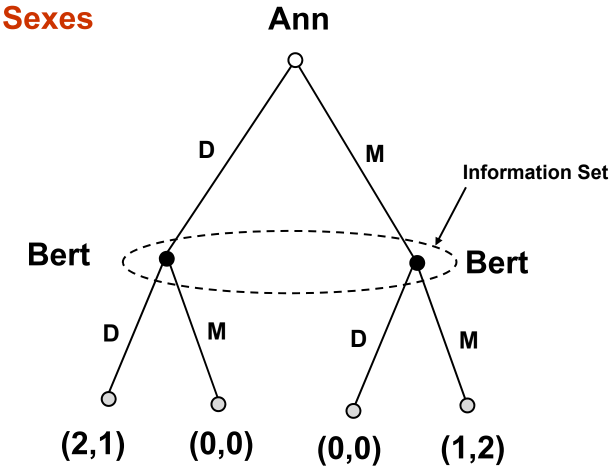
- Take into account the customers opinions
- Display the price of beers on website

Questions?

Source: Degouy, L., Leynaud-Kieffer, L., Matz, A. (2014) "The Great Escape: Generating Value by Optimizing Price and Inventory Cost," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

NORMAL-FORM GAMES CAN BE REPRESENTED IN EXTENSIVE FORM

Example: Battle of the Sexes

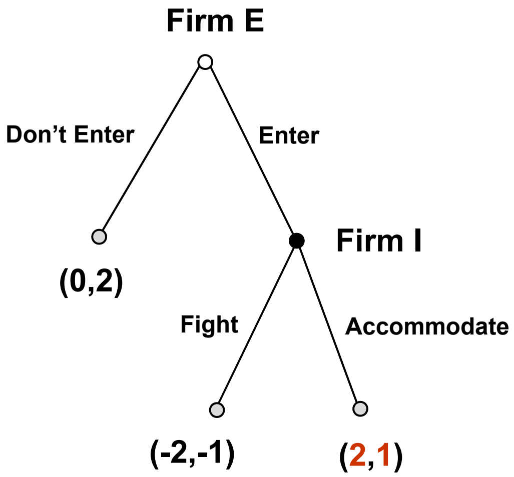


M: Movies
D: Dancing

Question: How would the game look if Ann could credibly communicate her move to Bert?

EXTENSIVE-FORM GAMES WITH PERFECT INFORMATION

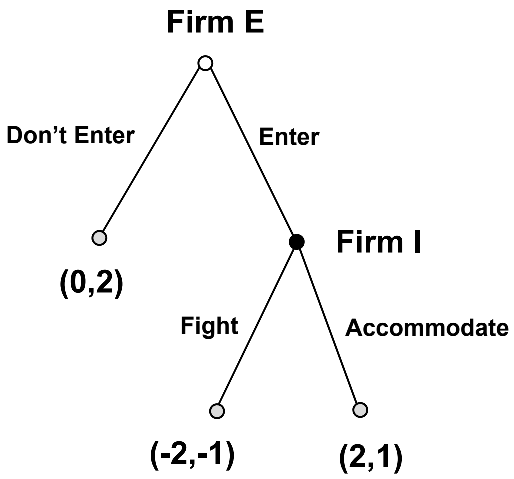
In many games the timing of players' actions matters. As an example, consider the following "entry game" played between an incumbent firm I and a potential entrant firm E.



Backwards induction leads to a unique prediction.

EXTENSIVE-FORM GAMES CAN BE REPRESENTED IN NORMAL FORM

Example: Let us re-examine the entry game.



		Firm I	
		F	A
Firm E	Enter	(-2,-1)	(2,1)
	Don't Enter	(0,2)	(0,2)

NE resulting from a noncredible threat. Why?

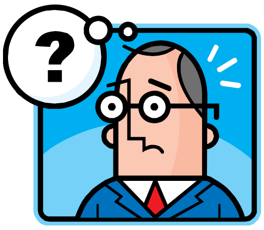
WHAT IS COMMITMENT?

Quick detour: Commitment (1/6)

Waterhouse (1891) “Ulysses and the Sirens” (National Art Gallery of Victoria, Melbourne)

WHAT IS COMMITMENT?

Quick detour: Commitment (2/6)



to commit

v. **com·mit·ted, com·mit·ting, com·mits**

v.tr.

1. To do, perform, or perpetrate: commit a murder.

2. To put in trust or charge; entrust: commit oneself to the care of a doctor; commit responsibilities to an assistant.

3. To place officially in confinement or custody, as in a mental health facility.

4. To consign for future use or reference or for preservation: commit the secret code to memory.

5. To put into a place to be kept safe or to be disposed of.

6. a. To make known the views of (oneself) on an issue: I never commit myself on such issues.

b. To bind or obligate, as by a pledge: They were committed to follow orders.

7. To refer (a legislative bill, for example) to a committee.

*v.intr.***To pledge or obligate one's own self: felt that he was too young to commit fully to marriage.**

[Middle English committen, from Latin committere : com-, *com-* + mittere, *to send*.]

Source: thefreedictionary.com

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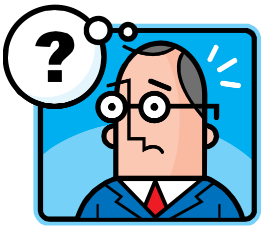

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Source: thefreedictionary.com

STANDARD COMMITMENT PROBLEM

Principal commits to a nonnegative adjustment cost $K(y-\alpha)$ and announces action α

Agent chooses action x

Principal chooses action y

Standard assumption:

either $K(y-\alpha) = 0$ (no commitment)

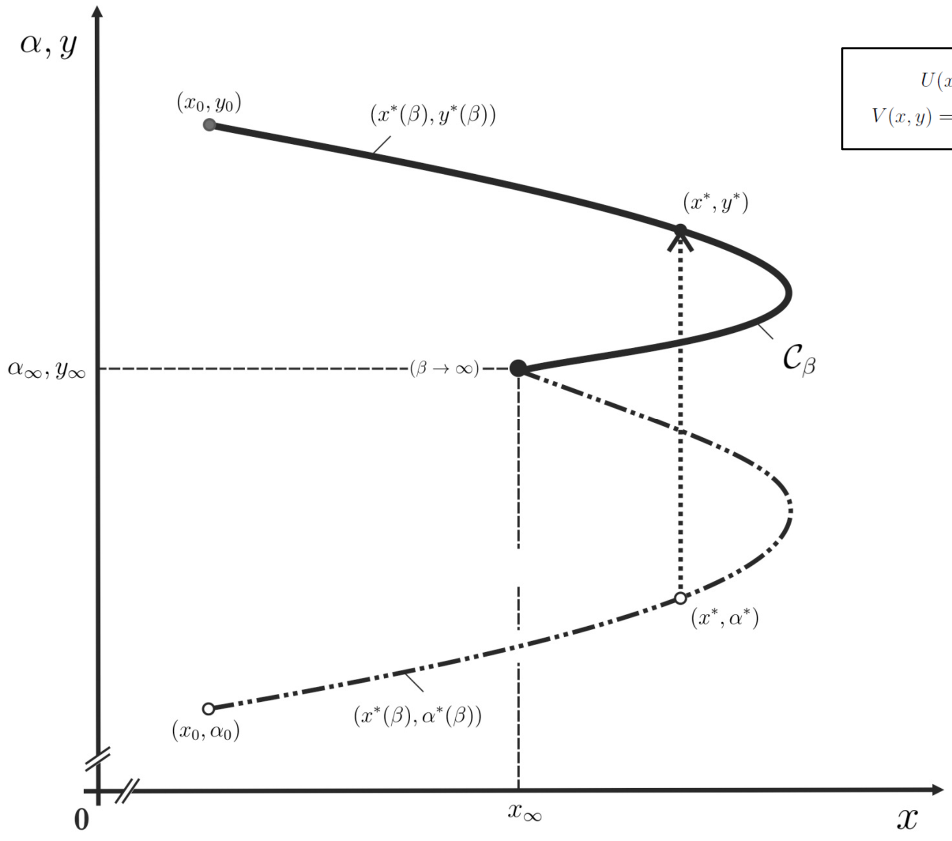
or $K(y-\alpha) = \beta\,(y - \alpha)^2$, for $\beta \rightarrow$ infinity (perfect commitment to announced action α)

Source: Weber, T.A. (2014) "A Continuum of Commitment," *Economics Letters*, Vol. 124, No. 1, pp. 67—73.

ADJUSTMENT WITH OPTIMAL COMMITMENT

$U(x,y) = (15 - 2x - 3y)x$

$V(x,y) = (2 - x - y)(y - 7) - (x^2/2)$



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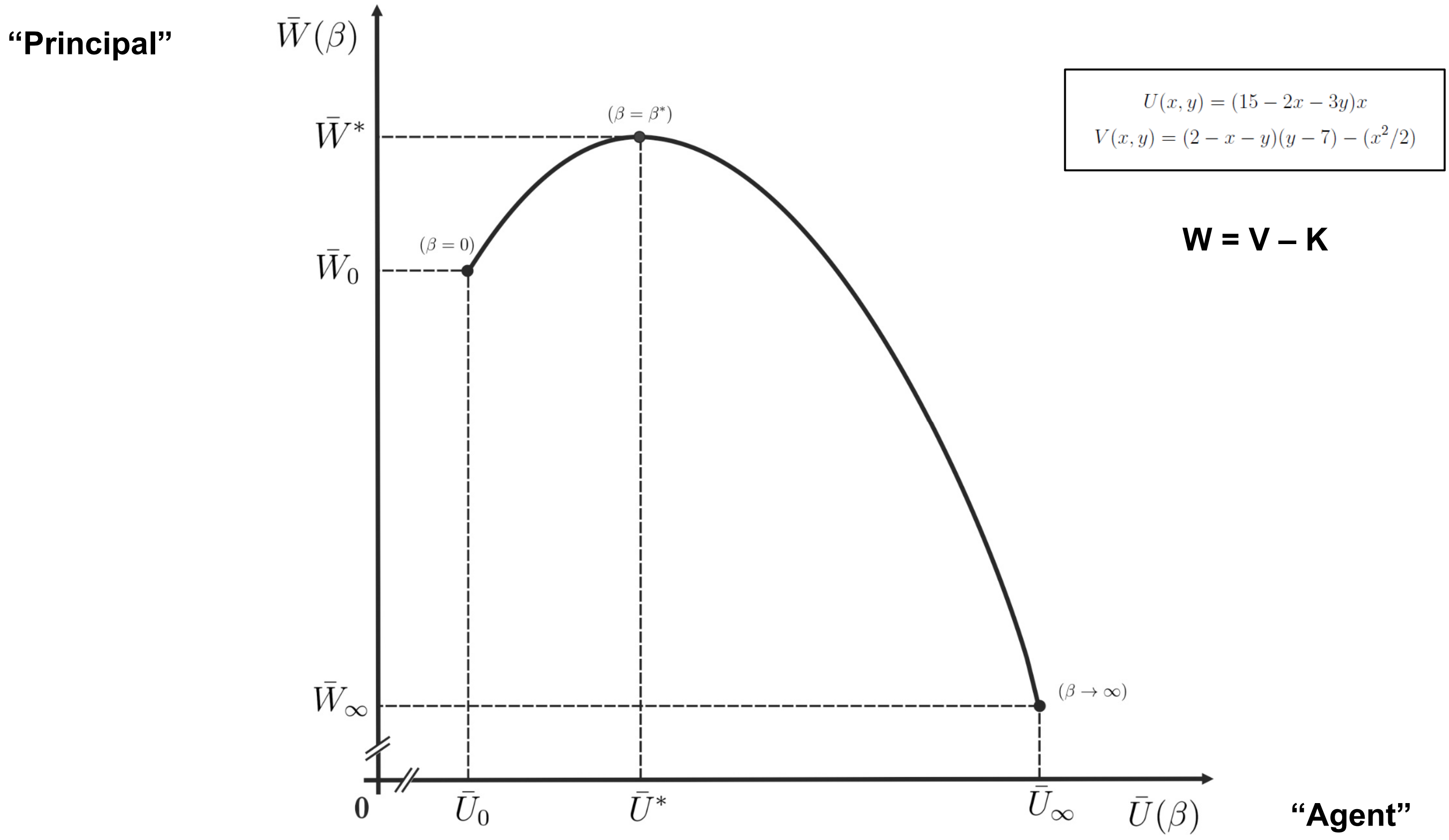
INTERIOR COMMITMENT IMPROVES PAYOFF

$U(x,y) = (15 - 2x - 3y)x$

$V(x,y) = (2 - x - y)(y - 7) - (x^2/2)$

W = V – K



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SUBGAME-PERFECT NASH EQUILIBRIUM

Definition: A subgame of an extensive-form game is a subset of the game with the following properties:

- **It begins with an information set containing a single decision node**
- **If a decision node is in the subgame, then all nodes belonging to its information set are also in the subgame.**

The subgame is called proper if it associated with a nonterminal history (i.e., with more than just a terminal node).

Definition: A strategy profile specifies for each player i's turn and each possible history of actions a choice for player i. It is therefore a complete contingent plan.

Definition: A strategy profile is a subgame-perfect Nash equilibrium (SPNE) of an extensive-form game, if it induces a Nash equilibrium in every proper subgame.

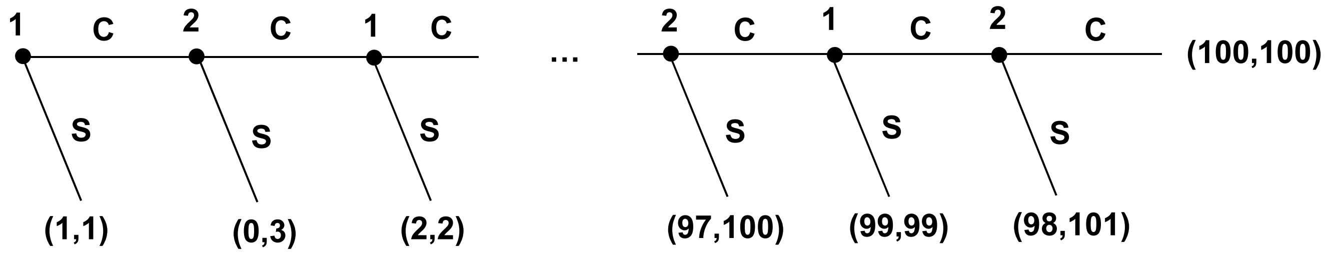
Note: An NE of Γ_E is defined analogously to an NE of a normal-form game Γ_N .

CENTIPEDE GAME

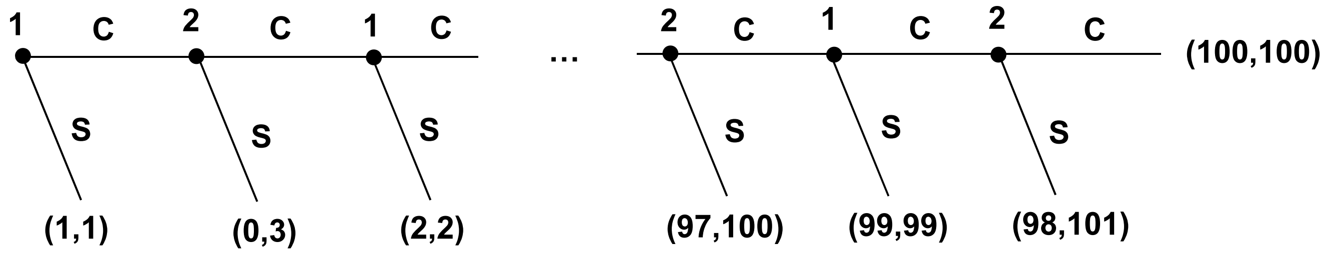
Consider the following game:

- **Two players, 1 and 2, start with one dollar in front of them, and they alternately announce "stop" (S) or "continue" (C).**
- **When a player chooses C, one dollar is taken by a referee from her pile and two dollars are put in her opponent's pile.**
- **The game is stopped when one player chooses S or if both players' payoffs reach $100.**

Extensive-Form Representation:



CENTIPEDE GAME (cont’d)



Via backward induction, we can find that the unique SPNE is [(S,S,…,S);(S,S,…,S)].

- **Does this raise doubts concerning the consequences of rationality?**
- **Note that backward induction only works for *finite* extensive-form games**

The game-theoretic prediction about the outcome of the Centipede Game highlights the consequences of assuming complete rationality of players

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FINITELY REPEATED GAMES

Example: T-times Repeated Prisoner’s Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(1,1)	(-1,2)
	Defect	(2,-1)	(0,0)

To obtain SPNE, can use backward induction starting in the last period t = T.

→ For any finite T, obtain unique SPNE

INFINITELY REPEATED GAMES

Main Points of the Analysis

- **Backward induction cannot be used to obtain SPNE**
- **Threat of a lower future payoff can be used to induce players to deviate from the myopic stage-game Nash equilibrium**
- **Depending on the threats used, different outcomes (in terms of the players average payoffs) can be attained**
- **Note: the game does not have to be really infinite: a positive probability of continuation in *each* period is enough to yield an equivalent analysis (e.g., if the continuation probability p is constant across periods then can use a discount factor of $\delta = p$ if there is no additional discounting)**

INFINITELY REPEATED PRISONER'S DILEMMA

Consider again the T-times repeated Prisoner's Dilemma game for T = infinity

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	**(1,1)**	**(-1,2)**
	Defect	**(2,-1)**	**(0,0)**

Claim: One SPNE of this game is both players choose D in every period

INFINITELY REPEATED PRISONER'S DILEMMA (cont'd)

If strategies depend on the histories (as they can by definition), then other SPNE outcomes are possible

Claim: If $\delta > 1/2$, then the following "grim trigger" strategy profile constitutes an SPNE:

- **Player i chooses C in the first period**
- **Player i continues to choose C as long as no player has deviated to D in any earlier period**
- **If the opponent chooses D, then player i plays D always (i.e., for the rest of the game)**

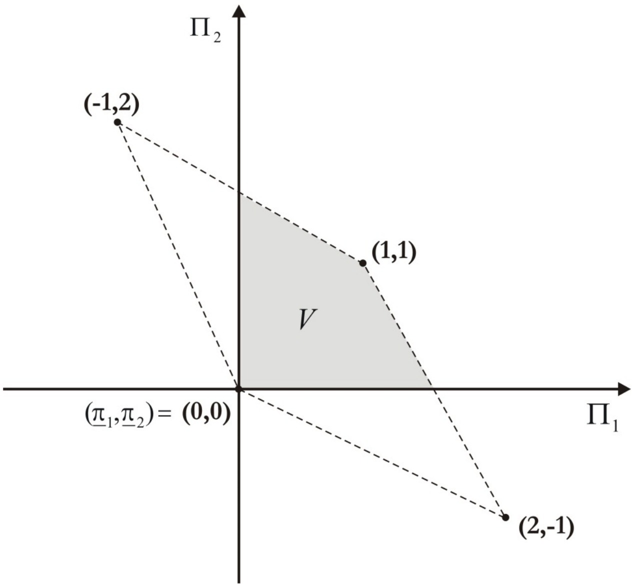
Reasoning: If both players conform to the grim trigger strategy, then their respective payoff is one. Consider now a single deviation in period t, which yields a one-time payoff of 2 instead of 1 for the deviating player. After that the game reverts to (D,D) yielding zero payoffs for all players forever. Hence, if all players think a payoff of 1 forever is worth more than a payoff of 2 once, then no player will deviate. The last condition can be written as follows:

$$2 = 2 + \sum_{\tau=1}^{\infty} \delta^{\tau} \cdot 0 < \sum_{\tau=0}^{\infty} \delta^{\tau} \cdot 1 = \frac{1}{1-\delta}$$

Hence $\delta > 1/2$ is the relevant condition, i.e., the claim is correct.

INFINITELY REPEATED PRISONER'S DILEMMA (cont'd)

Indeed all individually rational payoffs can be attained ("folk theorem")



Set of individually rational payoffs: $V = \{(\pi_1,\ldots,\pi_n) : \exists a \in A \ s.t. \ g_i(a) \geq \pi_i \geq \underline{\pi}_i \quad \forall i \in N\},$

where $\underline{\pi}_i = \min_{a_{-i} \in A_{-i}} \left\{ \max_{a_i \in A_i} g_i(a_i, a_{-i}) \right\}$ **is player i's minmax payoff**

NASH REVERSION FOLK THEOREM
-- Just for Information --
Friedman (1971)

Definition: **The set** $R = \left\{ (\pi_1,\ldots,\pi_n) : \exists NE\, e^* \, of \; G \; and \; \exists a \in A \; s.t. \; g_i(a) \geq \pi_i \geq g_i(e^*) \right\} \subseteq V$ **is the set of Nash reversion payoffs.**

Definition: **The infinitely repeated normal-form game** G **yielding individual average payoffs of**

$$\Pi_i(\sigma_i, \sigma_{-i}) = (1-\delta)\sum_{t=0}^{\infty} \delta^t g_i(\sigma_i^t, \sigma_{-i}^t)$$

given any strategy profile σ **is called a supergame,** $G^{\infty}(\delta)$ **.**

We can now formulate *Friedman's (1971) Nash reversion folk theorem*:(1)

Theorem: **For any Nash reversion payoff** $\pi \in R$ **there is a constant** $\underline{\delta} \in (0,1)$ **such that for any common discount factor** $\delta \in (\underline{\delta},1)$ **there exists an SPNE of the supergame** $G^{\infty}(\delta)$ **with payoffs equal to** π **.**

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(1) See Friedman, J. (1971) "A Non-Cooperative Equilibrium for Supergames," *Review of Economic Studies*, Vol. 28, No. 1, pp.1—12.

NASH REVERSION FOLK THEOREM (cont'd)

Proof (Outline):

Consider a (possibly correlated) stage game action profile a **such that** $\pi = (g_1(a),\ldots,g_n(a)) \in R$ **. The following strategy profile** s^* **induces an SPNE in the supergame:**

- **Start playing** a_i **and continue doing so, as long as** a **was played in the previous period;**
- **If in the previous period, at least one player deviated, then each player plays a dominated NE** e^* **for the rest of the game.**

This strategy profile constitutes indeed an SPNE, since

$$\max_{\hat{a} \in A} g_i(\hat{a}) + \frac{\delta e_i^*}{1-\delta} \leq \frac{g_i(a)}{1-\delta}$$

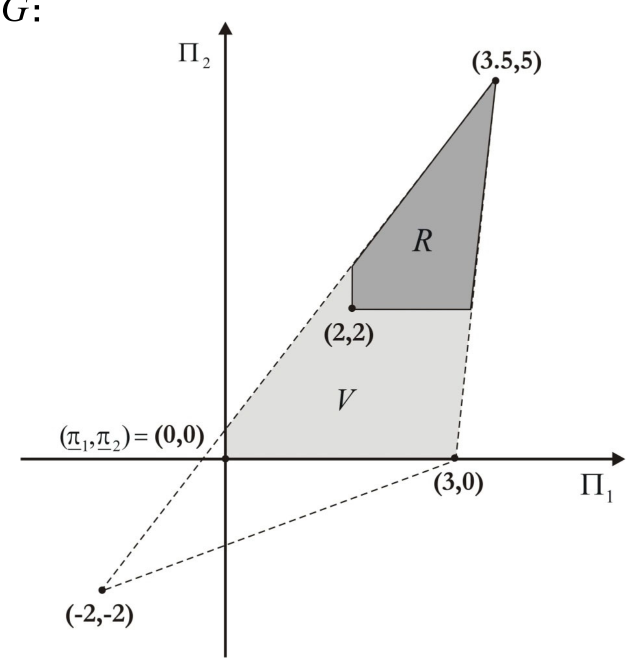
as long as $\delta \in (0,1)$ **is large enough. The rest follows using the one-shot deviation principle.**

EXAMPLE

Consider the following normal-form stage game G:

Player 2

		L	C	R
Player 1	U	0,0	0,0	3.5,5
	M	0,0	-2,-2	0,0
	D	2,2	0,0	3,0



- G has two pure-strategy NE: [D,L] and [U,R]
- Minmax payoffs are $(\underline{\pi}_1, \underline{\pi}_2) = (0{,}0)$

INFINITELY REPEATED COURNOT DUOPOLY

Stage Game

Two firms, 1 and 2, produce homogeneous widgets in respective quantities q_1 and q_2 .

- Firm i's production cost is $C(q_i) = cq_i$ (with constant marginal cost, $c > 0$)
- Inverse market demand is given by $P(Q) = a - Q$, where $a > c$ and $Q = q_1 + q_2$

COMPETITION

The unique NE of the stage game is given by $q_1^c = q_2^c = (a-c)/3$ yielding profits of for the firms $\pi_1^c = \pi_2^c = (a-c)^2/9$

MONOPOLY

If the two firms merge, they can improve stage game profits by producing half of the monopoly quantity each, i.e., they choose $q_1^m = q_2^m = (a-c)/4$ so as to obtain $\pi_1^m = \pi_2^m = (a-c)^2/8 > \pi_i^c$.

Note that the monopoly outcome is Pareto-dominant (from the firms' point of view); however, without a contract, each firm could improve its profit unilaterally by deviating (i.e., it is not a NE of the stage game: best response to monopoly quantity would be $B_i(q_{-i}^m) = \left((a-c) - q_{-i}^m\right)/2 = 3(a-c)/8 > q_i^c > q_i^m$ leading to deviation profits of $\overline{\pi}_i = 9(a-c)^2/64 > \pi_i^m$).

INFINITELY REPEATED COURNOT DUOPOLY (cont'd)
Dynamic Collusion

Question: Can the two firms collude in the supergame?

Answer: Yes, if they are both patient enough (i.e., if the firms' common discount factor δ is close enough to one)

Consider the following Nash reversion strategy for firm i:

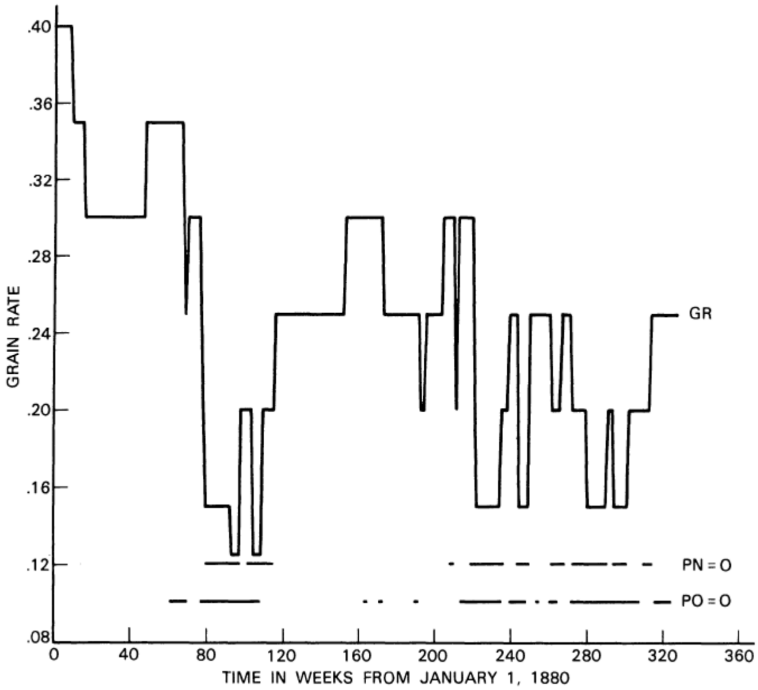
- Produce q_i^m in the first period and continue to produce q_i^m as long as the observed outcome in the previous period is (q_1^m, q_2^m)
- If the outcome in the previous period is different from (q_1^m, q_2^m), then choose q_i^c forever thereafter

Check that this strategy profile constitutes a SPNE using the one-shot deviation principle. Indeed, the payoff difference from a deviation,

$$\Delta_i = \left(\bar{\pi}_i + \frac{\delta \pi_i^c}{1-\delta}\right) - \frac{\pi_i^m}{1-\delta} = \left(\frac{9(a-c)^2}{64} + \frac{\delta(a-c)^2}{9(1-\delta)}\right) - \frac{(a-c)^2}{8(1-\delta)} < 0 \quad \Leftrightarrow \quad \delta > \frac{9}{17}$$

is negative, as long as δ is close enough to one, since $\pi_i^m > \pi_i^c$.

EXAMPLE: THE JOINT EXECUTIVE COMMITTEE
Cooperation (Collusion) with Price as Decision Variable



Source: Porter, R.H. (1983) "A Study of Cartel Stability: The Joint Executive Committee, 1880-1886," *Bell Journal of Economics* 14(2):301—314.

AGENDA

Dynamic Games: Coordination & Cooperation

Relational Contracts

Key Concepts to Remember

VOLUNTARY COOPERATION = RELATIONAL CONTRACT

Requirements

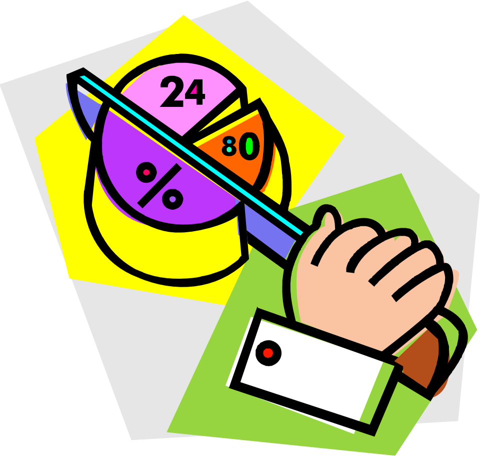
1. Common Interest
2. Repeated Interaction
3. Ability to Detect Defections
4. Ability to Punish Defections
5. Ability to Deal with Errors

Implementation

- Explicit Cooperation (e.g., via contract or public enforcement)
- Tacit Cooperation (e.g., using a mechanism of mutual punishments)

Cooperation, i.e., relational contracting, works when the (discounted) value of a continued relationship is larger than what any party can gain by breaking the agreement

REQUIREMENT 1: COMMON INTEREST



Incentives have to be aligned in some way and there has to be a value (at least on average) of cooperating.

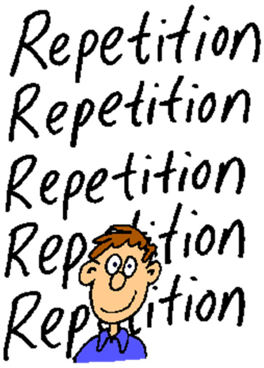
REQUIREMENT 2: REPEATED INTERACTION

Compare the one-shot Prisoners' Dilemma to the finitely repeated Prisoners' Dilemma and to the infinitely repeated Prisoners' Dilemma.

- What do you notice?

Positive probability of interacting again in general enough.

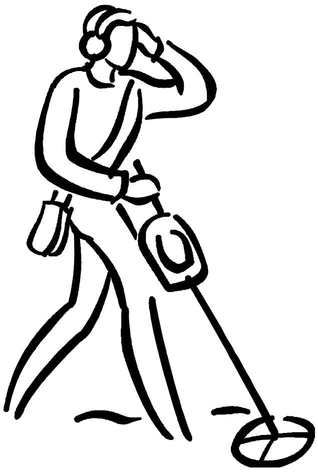
Without outside enforcement, "infinite" horizon of relationship is critical to providing incentives for cooperation.



REQUIREMENT 3: ABILITY TO DETECT DEFECTIONS

There are many problems with detecting defections

- **Imperfect observability**
- **Mixed strategy needed to implement the desired outcome**
- **Outside noise**
- **Misperceptions**
- **Lack of common knowledge**
- **"Law of increasing opaqueness" (Dixit/Nalebuff, 1991)**



REQUIREMENT 4: ABILITY TO PUNISH DEFECTIONS

- **Threats have to be credible → Subgame perfection**

What do we require from punishments?

- **They have to be "incentive compatible"**
- **They have to be "legal"**
- **They have to be "quick"**
- **They should be "forgiving"**
- **They have to be "individually rational"**

REQUIREMENT 5: ABILITY TO DEAL WITH ERRORS

ABILITY TO DEAL WITH ERRORS
… Renegotiating or Not

Stanley Kubrick: Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb.

AGENDA

Dynamic Games: Coordination & Cooperation

Relational Contracts

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- **Rationality and common knowledge**
- **Game (in normal form and extensive form)**
- **Nash equilibrium (in pure or mixed strategies)**
- **Cournot oligopoly**
- **Concept of commitment**
- **Credible threat and commitment devices**
- **Subgame perfection**
- **Supergame**
- **Grim Trigger Strategy**
- **Minmax Strategy**
- **Nash Reversion**
- **Folk Theorem**
- **Individually Rational Payoffs**
- **Relational Contract**