MGT 528 – OPERATIONS: ECONOMICS & STRATEGY

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6. Dealing with Risk

Autumn 2022

École Polytechnique Fédérale de Lausanne
College of Management of Technology

AGENDA

Risk Attitude

Risk Management

- **Value-at-Risk Measure**
- **Antifragility**
- **The Power of Inaction**
- **Qualitative Approaches**

Key Concepts to Remember

LOTTERIES ARE (DISCRETE) RANDOM VARIABLES

Let X be a **random variable** with possible outcomes in the set $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$ (the "outcome space"). Each outcome x_i occurs with probability p_i.

The random variable X is sometimes also called a **lottery** and denoted

$$X = [p_1, x_1; p_2, x_2; \dots; p_n, x_n]$$

If all outcomes x_i are real and measured in dollars (or any other currency), then X is commonly referred to as a **"money lottery."**

The set of all lotteries with outcomes in $\mathcal{X}$ is the **"lottery space" $L(\mathcal{X})$**.

Example: A coin-flip lottery X (with an unbiased coin) pays \$1 if heads and zero if tails. Then X = [0.5,\$1; 0.5, \$0].

PREFERENCES OVER LOTTERIES: EXAMPLE

Question. Jane likes to play ping pong and she wonders about how to respond to an opponent's serve.

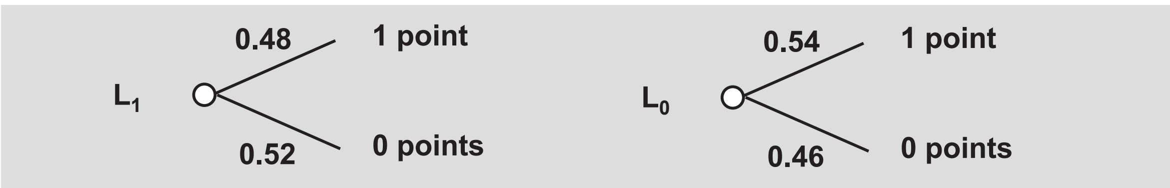
- If she hits a top spin (decision d_1), the ball is going to be on the table with probability 0.6 and given that it is, she is going to score with probability 0.8.
- If she does not play a top spin (decision d_0), the ball is going to land on the table with probability 0.9, but she is only going to score with probability 0.6.

What decision should she take?

Solution:

Jane needs to choose between the following two lotteries:

- $L_1 = [P(\text{score}|d_1), 1 \text{ point}; P(\text{don't score}|d_1), 0 \text{ points}]$
- $L_0 = [P(\text{score}|d_0), 1 \text{ point}; P(\text{don't score}|d_0), 0 \text{ points}]$



Thus, she should prefer L_0 (i.e., $L_0 \geq L_1$) which implies **"don't play top spin"** as her decision.

PREFERENCES OVER OUTCOMES
Utility Representation

Preferences over lotteries imply preferences over particular (certain) outcomes in *X*. Indeed, for any x and y in *X* one could just consider the lotteries X and Y that produce the outcomes x and y with probability one respectively.

Thus, x $\geq$ y if the outcome x is (weakly) preferred to the outcome y.

Definition: A real-valued function u with domain *X* represents the DM's preferences over outcomes in *X* if for any x,y in *X*:

x $\geq$ y if and only if u(x) $\geq$ u(y).

The function u is called the DM's utility function.

For some preferences no utility function representation exists (e.g., lexicographic preferences). We typically take a utility function as an input for a decision model.(1) A utility function always exists for finite sets of outcomes.

A *utility representation of a DM's preferences is generally* *not unique*: given any utility function u and a strictly increasing function ϕ (from real numbers to real numbers), the function v = ϕ(u) is an *equivalent* utility representation.

(1) For more details on utility theory see Kreps, D.M. (1988) *Notes on the Theory of Choice*, Westview Press, Boulder, CO, or Fishburn, P.C. (1970) *Utility Theory for Decision Making*, Wiley, New York, NY.

EXPECTED UTILITY MAXIMIZATION

Given a utility representation u of the DM's preferences over outcomes, we would like to infer his preferences over lotteries of outcomes (random variables), which corresponds to his preferences over actual decisions (e.g., at which speed to drive a car).

Under certain axioms (= assumptions on the DM's preferences over lotteries, typically referred to as the Von Neumann-Morgenstern axioms), the DM's expected utility of a particular decision d in D, which induces a lottery X(d) with probability distribution P(.|d), is

$$EU(X(d)) = \sum_{x \in X} P(x \mid d) u(x)$$

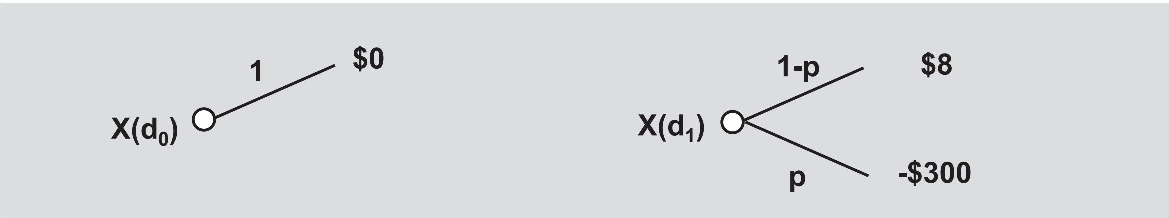
Thus, under uncertainty the DM maximizes expected utility, i.e., he solves

$$d^* \in \arg\max_{d \in D} EU(X(d))$$

EXPECTED UTILITY MAXIMIZATION: EXAMPLE

Question. Joe needs to decide how fast to drive on highway E25 from Lausanne to Geneva. Any minute saved he values at $1. At 120 km/h it takes about 40 min, and at 140 km/h about 32 min. However, if he drives 140 km/h there is a chance p that he gets pulled over and has to pay a ticket worth $280 plus a delay of 20 min (over the 120 km/h time).

Let d_0 : drive 120 km/h, d_1 : drive 140 km/h. He thus needs to choose between the lotteries $X(d_0)$ = [1,$0] and $X(d_1)$= [p, -$300; 1-p, $8].



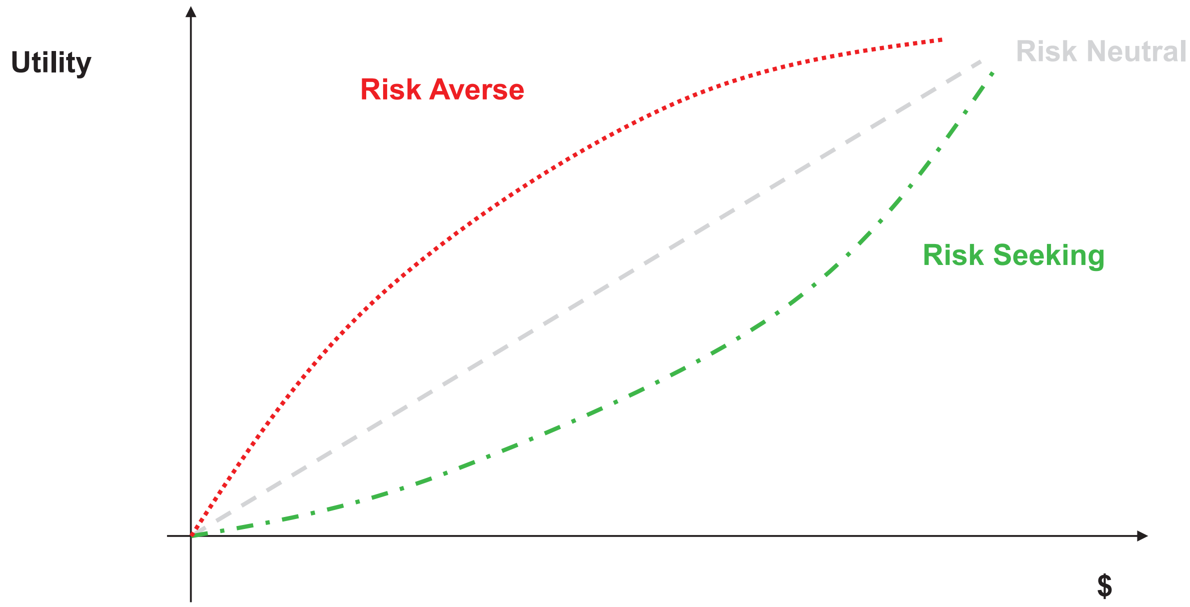
If Joe's utility function for money is u(x)=-exp(-x/1000), at what detection probability p would he be indifferent between d_0 and d_1?

Answer:

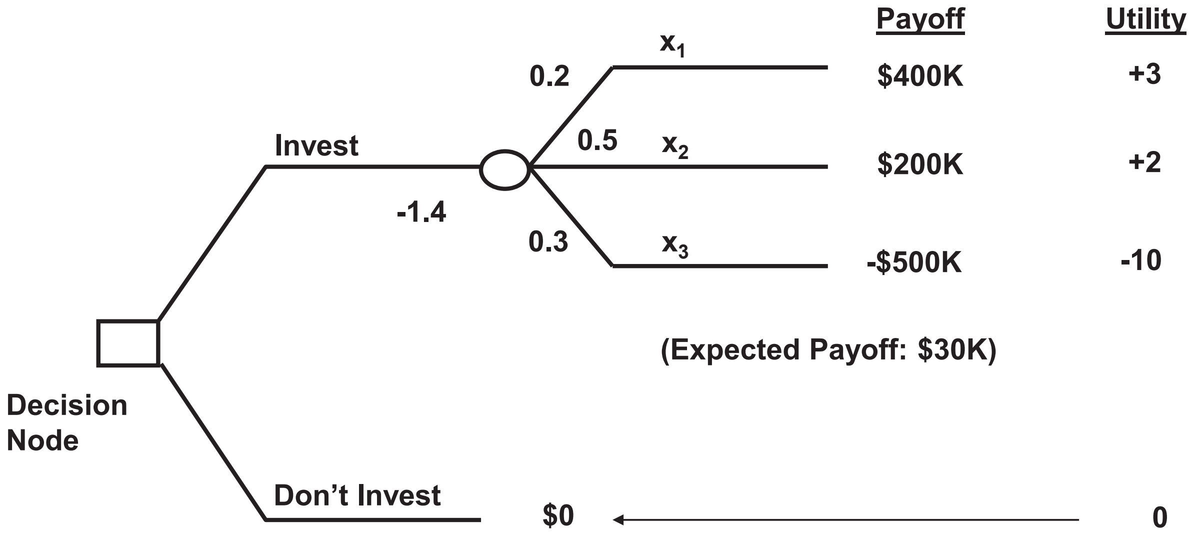
$$EU(X(d_0)) = u(\$0) = EU(X(d_1)) = pu(-\$300) + (1-p)u(\$8)$$

$$\Rightarrow\ p = \frac{u(\$8) - u(\$0)}{u(\$8) - u(-\$300)} = 2.2\%$$ (i.e., for p>2.2%, Joe drives 120 km/h!)

UTILITY FUNCTIONS: SOME COMMON SHAPES



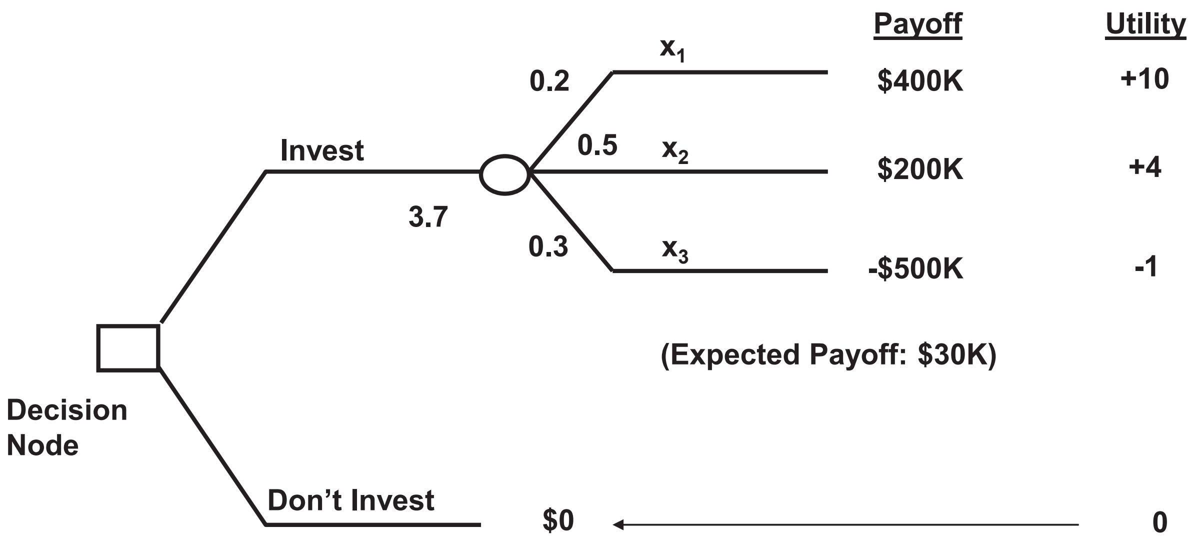
A SIMPLE DECISION: RISK AVERSE DM



Decision Criterion: Maximize Expected Utility

EU = (0.2)(3)+(0.5)(2)+(0.3)(-10) = -1.4 < 0 → Don't Invest!

A SIMPLE DECISION TREE: RISK-SEEKING DM



Decision Criterion: Maximize Expected Utility

EU = (0.2)(10)+(0.5)(4)+(0.3)(-1) = 3.7. Invest!

CERTAINTY EQUIVALENT

The certainty equivalent of a lottery is a single certain outcome for which the DM is indifferent between receiving the outcome for sure and participating in the lottery.

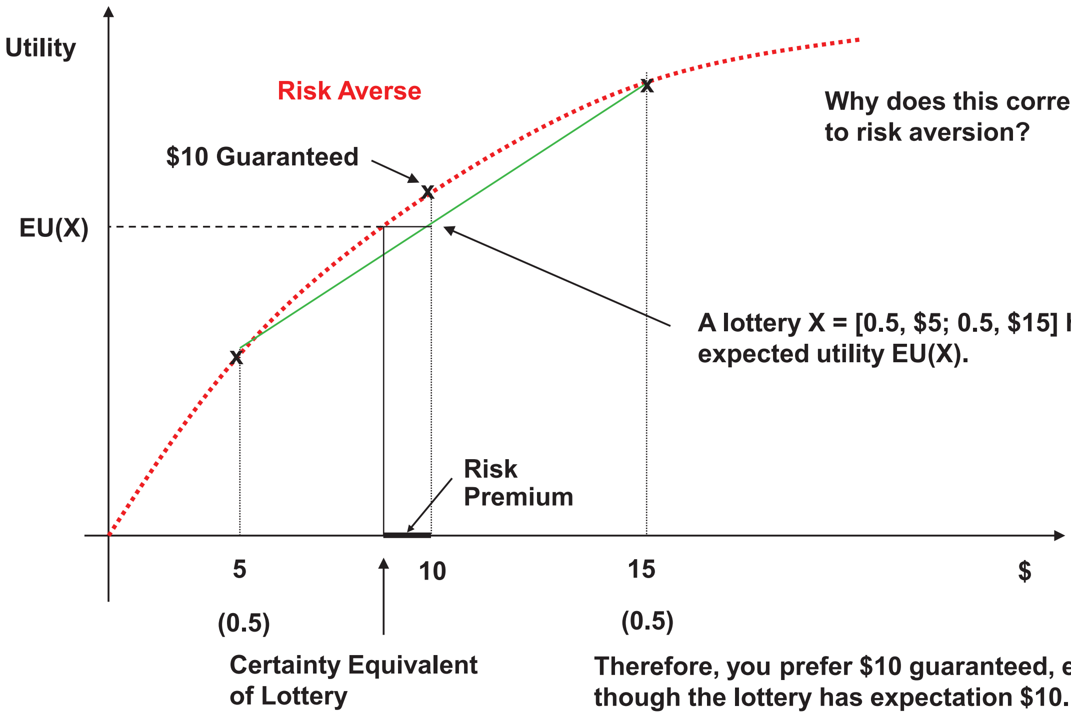
It represents the "selling price" of the lottery.

Denote the certainty equivalent of a lottery X by $CE(X)$

Then: $u(CE(X)) = EU(X)$ **(DM is indifferent)**

$$\Rightarrow CE(X) = u^{-1}(EU(X))$$

CERTAINTY EQUIVALENT: EXAMPLE



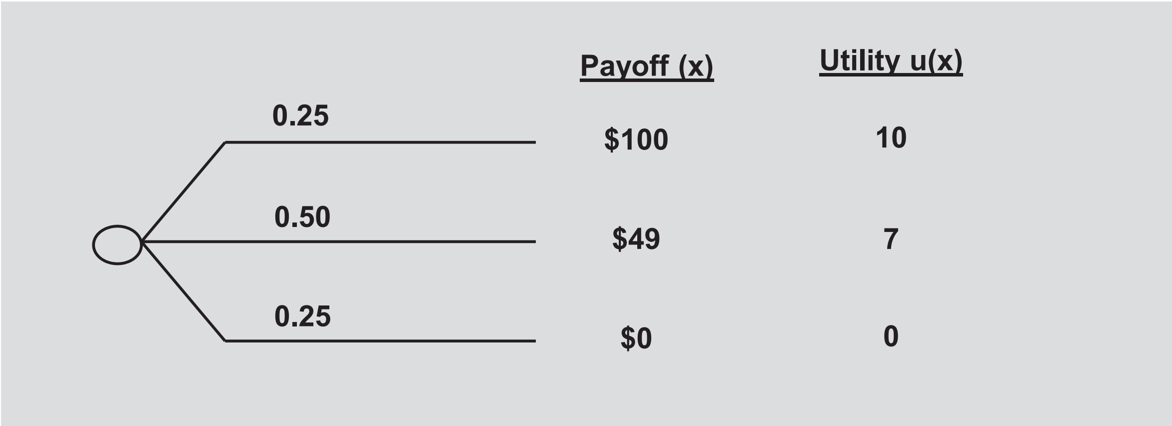
Why does this correspond to risk aversion?

A lottery X = [0.5, $5; 0.5, $15] has expected utility EU(X).

Therefore, you prefer $10 guaranteed, even though the lottery has expectation $10.

CERTAINTY EQUIVALENT: ANOTHER EXAMPLE

Consider the lottery X = [.25, \$100; .5, \$49; .25, \$0] and the utility function $u(x) = \sqrt{x}$



Expected utility: EU(X) = (0.25)(10) + (0.5)(7) + (0.25)(0) = 6

$$EU(X) = u(CE) = \sqrt{CE} \Rightarrow \sqrt{CE} = 6 \Rightarrow CE = \$36$$

ABSOLUTE AND RELATIVE RISK AVERSION DESCRIBE A DM'S RISK ATTITUTE

The level of risk aversion may be measured by the (Arrow-Pratt) absolute-risk-aversion coefficient,

$$R(x) = -\frac{u''(x)}{u'(x)}$$

or the relative-risk-aversion coefficient,

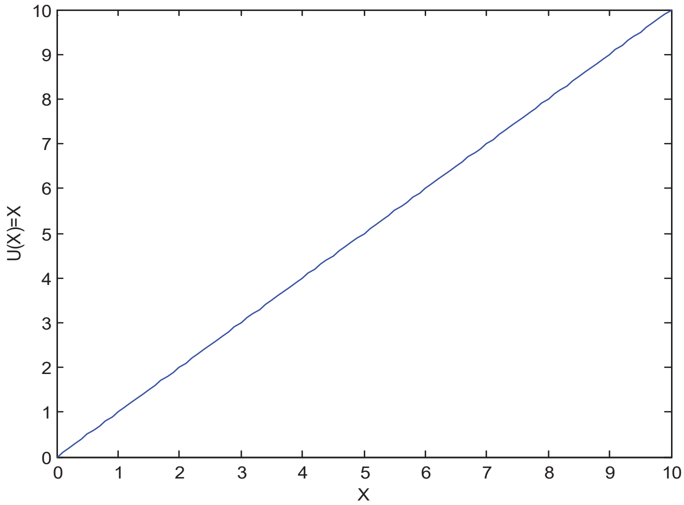
$$r(x) = xR(x)$$

If R(x) > 0, the DM is risk averse. Similarly, if R(x) < 0, the DM is risk seeking, while R(x)=0 for a risk-neutral DM.

Both absolute and relative risk aversion are local properties: they can vary for different outcomes.

RISK NEUTRALITY: LINEAR UTILITY

$$u(x) = \alpha\, x \qquad u'(x) = \alpha \qquad u''(x) = 0$$



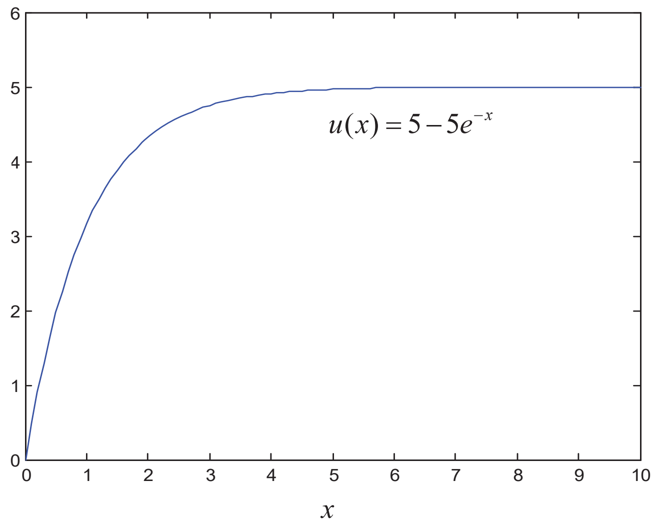
$$R(x) = -\frac{u''(x)}{u'(x)} = 0$$

CONSTANT ABSOLUTE RISK AVERSION

Exponential utility: $u(x) = \alpha - \beta e^{-\rho x}$

$$u'(x) = \rho \beta e^{-\rho x}$$

$$u''(x) = -\rho^2 \beta e^{-\rho x}$$



$$R_A(x) = -\frac{u''(x)}{u'(x)} = \rho$$

Exponential utility functions exhibit constant absolute risk aversion (CARA).(1)

(1) CARA utility functions are often used in financial modeling, since it allows obtaining conclusions free from wealth effects (adding a constant w to an individual's wealth just amounts to a positive linear transformation and thus leads to the same decisions, since the expected utility representation of the individual's preferences does not change).

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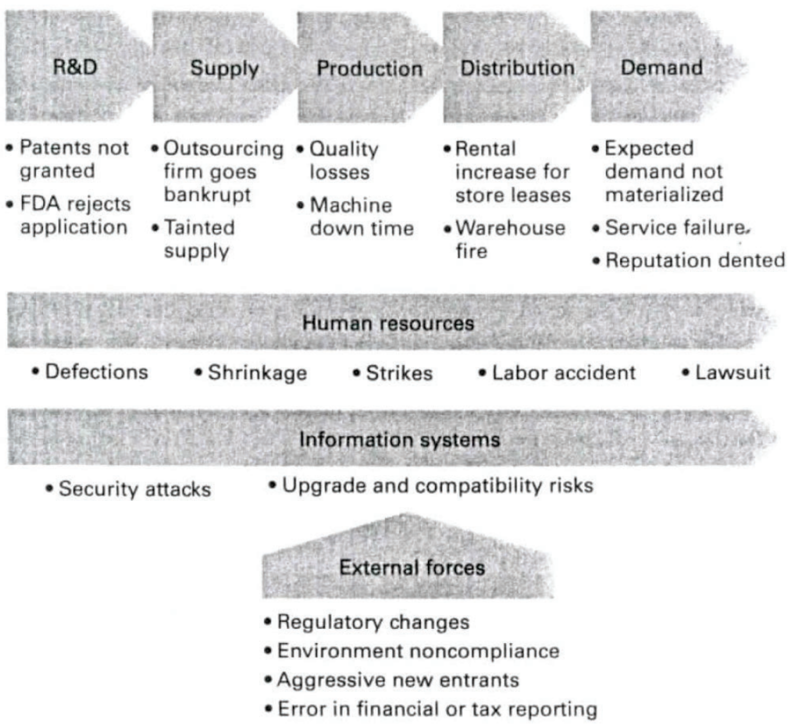
Key Concepts to Remember

RISKS EXISTS ALL ALONG THE VALUE CHAIN
Example: Pharmaceutical Company



Source: Lai, R. (2013) *Operations Forensics*, MIT Press, Cambridge, MA.

SYSTEMATIC RISK + SPECIFIC RISK = TOTAL RISK

Definition. Systematic risk (undiversifiable risk, market risk)

Risk that is inherent to an entire market or market segment. This type of risk is difficult to predict and cannot be diversified away by investors.

Definition. Specific risk (diversifiable risk, unsystematic risk)

This is the business-specific risk that can be diversified away, often measured by the "beta" of a company: beta = cov(business return, market return)/var(market return)

E[business return] = risk-free return + beta x (E[market return] – risk-free return)

(Capital Asset Pricing Model (CAPM))

Business-specific risk can be process risk (= recurring) or one-off risk (= low probability, high consequence events).

Which of these are systematic and specific depends on the situation.

MANAGERIAL ACTION DEPENDS ON PROBABILITY AND SEVERITY
Simple 2x2 Matrix is a First Step to Classify Risks

		Probability: Low	Probability: High
Severity	High	Transform, transfer, diversify, insure	Immediately diagnose and neutralize
Severity	Low	Accept, insure	Monitor, reduce probability, avoid

Source: Lai, R. (2013) *Operations Forensics*, MIT Press, Cambridge, MA.

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VALUE-AT-RISK MEASURE FOR ONE-OFF RISKS

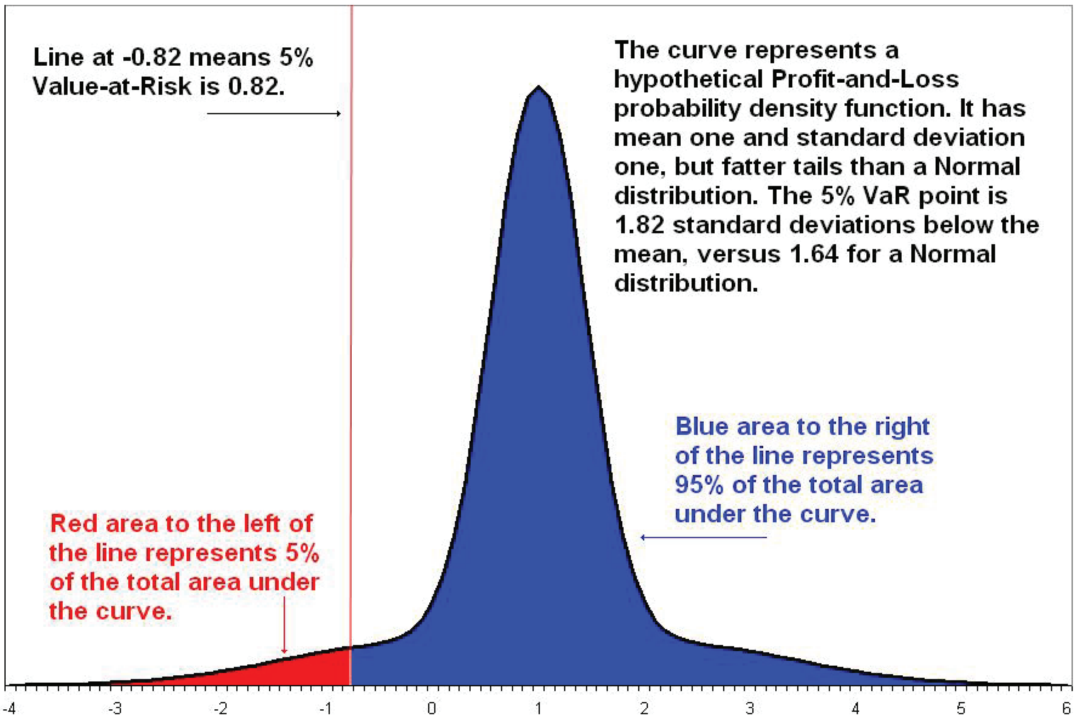
- **Let the random variable X represent the net monetary payoff for a company in a given period (for $X < 0$ the company makes a loss, for $X > 0$ a gain).**
- **Let F(x) be the cumulative probability distribution function (cdf), defined for all realizations x.**
- **Then $z = F(v(z)) = \mathrm{Prob}(X<v(z))$ defines the value at risk v(z) (VaR) at the z (percent)-level**

Example: Let z = 5% = 0.05. G(.) : cdf for a standard normal distribution, i.e., N(0,1); and F(.) cdf for a normal distribution with mean μ and standard deviation σ, i.e., $N(\mu, \sigma^2)$.

Then: $5\% = F(v(z)) = G((v(z)- \mu)/\sigma) = G(-1.96)$.

Hence: $v(z) = \mu - 1.96\ \sigma$.

QUANTIFY LOW-PROBABILITY HIGH-SEVERITY EVENTS
Value-at-Risk Measure



Source: Wikipedia.

EXAMPLE: DETERMINE VALUE AT RISK

Question. Define the value-at-risk (VaR) measure for a monopolist software vendor who faces a random demand curve of D(p) = X - p, with zero variable cost and fixed cost of K= $100,000, where the random variable X is distributed uniformly on the interval [0,2000]. Determine the company's VaR at the 10-percent level.

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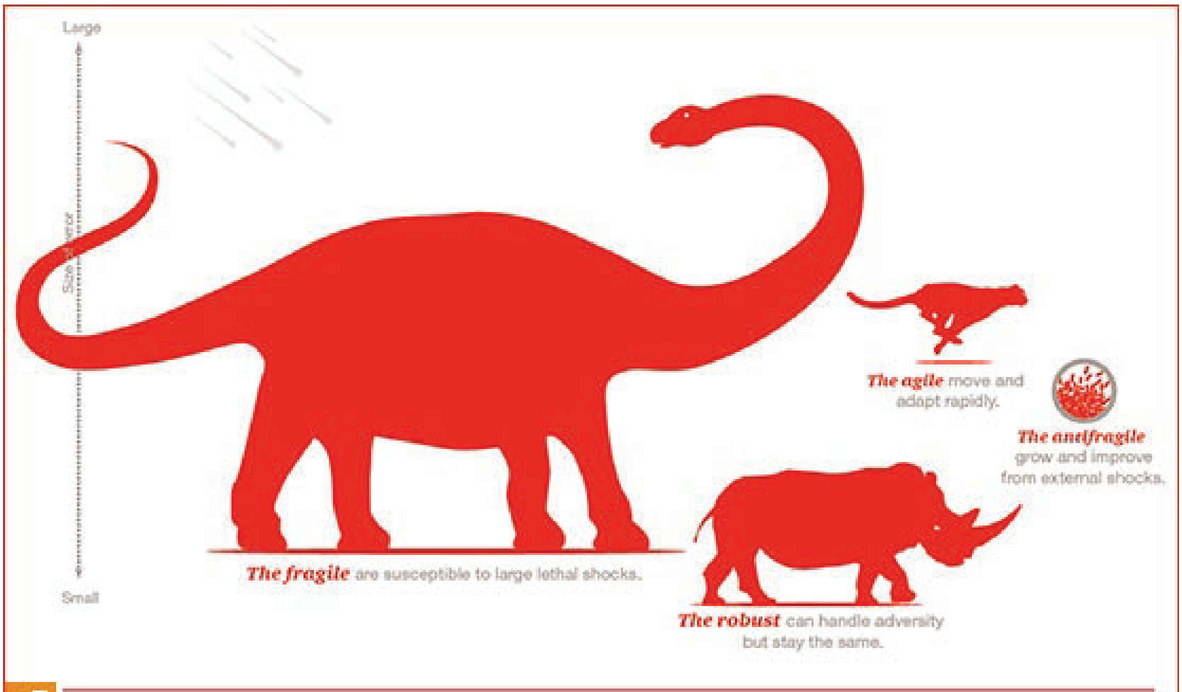
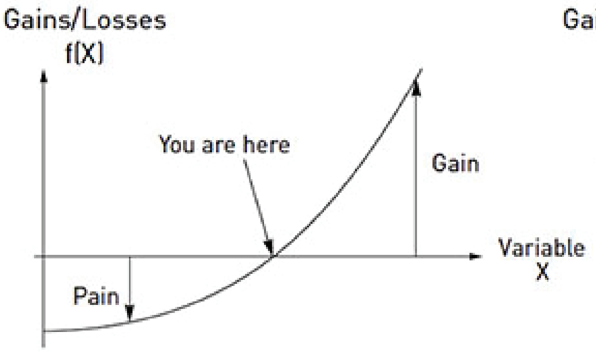
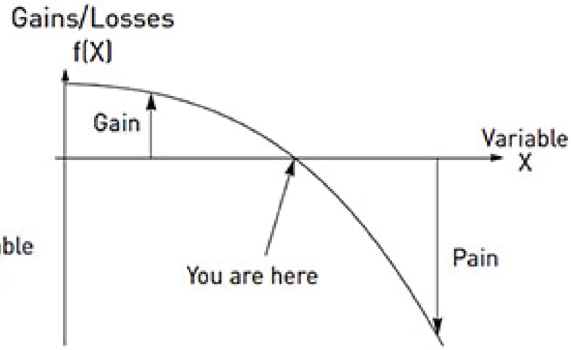
ANTIFRAGILITY



Source: Nassim Taleb, Rob England, and PwC, 2013

ANTIFRAGILITY: THE IDEA IS SIMPLE



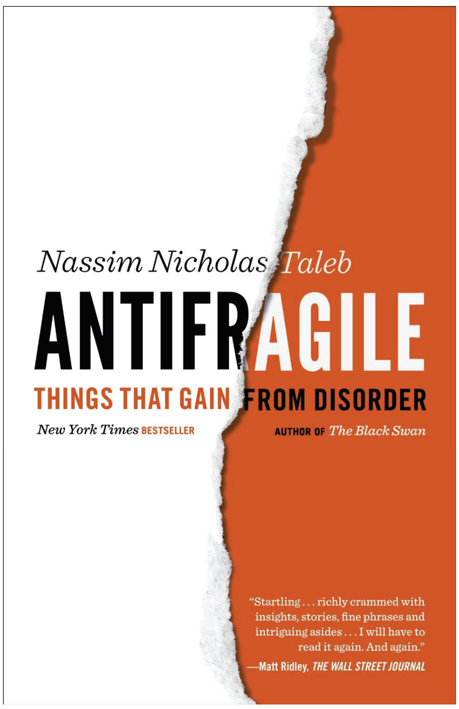


Positive asymmetric payoff (**antifragile**) anything that has more upside than downside from random events (variability)

Negative asymmetric payoff (**fragile**) anything that has more downside than upside from random events (variability)

Source: innolution.com

MORE ON THE SUBJECT



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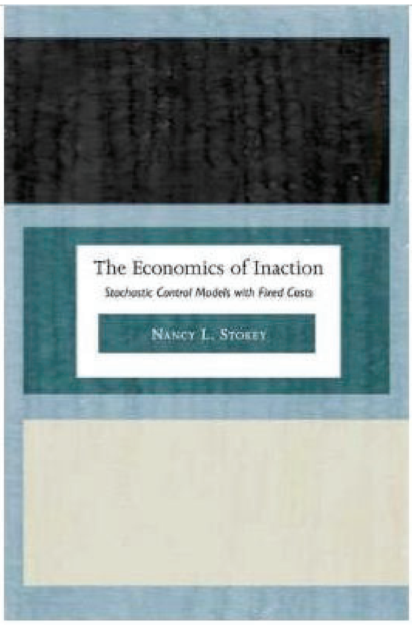
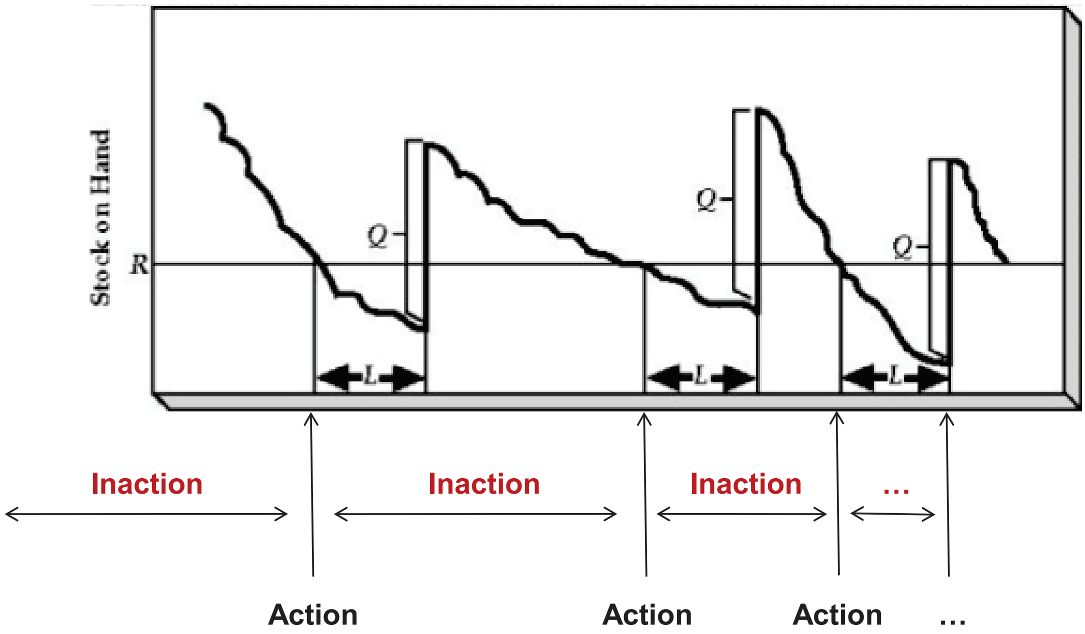
Key Concepts to Remember

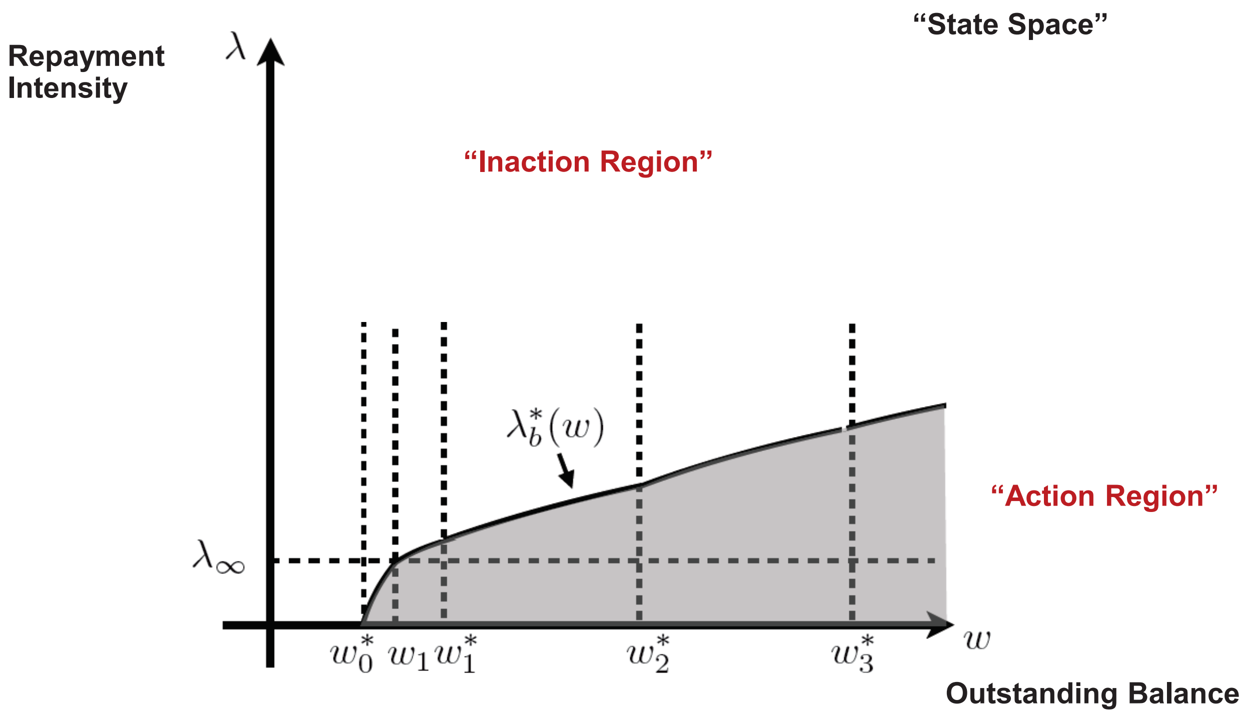
INACTION IS AN IMPORTANT CHOICE



EXAMPLE 1: STOCHASTIC INVENTORY CONTROL

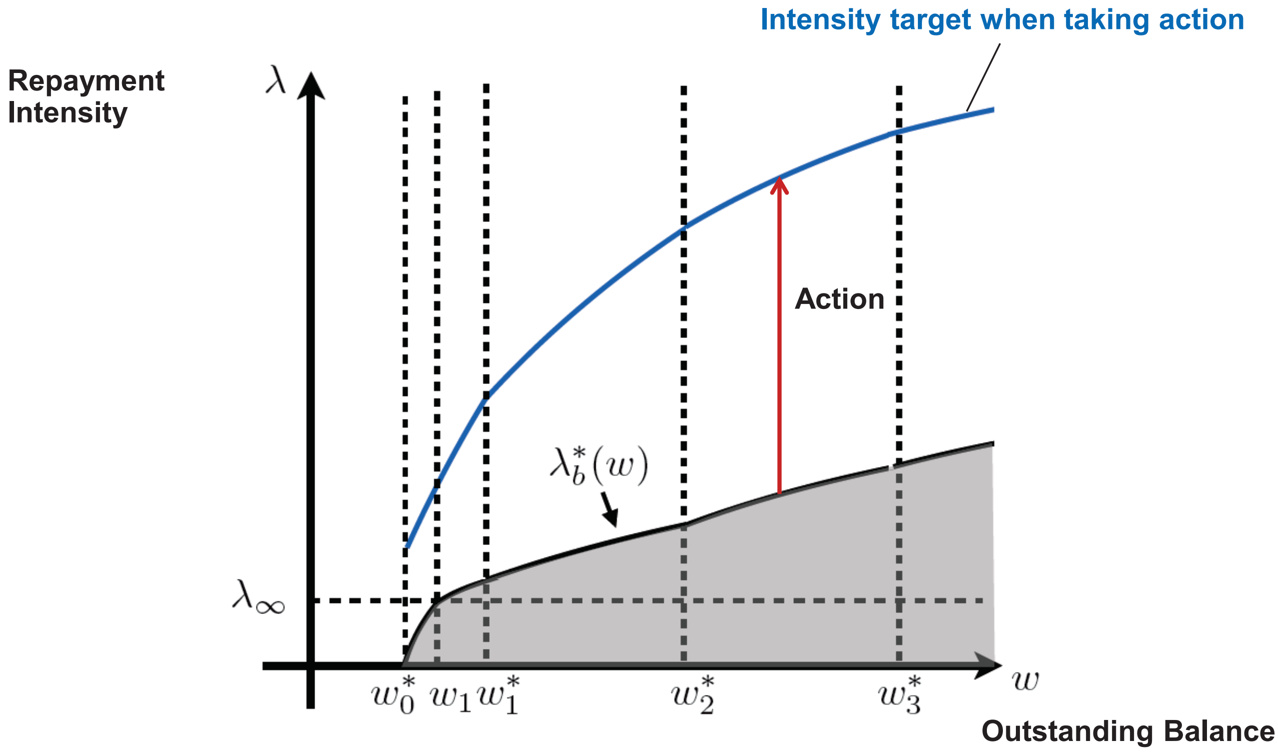


EXAMPLE 2: CREDIT COLLECTIONS



Source: Chehrazi, N., Glynn, P.W., Weber, T.A. (2015) "Dynamic Credit-Collections Optimization"

EXAMPLE 2: CREDIT COLLECTIONS (Cont'd)



Source: Chehrazi, N., Glynn, P.W., Weber, T.A. (2015) "Dynamic Credit-Collections Optimization"

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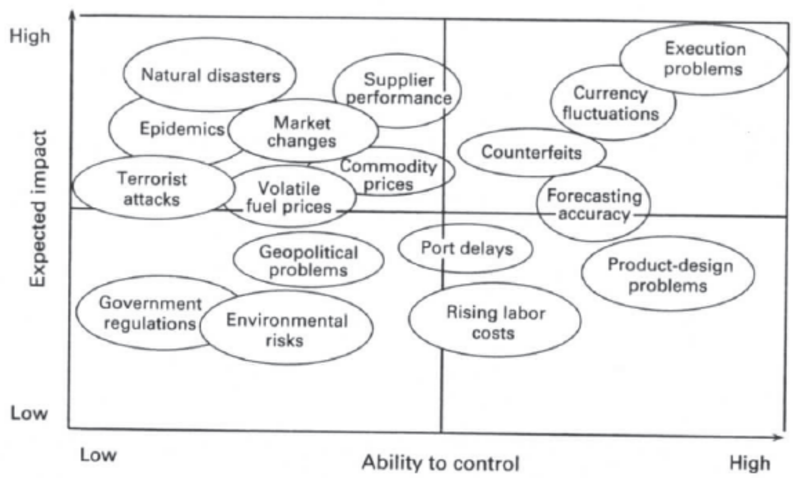
Key Concepts to Remember

CAN ALSO CLASSIFY RISKS BY ABILITY TO CONTROL

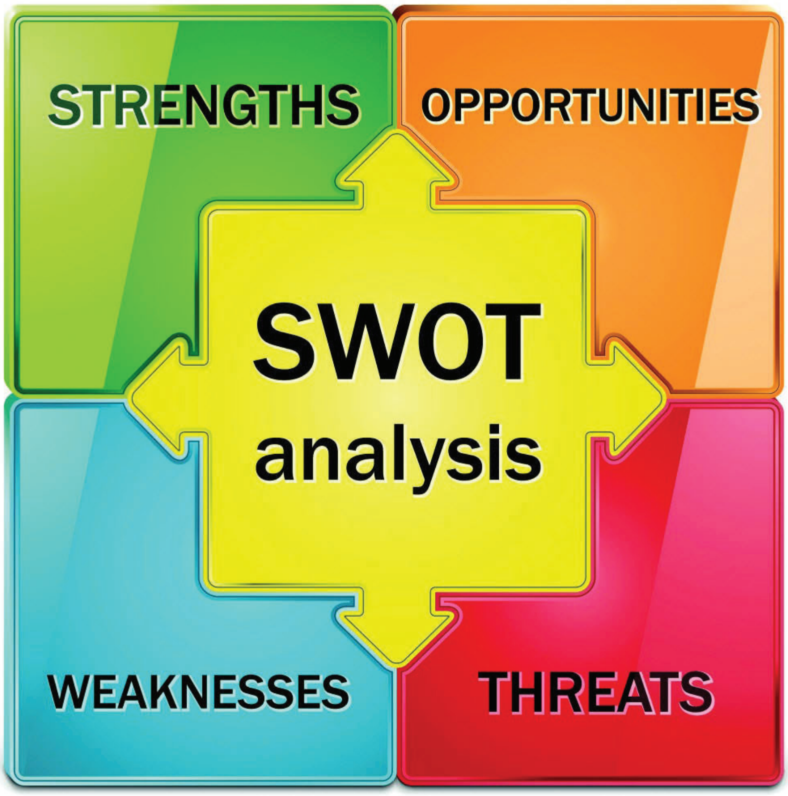
Source: Simchi-Levi, D. (2010) *Operations Rules*, MIT Press, Cambridge, MA.

ABILITY TO CONTROL VS. EXPECTED IMPACT (SEVERITY)

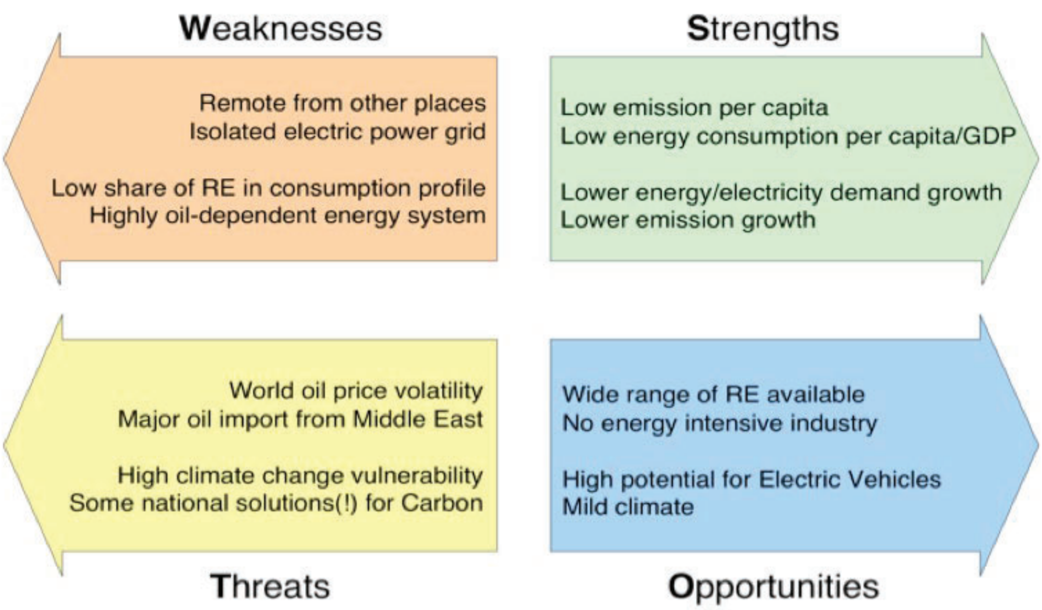


Source: Simchi-Levi, D. (2010) *Operations Rules*, MIT Press, Cambridge, MA.

SWOT ANALYSIS

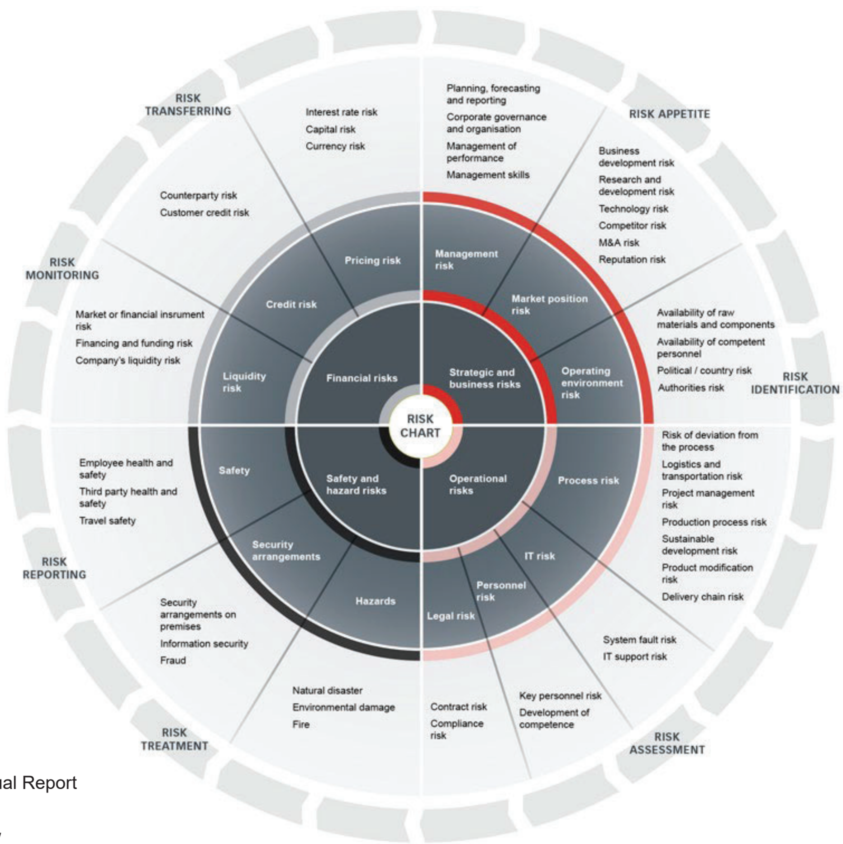


SWOT ANALYSIS: AN EXAMPE
Renewable Energy (RE) in Hawai'i



Source: UHERO (Economic Research Organization at the University of Hawai'i)

DIFFERENT TYPES OF BUSINESS RISK



Source: Cargotec, 2013 Annual Report

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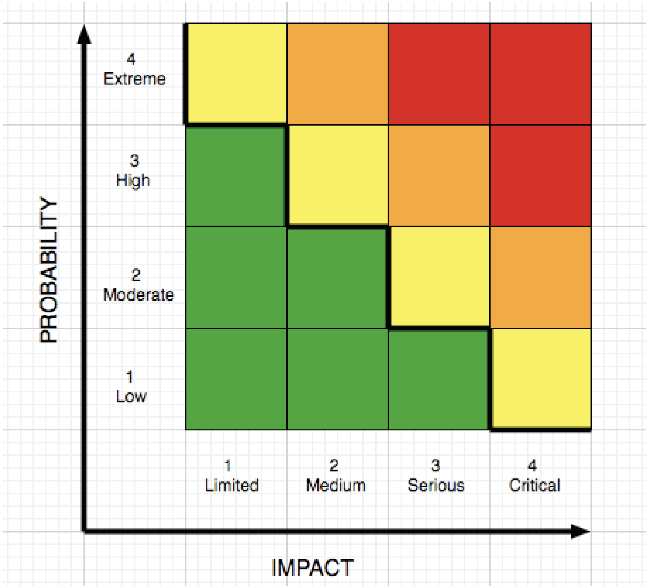
	Low	Accept, insure	Monitor, reduce probability, avoid

Source: Lai, R. (2013) *Operations Forensics*, MIT Press, Cambridge, MA.

PORTFOLIO OF RISKS IN (IMPACT,PROBABILITY)-SPACE



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KEY CONCEPTS TO REMEMBER

- Expected Utility Maximization
- Risk Aversion
- Certainty Equivalent
- Risk Premium
- Value-at-Risk Measure
- Antifragility
- Action Region/Inaction Region
- SWOT Analysis
- Classification of Risks (wrt Probability & Severity)
- Risk Mitigation