MGT 528 – OPERATIONS: ECONOMICS & STRATEGY

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5. Inventory Management: Basics

Handwritten Course Notes

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École Polytechnique Fédérale de Lausanne

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WHY HOLD INVENTORY AT ALL?
Possible Reasons (Selection)

- **Satisfy demand in a timely manner (at a sufficiently high service level, e.g., in terms of fill rate and/or probability of stockout)**
- **Take advantage of economies of scale, e.g., because of fixed cost (truck costs the same no matter how full) or nonlinear pricing by suppliers (quantity discounts)**
- **Comply with regulatory requirements (e.g., spare parts for military, pharmaceutical supply, water)**
- **Guard against disruptions: supply-chain resilience (e.g., 7-day water storage at Evian, dual sourcing by military)**
- **Smoothen production (decoupling of different production steps)**
- **Prepare for high seasonal demand given limited production capacity**
- **Safety against stochastic demand and supply (lead time) fluctuations**

DIFFERENT TYPES OF INVENTORY
Cachon/Terwiesch (2009)

1. **Pipeline Inventory**
   It takes time to process or service a unit (e.g., a patient in a hospital), which implies an inventory (I) by *Little's Law*: $I = R \times T$, where R = avg flow rate (e.g., 1 patient/hr) and T = avg flow time (e.g., 2 hrs, so I = 2 patients).
2. **Seasonal Inventory**
   If the sales (s) in a selling season (of length t) exceeds the production capacity (k), then it is necessary to produce a sufficient stock. For example, if t = 60 days, s = 200 widgets/day, and k = 50 widget/day, then one needs to start production 180 days ahead of time and produce for 240 days in total, for a maximum inventory of 9,000 widgets (at the beginning of the selling season) and therefore an average inventory of 4,500 widgets (over the entire 240 days of production activity).
3. **Cycle Inventory**
   To take advantage of scale economies it is often useful to process several flow units together. For example, the EOQ model can be used compute the "economic order quantity" in the presence of fixed ordering cost.
4. **Decoupling Inventory (Buffers)**
   To decouple different sequential production steps (e.g., piece manufacture and assembly), it is necessary to keep local queues. The reduction of these buffers is the goal of "lean manufacturing," e.g., using "*kanban*" produce-to-demand systems
5. **Safety Inventory**
   A safety stock (s) compensates for a random lead time (LT, with stdev σ_{LT}) and random demand (D, with stdev σ_D). A standard rule of thumb used (here without derivation) is: $s = z \, \mathrm{sqrt}(LT * \sigma_D^2 + D^2 * \sigma_{LT}^2)$, where z is the inverse of the normal distribution at the service-level (e.g., at 95% in-stock probability it is z = 1.64). The newsvendor problem (discussed below) can be solved to optimize the safety inventory given model specifics.

AGENDA

Inventory Management I: Economic Order Quantity

Inventory Management II: The Newsvendor Problem

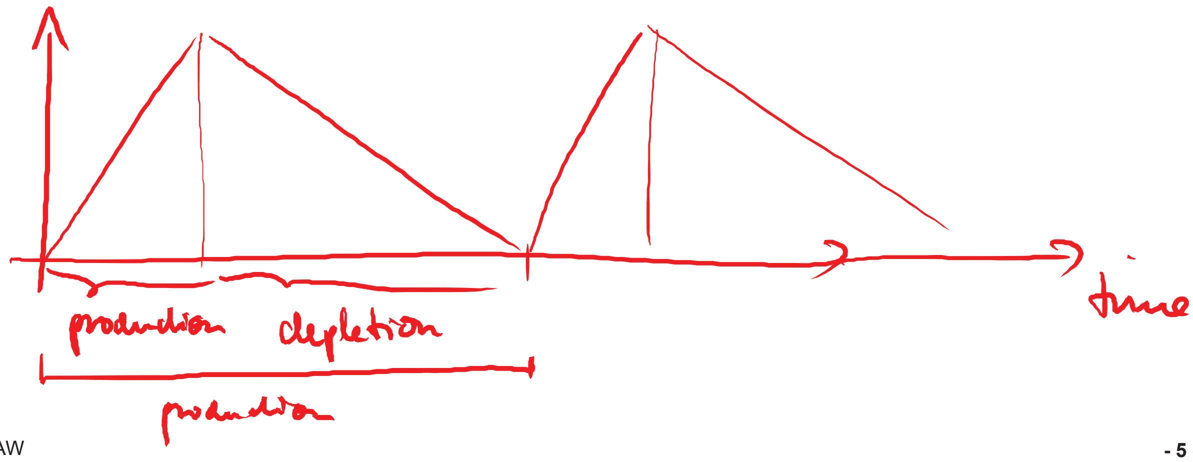
Key Concepts to Remember

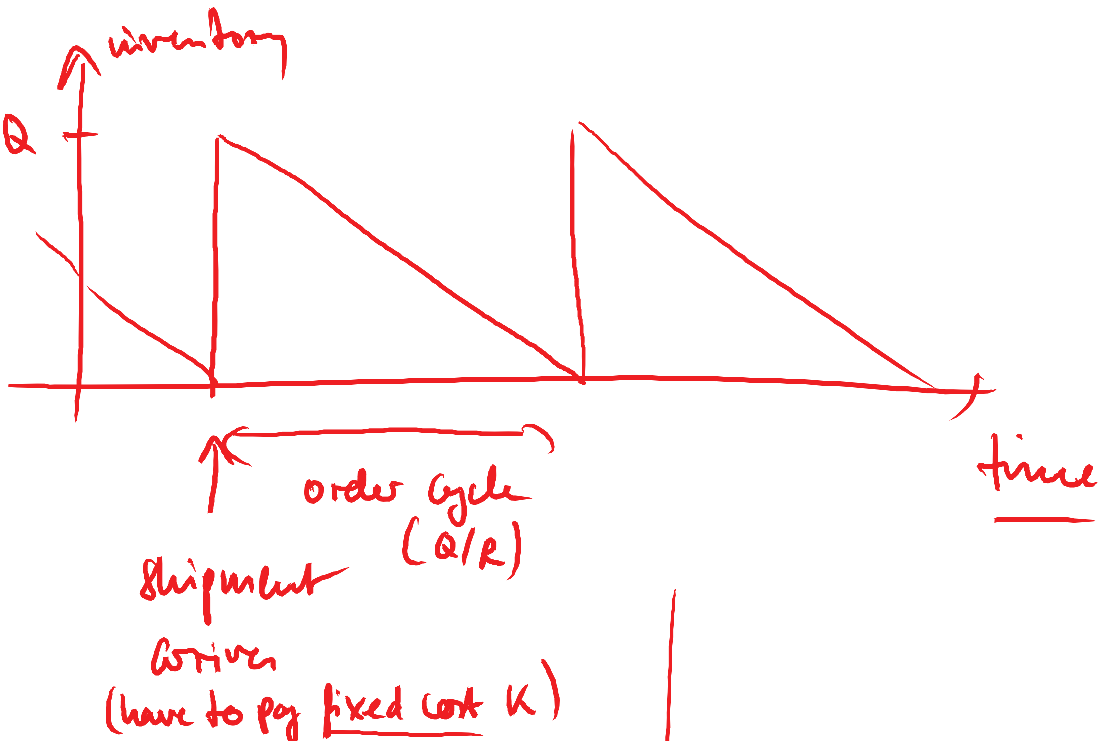
INVENTORY MANAGEMENT

EOQ (Economic Order Quantity)

X : process cycle time
(every X units of time one widget enters process)

in reality processes are not smooth





Question: Find optimal Q.

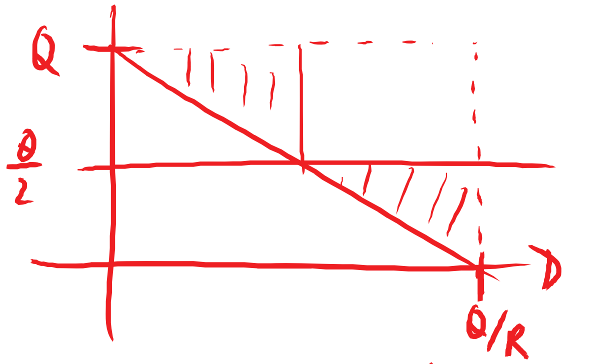
K : ordering cost [$]
h : inventory cost [$/widget/week]

$R = \frac{1}{X}$: flow rate [widgets/week]

Objective: minimize cost.

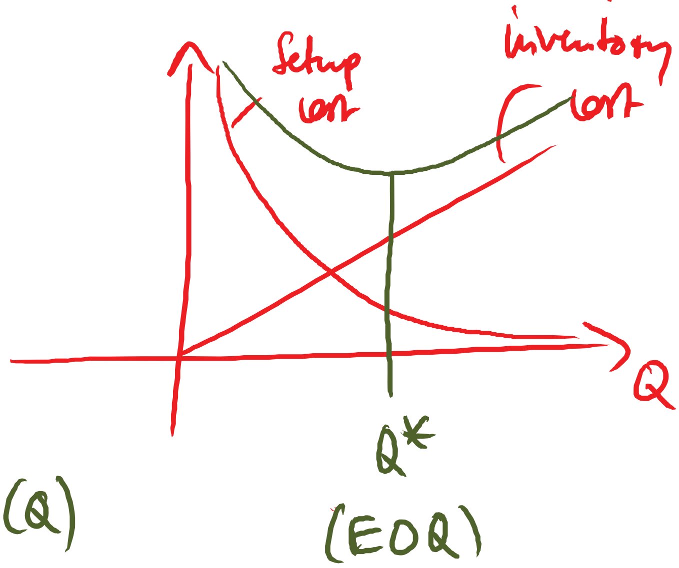
Average inventory: $\frac{Q}{2}$

$\Rightarrow$ Inventory cost (per week): $\frac{1}{2} Q \cdot h$



Setup cost per time (per week): $\frac{K}{Q/R}$

$\uparrow$ length of order cycle

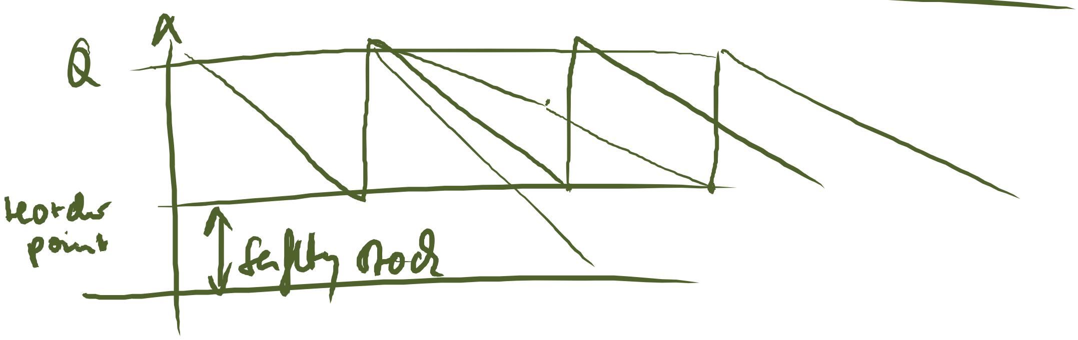


Total cost $= \frac{1}{2} Q h + \frac{KR}{Q} =: C(Q)$

$$C'(Q) = \frac{h}{2} - \frac{KR}{Q^2} \stackrel{!}{=} 0 \iff Q = Q^*,$$

$$\boxed{Q^* = \sqrt{\frac{2KR}{h}}} \quad \text{(EOQ)}$$

$$\boxed{C^* = C(Q^*) = \sqrt{2KRh}}$$



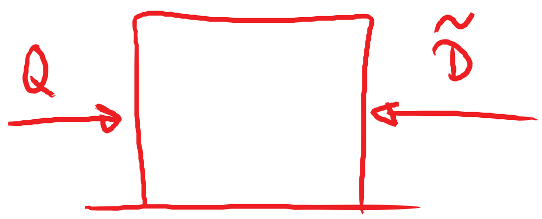
AGENDA

Inventory Management I: Economic Order Quantity

Inventory Management II: The Newsvendor Problem

Key Concepts to Remember

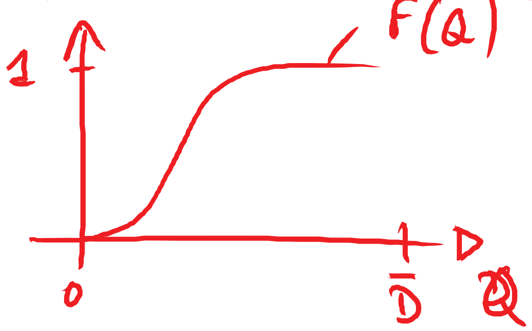
THE NEWSVENDOR PROBLEM

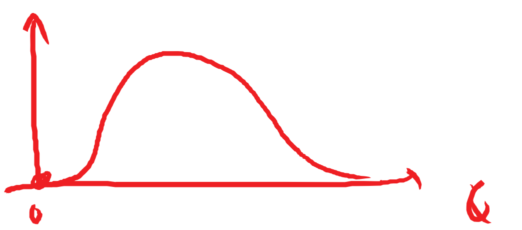


Newsstand

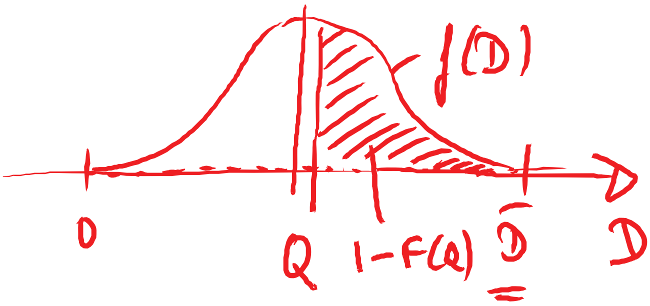
$\tilde{D}$: uncertain demand characterized by (no pricing)

$$\boxed{F(Q) = \text{Prob}(\tilde{D} \le Q)}$$





Cachon/Terwiesch: Matching Supply and Demand



$f = F'$, F

pdf, cdf.

- Spoilage (for perishable goods)
- inventory holding cost

$\left[Q > \tilde{D} \Rightarrow \right.$ overage cost (C_o)

$\tilde{D} \sim F : \underbrace{[0, \bar{D}]}_{\text{support}} \rightarrow [0,1]$

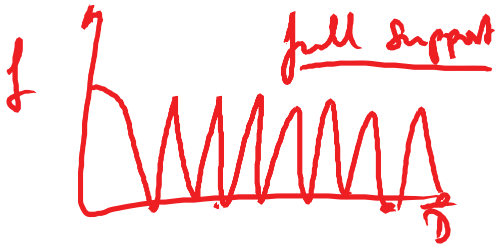
$Q < D \Rightarrow$ underage cost (C_u)] due to shortage (lost demand)

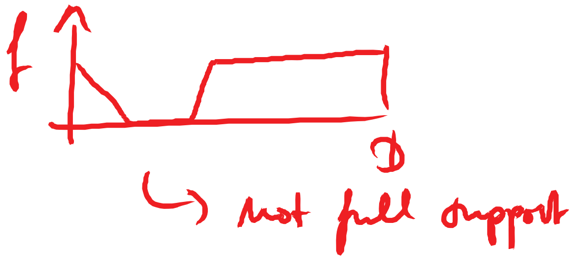
C_o high, $C_u = 0 \Rightarrow Q^* = 0$

$C_o = 0$, C_u high $\Rightarrow Q^* = \bar{D}$

Intuition

Expected overage cost = expected underage cost





$F(Q)\, C_o = (1 - F(Q))\, C_u$

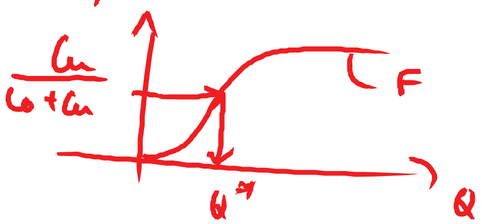
└ $\text{Prob}(\tilde{D} \le Q)$

$\Leftrightarrow \quad F(Q) = \dfrac{C_u}{C_o + C_u}$

$f^* > 0 \Rightarrow$ full support $\Rightarrow$ F invertible, i.e., F^{-1} $\exists$ ("exists")

$F' > 0$

$$\underline{\underline{Q^* = F^{-1}\left(\frac{C_u}{C_o + C_u}\right)}}$$



$$\underbrace{\bar{\pi}(Q)}_{\substack{\text{expected}\\ \text{profit}}} = \mathbb{E}\Big[\underbrace{p}_{\text{fixed}} \cdot \min\{Q, \underbrace{\tilde{D}}_{q}\}\Big] - \underbrace{cQ}_{\substack{\text{marginal}\\ \text{cost}\\ \text{(constant)}}}$$

marginal cost (constant)] contains everything variable (~~[illegible]~~)

$$= p\Big[\int_0^Q \underbrace{q\, \overbrace{dF(q)}^{F'(q)\,dq}}_{q f(q)\, dq} + \underbrace{\int_Q^{\bar{D}} Q\, dF(q)}_{Q \cdot \int_Q^{\bar{D}} f(q)\,dq \; - (1-F(Q))}\Big] - cQ$$

$$\bar{\pi}'(Q) = p\Big[\cancel{Qf(Q)} + 1 - F(Q) + Q(\cancel{-f(Q)})\Big] - c \overset{!}{=} 0$$

$$\Leftrightarrow \quad p(1-F(Q)) - c = 0 \quad \Leftrightarrow \quad \boxed{F(Q) = \frac{p-c}{p}}$$

$c_u = p - c$: margin (loss per ~~sold~~ demanded extra item, in terms of opportunity cost)

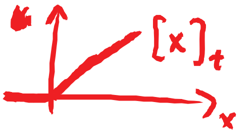
$c_o = c$: cost of unsold excess, in terms of marginal cost

$$\underbrace{(p-c)(1-F(Q))}_{\substack{\text{expected underage}\\ \text{cost}}} = \underbrace{cF(Q)}_{\substack{\text{expected overage}\\ \text{cost}}} \qquad \frac{p-c}{p} = \frac{p-c}{(p-c)+c}$$

$$\hat{\pi}(Q) = \mathbb{E}\Big[p \min\{Q, \tilde{D}\} - h[Q - \tilde{D}]_+\Big] - cQ$$

exp. profit w/ holding cost

$[x]_+ = \max\{x, 0\}$



$$\hat{\pi}' = \bar{\pi}' - h\Big[\int_0^Q (Q-q)\,dF(q)\Big]'$$

$$[\;]' = -\cancel{Qf(Q)} + F(Q) + \cancel{Qf(Q)} = F(Q)$$

$$p(1-F(Q)) - hF(Q) - c = 0$$

$$p - c - (p+h)F(Q) = 0$$

$$\boxed{F(Q) = \frac{p-c}{p+h}} \Rightarrow Q^* = F^{-1}\left(\frac{p-c}{p+h}\right)$$

$$\hat{C}_o = p+h-\hat{C}_u = \cancel{p}+h-(\cancel{p}-c)$$

$$\boxed{\begin{aligned} \hat{C}_u &= p-c \\ \hat{C}_o &= \underline{c+h} \end{aligned}}$$

Remark Leibniz Rule

$$D(x) = \frac{d}{dx}\int_0^{g(x)} f(\hat{x})\,d\hat{x} = ?$$

f, g differentiable

$$F(y) = \int_0^y f(\hat{x})\,d\hat{x}$$

$$D(x) = \frac{d}{dx}F(g(x)) = F'(g(x))\cdot g'(x)$$

$$D(x) = f(g(x))\cdot g'(x)$$

More generally:

$$\frac{d}{dx}\int_0^{g(x)} f(\hat{x},x)\,d\hat{x}$$

$$= f(g(x),x)\,g'(x) + \int_0^{g(x)} \frac{\partial}{\partial x} f(\hat{x},x)\,d\hat{x}$$

$$\boxed{\int_a^b = -\int_b^a}$$

Of course, you can now compute

$$\frac{d}{dx}\underbrace{\int_{h(x)}^{g(x)} f(\hat{x},x)\,d\hat{x}}_{\left\{\int_0^{g(x)} - \int_0^{h(x)}\right\} f(\hat{x},x)\,dx}$$

STUDENT PROJECT (2018): UCHITOMI

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

PART 01

Introduction

- Uchitomi Shop
- Uchitomi Restaurant

Overview

- Japanese specialty shop
- Founded by Mr. Uchitomi
- 3 shops: 2 in Geneva and 1 in Lausanne

Uchitomi Lausanne

Uchitomi Shop

Uchitomi Restaurant

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Key problems

Current problems?

How can we help?

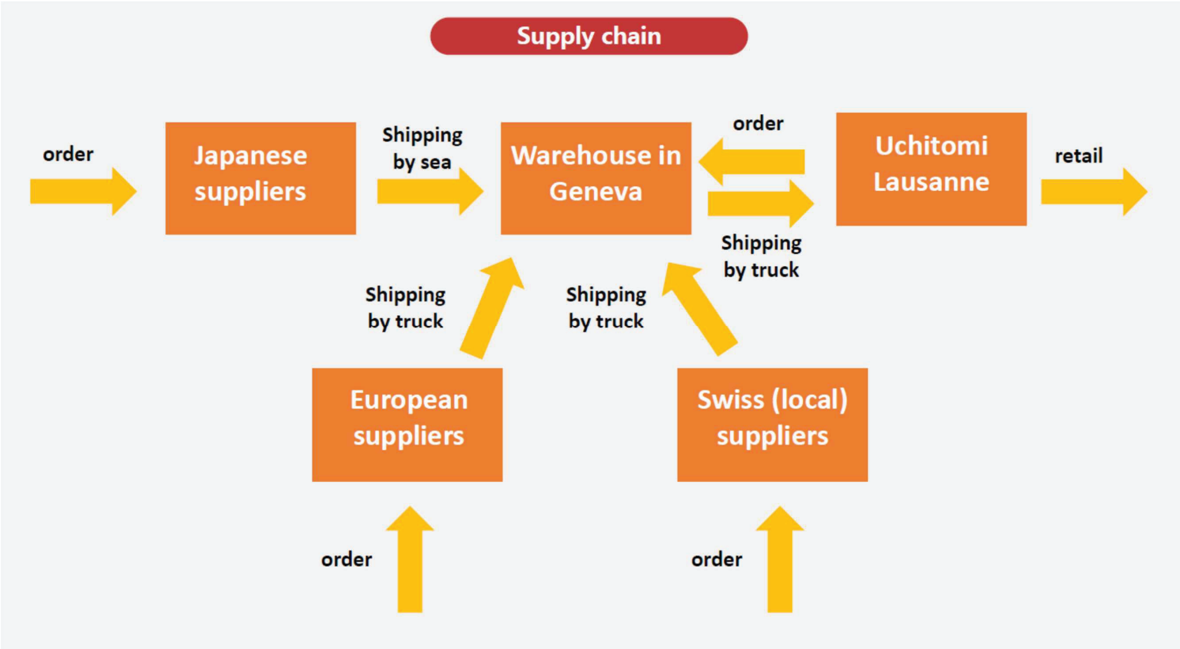
How to gain competitive advantages ?

- Problems of stock outs and under stocks
- Missing market segment of students
  - low popularity of student

- Optimize re-ordering quantity and minimize the associated costs ← Inventory Management
  - Analysis narrow down to the top 2 best selling products in each sector:
    - *Shop*: soy sauce & tofu—Economic Order Quantity (EOQ) Model
    - *Restaurant*: sushi and lunch box—Newsvendor Model
- Offer student discount ← Price discrimination
  - win-win situation

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)



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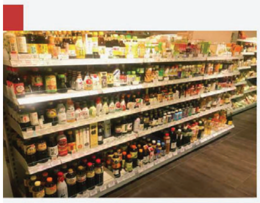


PART 02

Inventory Management

- Economic Order Quantity (EOQ) model
- Newsvendor model

EOQ model in Shop

$$Q = \sqrt{\frac{2kR}{h}}$$

$$C(Q) = \frac{Q}{2}h + \frac{kR}{Q}$$

Q: optimal ordering amount;
k: reordering cost;
R: flow rate in targeted period;
h: holding cost and C(Q) is the cost function.

Major Assumptions

- **Soy sauce and tofu have relatively constant demand**
  - High Costumer loyalty
- **Demand forecast is done by Simple Moving Average Method**
  - In order to to smooth out little variation
- **Implicit costs (depreciation) are not included**

Kikkoman soy sauce (500mL)

House Foods Premium Tofu (soft & medium firm)

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

EOQ model in Shop

	Reordering cost k	Holding cost h	Flow rate R	Current ordering quantity Q_0	Current cost C_0	Optimized ordering quantity Q*	Optimized costs C*
Soy sauce	CHF 131.81	12.5 CHF/ bot.	3.5 bot./ week	10 bot./ week	CHF 108.63	9 bot./ week	CHF 107.51
Tofu	CHF 131.81	11.58 CHF/ box	56 boxes/ week	24 boxes/ week	CHF 446.52	36 boxes/ week	CHF 413.48

Total savings = 136.64 CHF/ month

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Proposed Strategy

Reordering Frequency of Soy sauce:

Every week → **Every Month**

h decreases

→ More soy sauce are occupied in the same space, so the allocated rental price is averaged out.

	Holding cost h	Flow rate R	Optimized ordering quantity Q*	Optimized costs C*
Weekly order	12.5 CHF/ bot.	3.5 bot./ week	9 bot./ week	107.51 CHF/week
Monthly order	10.625 CHF/ bot.	14 bot./ month	19 boxes/ month	123.76 CHF/ month

Total savings = 306.28 CHF/ month

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Newsvendor model in restaurant

Theory

$$\Pi = E[p\ min\{Q, D\}] - cQ$$

$$Q^* = \mu + z^*\sigma$$

$$z^* = F^{-1}\left(\frac{C_u}{C_O + C_u}\right)$$

$$C_u = p - c;\ C_O = c$$

p: price, D: demand, c: marginal cost;
Q: optimal order quantity;*
μ, σ : mean and standard deviation of the demand under assumption of normal distribution;
Cu: underage cost, Co: overage cost.

Major Assumptions

- Focus on 2 major products
- Demands of sushi and lunch box have uncertainty
- Demand forecast is assumed as normal distribution based on the current sales average and standard deviation

Sushi

Lunch box

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Newsvendor model in restaurant

Current case

	Current quantity (/day)	Expected profit (CHF/day)	Fill rate
Sushi	142	38.95	86.8%
Lunchbox	24.3	59.53	95.2%

Results for 3 cases

	case	computed quantity	Expected profit (CHF/day)	Fill rate
Sushi	(a)optimal	96.34	109.5	65.0%
	(b)fill rate = 95%	172.41	-93.2	95.0%
	(c)in-stock prob. =95%	218.19	-418.3	98.4%
Lunchbox	(a)optimal	20.51	67.6	86.3%
	(b)fill rate = 95%	24.18	60.0	95.0%
	(c)in-stock prob. =95%	30.00	21.1	99.6%

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Proposed Strategy

Reducing ordering amount of <u>Sushi</u>

- **Effective ordering(preparing) amount of lunchbox**

  In theory, the fill rate is already 95 %

- **More profitable amount of sushi**

  It can produce more profits per day **38.95→109.5** by reducing ordering quantity **142→96.34**

  However fill-rate remains too small due to the large standard deviation

Put more appropriate assumption of demand distribution
- the demand might depend on the day of the week

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

PART 03

Revenue Enhancement

Price discrimination

Theory ~ Third-degree price discrimination

The optimal quantities and prices can be obtained by maximizing sum of the profits for each product:

$$max\,\{p_1(q_1)q_1 + p_2(q_2)q_2 - c(Q)\},\, Q = q_1 + q_2,\; q_1, q_2 > 0$$

q_i: quantity for each customer, i={1,2};
$p_i(q_i)$: demand curve.

Major Assumptions

- **Focus on only lunch box**
- **Demand curves for each consumer group are linear**
- **Each group can be legally charged different prices for same products**
- **Uchitomi is monopoly in Japanese lunch box market in Lausanne**

Lunch box

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Price discrimination

Current case

	General people	Student
Price	12.8	12.8
Quantity sold	22.83	1
Avg. number visited	120	20

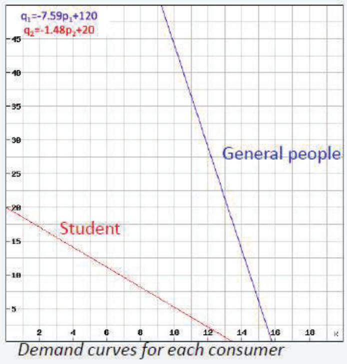
$\Pi = p_1(q_1)q_1 + p_2(q_2)q_2 - c(Q)$

Where $q_1 = -7.59p_1 + 120$

$q_2 = -1.48p_2 + 20$

$c(Q) = 9(q_1 + q_2)$

Under this model, current profit = **90.6** CHF/day



Demand curves for each consumer

Results

	General people	Student
Optimal price	12.40	11.24
Optimal quantity sold	25.84	3.32

9% discount

Π^*=**95.37** CHF/day : More sales can be achieved by price discrimination

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Proposed Strategy

Implementing the price discrimination between students and others
"Student discount"

Revenue enhancement can be expected

In theory, discount for students is effective due to the difference in elasticity

Problem to be solved: limited number of students visiting

Proposed strategy with information technology

- Advertisement using the coupon which gives incentive for current customers to introduce to their friends
  : Lack of knowledge
- Starting the delivery system
  : Too far from universities/ schools
- Organizing the app for ordering

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

PART 04

Discussions

- Results Summary & Limitations
- Next Step

Results summary and Limitations

Inventory Management

1. EOQ model in Uchitomi Shop

- Total cost savings of 136.64 CHF/month
- Further savings in costs of 306.28 CHF/month for soy sauce

- Oversimplified cost estimations
- Optimal quantity of soy sauce is not a multiple of 10 → additional packaging and checking costs

2. Newsvendor model in Uchitomi Restaurant

- profit increase by 70.25 CHF/day

- Limited sales data
- Demand forecast needs deeper considerations (could be more periodic)

Revenue Enhancement

Price discrimination in Uchitomi Restaurant

- Student discount
- Quantity increase by 6.33 /day
- Profit increase by 4.77 CHF/day .

- Limited costs / sales / consumer data
- Demand curve needs deeper considerations
- Results are highly dependent on costs

Source: Tatara, S., Wan, Z. (2018) "Uchitomi Lausanne: Inventory Management and Price Discrimination," MGT-528 Course Project, EPFL, Lausanne, Switzerland.

STUDENT PROJECT (2018): UCHITOMI (Cont'd)

Next Step

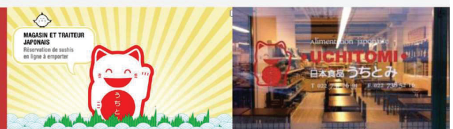
Make models closer to reality by using precise data of costs and sales.

Use more complicated model to forecast demand distribution for the EOQ and the newsvendor model. For example: take account into periodic or seasonal fluctuation of demand.

Deeper evaluation of costs and effects for advertisement / delivery system / app development in order to implement student discount.

THANK YOU

Any questions?



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AGENDA

Inventory Management I: Economic Order Quantity

Inventory Management II: The Newsvendor Problem

Key Concepts to Remember

KEY CONCEPTS TO REMEMBER

- Reasons for keeping inventory
- Different kinds of inventory
  - Cycle inventory
  - Pipeline inventory
  - Seasonal inventory
  - Decoupling inventory (buffer)
  - EOQ (Economic Order Quantity)
  - Safety inventory
- Standard newsvendor problem
- Leibniz' rule