École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-528 Operations: Economics and Strategy (Prof. Weber)

Final Exam – Solutions

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Problem 1 (35 Points + 7 Bonus Points)

(i) A competitive advantage is an asset (including know how) that allows a company to earn supra-normal profits (i.e., to earn more than another firm, all else equal). If not nurtured, a competitive advantage tends to disappear over time. The reason for this is that competitors are catching up by reengineering the asset underlying the competitive advantage, so as to eventually improve the capability. Hence, in order to make keep the distance to the competition and thus to sustain the current competitive advantage, the firm needs to make investments (e.g., in research and development of new products). Those investments may do nothing to actually increase the advantage, whence the analogy to the red-queen principle where despite "running" the red queen in Lewis Carroll's "Alice in Wonderland" does not move.

(ii) The standard product lifecycle explains how the diffusion (i.e., sales) of a product varies with its installed base (i.e., the number of users). The phases of the product lifecycle are 1. Introduction, 2. Growth, 3. Maturity, and 4. Decline. The best position in the (cost effectiveness, responsiveness)-space may vary with the phase. For example, in the introduction stage responsiveness might be more important than in the maturity stage. Depending on the competitive dynamics and the product lifecycle it can be more important to be cost effective (in order to maintain profit margin as monopolist) or else to be responsive (in order to maintain profit margin as an oligopolist).

(iii) The bullwhip effect describes the empirical regularity that uncertainties in the supply chain are amplified from downstream to upstream stages. Some potential causes are demand variability, lags in processing the demand signal, and order batching, price fluctuations (e.g., because of sales). Possible remedies include information technology (to provide transparency about data and decisions in the supply chain, and therefore reduce informational delays and enable the effective coordination of actions), effective contracting (to align incentives via revenue and/or cost sharing), and a transition to lean operations (to decrease the overall supply-chain inventory, e.g., by decreasing lead times, reducing the fixed ordering costs, smaller buffers in manufacturing via kanban).

(iv) Porter's five forces are: entrants, suppliers, buyers, substitutes, and market rivalry. These forces describe the competitive environment. A strong competition, as manifested, for instance, by a high number of entrants or many close substitutes or a strong rivalry (e.g., because of transparency) in the market, usually leads to more extreme strategic positioning, in the sense that either the company needs to improve its cost position or further differentiate its product and thus increase responsiveness. A larger number of buyers and/or lower number of suppliers increases the firm's market power and therefore allows it to increase its margin. The implications on the design of the supply chain are that it may therefore pay to increase the number of suppliers (with the side-benefit of resilience) and to increase the proximity to customers (so as to increase the customer base as for them the cost of getting to the firm's product decreases).

The key addition of the value-net framework is the complementor, which can have important implications for the value of the product (and its complement). For example, encouraging entry by companies that provide complementary products (e.g., apps for the iPhone) can significantly increase the value of the firm's product (e.g., the iPhone).

(v) Economies of scale are present if the unit cost decreases as a function of a firm's aggregate output, that is, if its average costs are decreasing. A company's average cost curve for a given product is often U-shaped because for small quantities all the fixed costs drive up unit costs, while for very large quantities unit costs are increasing because the complexity of the production process increases disproportionately.

(vi) Risk pooling refers to the mutual offsetting of the realizations of imperfectly correlated scalar random variables when the interest is on the sum of the realizations. For example, the standard deviation of the sum of N uncorrelated identically distributed random demands (say, in geographically dispersed regions) increases only sublinearly, proportional to the square root of N. As a result, a cost-effective risk-management strategy (e.g., to cut down on the safety stock in an inventory) can be aggregation. This can be done by centralizing certain warehouses, but also in more subtle ways, such as delayed differentiation when generic products are differentiated as late as possible in a given supply chain.

(vii) According to Cachon and Terwiesch (2009, Ch. 2) there are five different types of inventory: pipeline inventory, seasonal inventory, cycle inventory, decoupling inventory (buffers), and safety inventory. All of these were discussed in class. To be relevant for performance measurement the KPIs should measure the magnitude, flow, or degree of coupling (e.g., via forecast accuracy) of the different inventories in the overall operations. Tracking the KPIs over time provides the company with valuable feedback for performance improvement. Ideally, these KPIs will then also feed into a value map for the company as a whole and therefore have a defined relation to the overall objective of maximizing shareholder value.

(viii) The "value at risk" (VaR) for a real-valued random variable $\tilde{x}$ (with increasing distribution function $F : \mathbb{R} \to [0, 1]$) and a level $\alpha \in [0, 1]$ is defined implicitly as

$$\text{Prob}(\tilde{x} \leq \text{VaR}_\alpha) = F(\text{VaR}_\alpha) = \alpha.$$

For example, if $\tilde{x}$ is a company's profit in a given year, VaR_α for $\alpha = 5\%$ describes the company's profit best profit for the 5% of the worst possible profit outcomes. If VaR_α is negative and has a large magnitude, the company may want to seek the services of a reinsurance company to hedge against this downside risk.

Problem 2 (45 Points)

(i) The picture can be based on the lecture notes.

(ii) Since Firm 1 has to pay the constant price of p_2 upstream, this transfer price together with the processing cost c represents its (constant) marginal cost. On the other hand, its revenue function is $R_1(q) = (2000 - 2q)q$, so the optimal output (as a function of p_2) follows from the first-order necessary optimality condition

$$\mathrm{MR}_1(q) = p_2 + c,$$

or equivalently,

$$2000 - 4q = p_2 + c,$$

so the optimal order quantity is

$$q^*(p_2) = (2000 - p_2 - c)/4.$$

and firm 1's optimal price (as a function of p_2) becomes

$$p_1^*(p_2) = p(q^*(p_2)) = 2000 - 2q^*(p_2) = 1000 + (p_2 + c)/2.$$

(iii) Using the same logic as before, firm 2 faces the constant marginal cost of $p_3 + c$. Because its revenue is $R_2(q) = (2000 - 4q - c)q$, the first-order condition becomes

$$\mathrm{MR}_2(q) = 2000 - 8q - c = p_3 + c,$$

so the optimal order quantity is

$$q^*(p_2^*(p_3)) = (2000 - p_3 - 2c)/8,$$

and the optimal price

$$p_2^*(p_3) = 2000 - 4q^*(p_2^*(p_3)) - c = 1000 + (p_3/2).$$

(iv) Firm 3 sees the demand $q = (2000 - p_3 - 2c)/8$, or equivalently, the inverse demand $p_3(q) = 2000 - 8q - 2c$. Thus, its revenues are $R_3(q) = (2000 - 8q - 2c)q$, and the first-order condition for its profit-maximization problem yields

$$\mathrm{MR}_3(q) = 2000 - 16q - 2c = w + c,$$

so $q^* = (2000 - w - 3c)/16 = 1336/16 = 83.5$. As a result, firm 3's optimal price becomes

$$p_3^* = 2000 - (8)(83.5) - 400 = 932.$$

(v) It is $p_2^* = 1000 + (932/2) = 1466$ and $p_1^* = 1000 + (1666/2) = 1833$. Thus, $\Pi_1^* = (p_1^* - p_2^* - c)q^* = (167)(83.5) = 13{,}944.5$, $\Pi_2^* = (p_2^* - p_3^* - c)q^* = (334)(83.5) = 27{,}889$, and $\Pi_3^* = (p_3^* - w - c)q^* = (668)(83.5) = 55{,}778$. The total profit is

$$\Pi^* = \Pi_1^* + \Pi_2^* + \Pi_3^* = 97{,}611.5.$$

The profit comparison is explained in the answer under point (vi) below.

(vi) The sketch of the demand and marginal revenue curves is in the same spirit as in the problem set. As we go further upstream the demand curves become flatter and flatter, and therefore demand becomes more and more elastic (that is, the variation of quantity given a unit price increase goes up). Upstream profits are higher than downstream profits because the increase in price elasticity does not offset the increase in price that upstream firms are able to implement.

(vii) The profit Π^{**} that one could obtain if the firms were owned by the same entity with marginal cost $w + 3c$ and (inverse) demand $p(q)$ can be obtained by solving the associated simple monopoly problem, maximizing $\Pi(q) = (p(q) - w - 3c)q = (2000 - 2q - 64 - 600)q$, so $q^{**} = 334$ and $\Pi^{**} = 223{,}112 > 97{,}611.5 = \Pi^*$. Thus, the profit of the coordinated supply chain is by a factor of about 2.28 larger than the profit of the supply chain with triple marginalization. Triple marginalization occurs, because each supply-chain stage tends to withhold quantity to the next stage downstream in order to exercise its market power.

(viii) It was shown in the lecture that the 2-stage supply chain can be coordinated by writing a contract that shares revenues (at any nontrivial proportion) and sets the transfer price between firms to a weighted sum of their marginal costs. This remains true for 3 firms. Take the most upstream firm (firm 3), which has marginal cost $w + c$, and its downstream partner (firm 2), with marginal cost c. A revenue-sharing contract which secures a fraction of $(1 - \phi_{23})$ of firm 2's revenue to firm 3 coordinates firm 2 and 3 at the (per-unit) transfer price of $t_{23} = \phi_{23}(w + 2c) - c$.

Then firm 1 and the combined entity of firms 2 and 3 can be coordinated with firm 1 agreeing to transfer the share $(1 - \phi_{12})$ of its revenue to firm 2 (whose revenue is already shared with firm 3) and provided the transfer price between firms 2 and 1 is set to $t_{12} = \phi_{12}(w+3c) - c$. The procedure can be continued and thus coordinate an N-stage supply chain through "cascaded revenue sharing." For the case of three firms, the coordinated profits are equal to

$$\begin{aligned}
\hat{\Pi}_1 &= \phi_{12}\left(R_1(q^{**}) - (w + 3c)q^{**}\right) = \phi_{12}\Pi^{**}, \\
\hat{\Pi}_2 &= \phi_{23}(1 - \phi_{12})\left(R_1(q^{**}) - (w + 3c)q^{**}\right) = \phi_{23}(1 - \phi_{12})\Pi^{**}, \\
\hat{\Pi}_3 &= (1 - \phi_{23})(1 - \phi_{12})\left(R_1(q^{**}) - (w + 3c)q^{**}\right) = (1 - \phi_{23})(1 - \phi_{12})\Pi^{**}.
\end{aligned}$$

(ix) If we consider a coordinated supply-chain as it was discussed in part (viii), the profit that can be gained in the supply chain is maximized and is equal to the monopoly profit computed in part (vii). Hence, the supply chain becomes unprofitable if $p(q^*) = 2000 - 2q^* = w + 3\bar{c}$. Solving for $\bar{c}$, we get:

$$2000 - 2\frac{2000 - w - 3\bar{c}}{4} = w + 3\bar{c} \Rightarrow \bar{c} = 645.33.$$

Remark. For the firms to be willing to participate in coordination through revenue sharing, each of them needs to gain at least as much as without coordination, i.e., $\hat{\Pi}_i \geq \Pi_i^*$, for all $i \in \{1, 2, 3\}$. Thus, given the results in parts (v), (vii), and (viii), this imposes the following three inequality conditions on the coefficients ϕ_{12} and ϕ_{23} in $[0, 1]$:

$$\begin{aligned}
\phi_{12} &\geq \Pi_1^*/\Pi^{**} = (13{,}944.5)/(223{,}112) = 1/16, \\
\phi_{23}(1 - \phi_{12}) &\geq \Pi_2^*/\Pi^{**} = (27{,}889)/(223{,}112) = 1/8, \\
(1 - \phi_{23})(1 - \phi_{12}) &\geq \Pi_3^*/\Pi^{**} = (55{,}778)/(223{,}112) = 1/4.
\end{aligned}$$

Combining the three inequalities we find that

$$(\phi_{12}, \phi_{23}) \in \left\{ (\varphi_{12}, \varphi_{23}) \in \left[\frac{1}{16}, \frac{5}{8}\right] \times \left[\frac{1}{30}, \frac{11}{15}\right] : \varphi_{23} \in \left[\frac{(1/8)}{1-\varphi_{12}}, 1 - \frac{(1/4)}{1-\varphi_{12}}\right]\right\}.$$

For example, $\phi_{12} = \phi_{23} = 1/2$ works, with firm 3 experiencing no improvement over the case without coordination, while firm 2 and firm 1 enjoy a doubling and quadrupling of their uncoordinated profits, respectively.

Problem 3 (35 Points)

(i) At price p a type-θ consumer buys a cup of coffee of size q if and only if $2\theta\sqrt{q} - p \geq 0$. Since $\theta_L < \theta_H$, Frédéric's optimal price (as a function of q) is therefore either $2\theta_L\sqrt{q}$ or $2\theta_H\sqrt{q}$. Hence, the optimal quantity $\hat{q}$ is such that

$$\hat{q} = 1 \in \arg\max_{q \geq 0} \left\{ \max\left\{2\theta_L\sqrt{q} - cq, \mu\left(2\theta_H\sqrt{q} - cq\right)\right\}\right\} = \begin{cases} \{(\theta_L/c)^2\}, & \text{if } \theta_L \geq \sqrt{\mu}\theta_H, \\ \{(\theta_H/c)^2\}, & \text{otherwise.} \end{cases}$$

In sum, the optimal price-quantity tuple for the single-product monopolist is

$$(\hat{p}, \hat{q}) = \begin{cases} (2\theta_L^2/c, (\theta_L/c)^2), & \text{if } \theta_L \geq \sqrt{\mu}\theta_H, \\ (2\theta_H^2/c, (\theta_H/c)^2), & \text{otherwise.} \end{cases}$$

That is, $(\hat{p}, \hat{q}) = (2, 1)$.

(ii) First-degree price discrimination is achieved when Frédéric is able to extract all surplus from each consumer type, given that he is able to separate them. In that case he would charge $p(\theta) = u(q, \theta)$ to type $\theta \in \{\theta_L, \theta_H\}$, and the optimal quantity $\bar{q}$ becomes

$$\bar{q} = 0.49 \in \arg\max_{q \geq 0} \left\{2\left[(1-\mu)\theta_L + \mu\theta_H\right]\sqrt{q} - cq\right\} = \left\{\left(\frac{(1-\mu)\theta_L + \mu\theta_H}{c}\right)^2\right\}.$$

Hence, if $\bar{\theta} = (1-\mu)\theta_L + \mu\theta_H = 0.7$ denotes the expected type, then $p(\theta) = 2\theta\bar{\theta}/c = 1.4\,\theta$ for all $\theta \in \{\theta_L, \theta_H\}$, and $\bar{q} = (\bar{\theta}/c)^2 = 0.49$.

(iii) Given a menu $M = \{(p_L, q_L), (p_H, q_H)\}$, the respective consumer types participate if and only if the following *individual-rationality constraints* are satisfied:

$$u(q_L, \theta_L) - p_L \geq 0, \tag{IR-L}$$

for type θ_L, and

$$u(q_H, \theta_H) - p_H \geq 0, \tag{IR-H}$$

for type θ_H. Each type prefers its own product over the other type's product if the following *incentive-compatibility constraints* are satisfied:

$$u(q_L, \theta_L) - p_L \geq u(q_H, \theta_L) - p_H, \tag{IC-L}$$

for type θ_L, and

$$u(q_H, \theta_H) - p_H \geq u(q_L, \theta_H) - p_L, \tag{IC-L}$$

for type θ_H.

(iv) As in the lecture notes, it is straightforward to show (not required) that the constraints (IR-L) and (IC-H) are binding, provided that both types are participating.[1] The constraints (IR-H) and (IC-L) are implied by the other two constraints (as well as the properties of the given utility representation).

(v) The binding constraints (IR-L) and (IC-H) can be used to compute the prices as a function of the quantities, i.e.,

$$p_L = u(q_L, \theta_L) = 2\theta_L \sqrt{q_L},$$

and

$$p_H = u(q_H, \theta_H) - (u(q_L, \theta_H) - u(q_L, \theta_L)) = 2\left[\theta_H \sqrt{q_H} - (\theta_H - \theta_L)\sqrt{q_L}\right].$$

The firm's expected-profit maximization problem is therefore

$$\max_{q_L, q_H} \left\{(1-\mu)\left(u(q_L, \theta_L) - cq_L\right) + \mu\left(u(q_H, \theta_H) - (u(q_L, \theta_H) - u(q_L, \theta_L)) - cq_H\right)\right\},$$

so that

$$q_H^* = 1 \in \arg\max_{q_H \geq 0} \left\{2\theta_H \sqrt{q_H} - cq_H\right\} = \left\{\left(\frac{\theta_H}{c}\right)^2\right\},$$

and

$$q_L^* = \frac{1}{36} \in \arg\max_{q_L \geq 0} \left\{2\theta_L \sqrt{q_L} - \frac{2\mu}{1-\mu}(\theta_H - \theta_L)\sqrt{q_L} - cq_L\right\} = \left\{\left(\frac{\left[\theta_L - \frac{\mu}{1-\mu}(\theta_H - \theta_L)\right]_+}{c}\right)^2\right\}.$$

Using the earlier relations for the prices, we conclude that

$$p_L^* = \frac{2\theta_L}{c}\left[\theta_L - \frac{\mu}{1-\mu}(\theta_H - \theta_L)\right]_+ = \frac{1}{6},$$

and

$$p_H^* = \frac{2}{c}\left(\theta_H^2 - (\theta_H - \theta_L)\left[\theta_L - \frac{\mu}{1-\mu}(\theta_H - \theta_L)\right]_+\right) = \frac{11}{6}.$$

(vi) The first-best two-product menu $M^{\text{FB}} = \{(p_L^{\text{FB}}, q_L^{\text{FB}}), (p_H^{\text{FB}}, q_H^{\text{FB}})\}$ is such that

$$q_L^{\text{FB}} = 0.25 \in \arg\max_{q_L \geq 0} \left\{2\theta_L \sqrt{q_L} - cq_L\right\} = \left\{\left(\frac{\theta_L}{c}\right)^2\right\},$$

$$q_H^{\text{FB}} = 1 \in \arg\max_{q_H \geq 0} \left\{2\theta_H \sqrt{q_H} - cq_H\right\} = \left\{\left(\frac{\theta_H}{c}\right)^2\right\},$$

and

$$\begin{aligned} p_L^{\text{FB}} &= u(q_L^{\text{FB}}, \theta_L) = 2\theta_L^2/c = 0.5, \\ p_H^{\text{FB}} &= u(q_H^{\text{FB}}, \theta_H) = 2\theta_H^2/c = 2. \end{aligned}$$

[1]For high values of μ it might be in the firm's interest to "shut down" the low-volume types and sell only to the high-volume types, extracting their entire surplus.

Frédéric's expected profit when using the first-best menu M^{FB} is

$$\Pi^{\text{FB}} = \frac{100}{c}\left[(1-\mu)\theta_L^2 + \mu\theta_H^2\right] = 55.$$

On the other hand, his expected profit under the second-best menu M^* is

$$\Pi^* = \begin{cases} 100\mu\theta_H^2/c, & \text{if } \mu \geq \mu_0, \\ 100\left(\theta_L^2 + \mu\theta_H^2 - 2\mu\theta_L\theta_H\right)/((1-\mu)c), & \text{otherwise,} \end{cases}$$

where $\mu_0 = \theta_L/\theta_H = 0.5 \in (0,1)$. That is, $\Pi^* = 500/12 \approx 41.67$. The high-volume type is charged less than his willingness to pay. The difference $u(q_H^*, \theta_H) - p_H^* = 2 - (11/6) = 1/6$ is referred to as the "information rent."

(vii) [Bonus] Assume now that each cup costs $k > 0$. If the firm uses the second-best menu M^* as specified in part (v) its additional expected cost is μk for $\mu \geq \mu_0$ and k for $\mu \leq \mu_0$. But this means that the firm's profit (as a function of μ) changes discontinuously at $\mu = \mu_0$. Hence, using the expression for Π^* in part (vi) we find that μ_0 should be replaced by $\hat{\mu}_0 < \mu_0$ such that

$$\frac{\hat{\mu}_0\theta_H^2}{c} - \hat{\mu}_0 k = \frac{\theta_L^2 + \hat{\mu}_0\theta_H^2 - 2\hat{\mu}_0\theta_L\theta_H}{(1-\hat{\mu}_0)c} - k,$$

so that

$$\hat{\mu}_0 = \frac{\theta_H\theta_L - kc + \sqrt{kc}\,(\theta_H - \theta_L)}{\theta_H^2 - kc} = \frac{\theta_L + \sqrt{kc}}{\theta_H + \sqrt{kc}} \in \left(\frac{\theta_L}{\theta_H}, 1\right).$$

For $k = 0.1$ we obtain that $\hat{\mu}_0 = \left(8 - \sqrt{10}\right)/18 \approx 0.2688 < \mu$, so that it is best for Frédéric to serve only his high-volume clientele offering only the large size, $q_H = 1$, at the price $p_H = 2$, for an expected profit of $40 - 4 = 36$ (instead of about $41.67 - 10 = 31.67$ when serving both types as in parts (v) and (vi)).

Problem 4 (25 Points)

(i) The cumulative distribution function (cdf) for the uniform demand distribution is

$$F(x) = P(\text{demand} \leq x) = \frac{x - 1000}{5000 - 1000} = \frac{x}{4000} - \frac{1}{4}, \quad x \in [1000, 5000].$$

Hence, the probability that TGO sells less than 2,700 units of the smart watch is $1700/4000 = 42.5\%$.

(ii) The unit overage cost c_o is the difference between the wholesale price of \$200 and the salvage value of \$100. The unit underage cost c_u is the profit margin, i.e., the difference between the selling price of \$500 and the wholesale price of \$200. The order quantity Q should be chosen so as to equate overage and underage cost in expectation;[2] hence,

$$P(\text{demand} \leq Q)\, c_o = P(\text{demand} > Q)\, c_u.$$

[2] It is also possible to explicitly write down the firm's expected profit as a function of Q and then set the derivative of this function to zero, which leads to the same result (see the course notes for details).

In other words, using the above definition of the cdf and the fact that $P(\text{demand} > Q) = 1 - F(Q)$,

$$F(Q) = \frac{Q}{4000} - \frac{1}{4} = \frac{c_u}{c_o + c_u} = \frac{3}{4}.$$

Hence, the optimal order quantity becomes

$$Q^* = 4000 \text{ units.}$$

(iii) Let the fill rate $\varphi = 95\%$. The corresponding order quantity Q_φ is then such that

$$\begin{aligned}
\varphi &= \frac{1}{3000} \int_{1000}^{5000} \min\{x, Q_\varphi\} dF(x) \\
&= \frac{1}{3000} \left(\int_{1000}^{Q_\varphi} x dF(x) + (1 - F(Q_\varphi)) Q_\varphi \right) \\
&= \frac{1}{3000} \left(\left[\frac{x^2}{8000} \right]_{1000}^{Q_\varphi} + \left(\frac{5}{4} - \frac{Q_\varphi}{4000} \right) Q_\varphi \right) \\
&= \frac{1}{3000} \left[\left(\frac{5}{4} \right) Q_\varphi - 125 - \frac{Q_\varphi^2}{8000} \right],
\end{aligned}$$

or equivalently (noting that $Q_\varphi \in [1000, 5000]$ and $\varphi = 95\%$),

$$Q_\varphi \approx 3{,}905 \text{ units.}$$

(iv) To ensure an in-stock probability $p = 95\%$, TGO needs to order a quantity Q_p such that

$$p = P(\text{demand} \leq Q_p) = \frac{Q_p}{4000} - \frac{1}{4},$$

so

$$Q_p = 4000\,(0.25 + p) = 4{,}800 \text{ units.}$$

(v) Note first that when $p = \varphi$ we always have that Q_p exceeds Q_φ (at least weakly). The fill rate φ is a proxy for average customer service, whereas the in-stock probability is more conservative, measuring the likelihood that *some* customers do not get served. So it is possible that the fill rate is high while the in-stock probability p is low in the sense that an eventual stockout is likely. In our case, we see that in order for TGO to obtain a 95% in-stock probability it needs to order about 22.91% more units than when contenting itself with a 95% fill rate. At that fill rate its in-stock probability is about 72.62%, which means that by the end of the season with probability of about 27.38% *some* consumers will have unsuccessfully looked for the smart watch in a TGO store.