École Polytechnique Fédérale de Lausanne

College of Management of Technology

MGT-528 Operations: Economics and Strategy (Prof. Weber)

Final Exam

Autumn 2020

Monday, January 25, 2021 (16h15 CET – 19h15 CET)

This is a *take-home exam*. You will have **180 minutes** to complete it. Use the point totals listed below to allocate your time appropriately. Please write all answers and supporting work **by hand** on A4 sheets of paper, which you *scan and submit electronically* (as per email instructions; **deadline**: **20h15 CET**).

The answer pages need to be numbered and they each need to carry your name. Please also scan the cover page of the exam with your signature. [*If you cannot print out the exam cover sheet (e.g., because you do not have a printer), then you will need to write by hand the following sentence on page 1 of your exam answer: "I hereby certify that this exam is based solely on my own efforts, and is otherwise also in accordance with the EPFL Honor Code as put forth in the course syllabus." <Your Signature>*]

The distribution of points for the four questions in this exam is as follows:

- Question 1: 30 points
- Question 2: 30 points
- Question 3: 30 points
- Question 4: 30 points (+10 bonus)

Total: 120 points (+10 bonus)

Good luck!

I hereby certify that this exam is based solely on my own efforts, and is otherwise also in accordance with the *EPFL Honor Code* as put forth in the course syllabus.

Name: ....

Signature: ....

Problem 1 (30 Points)

(i) According to Wikipedia (as of January 25, 2021),

> "[t]he Fourth Industrial Revolution (or Industry 4.0) is the ongoing automation of traditional manufacturing and industrial practices, using modern smart technology."

Define the concept of a "push-pull frontier" in a supply chain. All else equal, what will be the impact of Industry 4.0 on this push-pull frontier? Explain.

(ii) Let $Q > 0$ be a firm's output and $C(Q) > 0$ its corresponding production cost. Assuming the firm knows its inverse demand $p(Q) > 0$, write the firm's profit as a function of its average cost. For the firm to be viable (i.e., to make nonnegative profits), how does its output need to be chosen? Compare this to the necessary optimality condition for maximizing the firm's profit. Does one condition imply the other? If "yes," explain. If "no," how do the conditions complement each other?

(iii) Is a risk-neutral firm indifferent to a pure increase in risk? If "yes," explain. [Hint: To answer the preceding question, think of just one parameter θ in the firm's objective function; instead of having the deterministic value $\theta = 1$, the now random parameter $\tilde{\theta}$ takes on the values 0.5 or 1.5 with equal probabilities.] If you answered "no," what property of the firm's objective function (with respect to θ) would make the firm strictly prefer *any* random distribution of $\tilde{\theta}$ on the interval $[0.5, 1.5]$ (with expected value $E[\tilde{\theta}] = 1$) over a deterministic situation where $\theta = 1$ for sure?

(iv) Which significant force is missing in Porter's "five forces" diagram that can be captured quite well in Brandenburger and Nalebuff's "value net"? Using a practical example, describe how that force (i.e., by having more of it) might lead to an *increase* in the retail price of a firm's product.

(v) When is having more information worse for a firm? Provide an example that does *not* include an information-processing argument (e.g., it is more difficult, or even impossible, to read 10,000 different newspapers than reading just 1 newspaper). [Hint: You need to outline a situation where a perfectly rational company would strictly prefer to have less information.]

(vi) Explain the statement "social networks might enable firms to substitute third-degree price discrimination for second-degree price discrimination." Operationally, what is the key difference between the two modes of price discrimination, and why might firms be in favor of this substitution (so that they may be willing to pay the social networks for helping them with the transition)?

Problem 2 (30 Points)

Consider a fine dining restaurant with a single supplier who does not always provide high-quality ingredients. Any given order of Q units of the ingredients arrives *either* all in low quality $q_L > 0$ (with probability $p \in [0, 1)$) *or* all in high quality $q_H > q_L$ (with probability $1 - p$). The restaurant can use only high-quality ingredients; low-quality ingredients need to be discarded. If the delivery is spoilt (i.e., of low quality), the monopolist needs to place a new order.[1] Luckily the lead time is zero, so no time is lost, only money. The ordering cost is $K = 10$ (the monetary unit is CHF, omitted for convenience). The wholesale price, per unit of the ingredient, is $w = 1$. Each "cover" (customer order) takes 1 unit of the ingredients to prepare. The retail price charged for each cover is $r = 60$. The restaurant's holding cost is $h = 0.1$ per day (*after* the first day) and per unit of the ingredient.

Part I. *Deterministic Demand / Nonperishable Goods.* Assume that the restaurant serves $D = 50$ covers every day and that leftover ingredients can be used forever.

(i) Determine the expected total cost $\text{TC}(Q, p)$ (per day) for Q units of the ingredient that arrives in the high-quality state at the restaurant. [Hint: Determine first the cost $\text{TC}(Q, 0)$ when the supplier's failure probability vanishes. This is a flow cost, evaluated per day.]

(ii) Compute the restaurant's optimal ordering quantity $Q^*(p)$, and the resulting expected total cost per order, $\text{TC}^*(p) = \text{TC}(Q^*(p), p)$. Note that orders (including repeat orders) are placed (and delivered) in the morning of each day. Covers are served only in the evening, and new orders cannot be placed during service, as preparation of the meals takes several hours and the ingredients are needed from the beginning of the preparation. [Hint: You will need to round the results appropriately to ensure that there are enough ingredients for any given day.]

(iii) Compute the restaurant's daily profit $\Pi(p)$ as a function of p, given that the manager applies the optimal ordering policy determined in part (ii). Up to what failure probability $\bar{p}$ of the supplier can the restaurant survive (i.e., have nonnegative profit) given its current pricing? [Hint: For simplicity, assume that all the other costs are fixed and covered by the owner's hotel business.]

(iv) Compute the "inventory turnover" (i.e., sales (demand) divided by average inventory).

Part II. *Deterministic Demand / Perishable Goods.* Using otherwise the same assumptions as in Part I, the restaurant manager now realizes that leftover ingredients turn to low quality the next day and must therefore be discarded.

(v) Determine the restaurant's optimal ordering quantity $\hat{Q}^*(p)$, the expected total cost per order (per day) $\widehat{\text{TC}}^*(p)$, and the restaurant's daily profit $\hat{\Pi}^*(p)$. Describe how your results depend on p.

Part III. *Stochastic Demand / Perishable Goods.* In addition to the knowledge incorporated up to Part II, the restaurant manager (after having analyzed the point-of-sales data) now realizes that the number of covers actually varies randomly between 40 and 60, so that $\tilde{D}$ is in fact a random variable, uniformly distributed on the interval $[40, 60]$.

(vi) Find the quantity $Q^{**}(p)$ which maximizes the restaurant's expected profit, and determine the optimal expected daily profit $\bar{\Pi}^{**}(p)$. Explain any systematic differences to the results in Part II.

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[1] There is no serial correlation of quality realizations across different orders.

Problem 3 (30 Points)

A monopolist producing a uniform good faces a demand composed of heterogeneous consumers. A consumer of type $\theta \in \{\theta_L, \theta_H\}$ (where $0 < \theta_L < \theta_H$) has a net utility of

$$u(q, \theta) - t(q),$$

when buying $q \in [0, 1]$ units of the good, where $t(q)$ is the total transfer paid to the monopolist and

$$u(q, \theta) = \theta \left(1 - (1 - q)^2\right) / 2.$$

The net utility of any consumer that decides to buy nothing is zero. There are μ type-θ_H consumers and $1 - \mu$ type-θ_L consumers, where μ is given constant with $0 < \mu < 1$.[2] The (constant) marginal cost for producing the good is $c \in (0, \theta_L)$.

(i) Assume initially that the monopolist is restricted to charge a fully linear price schedule of the form $t(q) = p \cdot q$, where $p > 0$ is a constant per-unit price. Determine the demand $D_L(p)$ and $D_H(p)$ of the two consumer groups and show that the aggregate demand $D(p)$ is of the form

$$D(p) = 1 - (p/\bar{\theta}),$$

where $\bar{\theta}$ is an appropriate 'average' type. [Hint: $\bar{\theta}$ depends on θ_L, θ_H, and μ.] Sketch the demand curves $D_L(p)$, $D_H(p)$, and $D(p)$ in (q, p)-space. What is the profit-maximizing monopoly price p^m and what is the resulting monopoly profit $\Pi^m(\mu)$?

(ii) Consider first the benchmark case in which the monopolist is able to distinguish the two consumer types and charge a different price to each consumer. Determine profit-maximizing price schedules $\hat{t}_L(q)$ (charged to type-θ_L consumers) and $\hat{t}_H(q)$ (charged to type-θ_H consumers).[3] Determine the quantities $\hat{q}_L$ and $\hat{q}_H$ consumed by the two consumer types respectively, and find the transfers $\hat{t}_L(\hat{q}_L)$ and $\hat{t}_H(\hat{q}_H)$ paid to the monopolist.

(iii) Assume now that the monopolist cannot distinguish the two consumer types, and that he is restricted to charging a two-part tariff, i.e., a price schedule of the form $t(q) = pq + f$, where p and f are appropriate constants. Determine the profit-maximizing p^* and f^* as a function of μ. What is the resulting profit $\Pi^*(\mu)$?

(iv) Compare the total transfers and quantities paid by each consumer type under an optimal two-part tariff (in part (iii)) with those under perfect price discrimination (in part (ii)). For each term, explain the similarities and differences.

(v) Can the monopolist do better in part (iii) when using a more general pricing schedule $t(q)$ that may not be affine? Explain. What do you think happens when the number of different consumer types increases?

[2] You can think of μ as either a probability that any given consumer is of type θ_H or as the number of type-θ_H consumers in a sufficiently large population (e.g., $\mu = 0.5$ could correspond to 0.5 million type-θ_H consumers). We chose the latter interpretation here, so that we can talk about the firm's profits instead of its *expected* profits in this problem.

[3] Without any loss in generality you can assume in this part that the price schedules are affine in q.

Problem 4 (30+10 Points) Consider two firms in a Cournot duopoly. If firm $i \in \mathcal{N} = \{1, 2\}$ chooses a production quantity $q_i \geq 0$, its cost is $C(q_i) = cq_i$, where $c \geq 0$ is a given constant. Provided that at least one firm has a positive output, the inverse market demand is

$$p(Q) = \frac{100}{Q^{3/2}},$$

where $Q = q_1 + q_2 > 0$ is the firms' aggregate production quantity.

(i) Let $i \in \mathcal{N}$. Characterize firm i's best response $\mathrm{BR}_i(q_{-i})$ to the other firm's action q_{-i}. [Hint: there is no need to compute the best response explicitly.]

(ii) Find a symmetric Nash equilibrium of the simultaneous-move quantity-setting game, and compute the firms' equilibrium profits as a function of $c \geq 0$.

(iii) Compare the equilibrium profits you obtained in part (i) for $c = 10$ and $c = 20$. Explain your finding by examining production quantities and profits in equilibrium for all $c \geq 0$. Provide a real-world example for such an industry and discuss the value of technological progress in this environment.

(iv) Which cost $\check{c}$ would minimize the industry's aggregate profit? Explain.

(v) [Bonus] (10 points) Assume that if one firm gets to choose its production quantity before the other firm and could obtain a first-mover advantage of $\Delta = \Pi_{\text{first}} - \Pi_{\text{second}}$. If the two firms could spend money on being first, determine how much each firm would be willing to spend, and try to find the firms' spending strategies in equilibrium under the assumption that if they spend the same amount, then either firm comes first with probability 1/2.