Intermediation in a Sharing Economy: Insurance, Moral Hazard, and Rent Extraction

Thomas A. Weber∗

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Abstract

Electronic intermediaries have become pervasive in sales transactions for many durables, such as cars, power tools, and apartments. Yet only recently have they successfully tackled the challenge of enabling parties to share such goods. A key impediment to sharing is a lender’s concern about damage due to unobservable actions by a renter, usually resulting in moral hazard. This paper shows how an intermediary can eliminate the moral-hazard problem by providing optimal insurance to the lender and first-best-incentives to the renter to exert care, as long as market participants are risk-neutral. The solution is illustrated for the collaborative housing market but applies in principle to any sharing market with vertically differentiated goods. A population of renters, heterogeneous both in their preferences for housing quality and with respect to the amounts of care they exert in a rental situation, faces a choice between collaborative housing and staying at a local hotel. The private hosts choose their prices strategically and the intermediary sets commission rates on both sides of the market as well as insurance terms for the rental agreement. The latter are set so as to eliminate moral hazard. The intermediary is able to extract the gains the hosts would earn compared to transacting directly. Finally, even if hotels set their prices at the outset so as to maximize collusive profits, we find that collaborative housing persists at substantial market shares, regardless of the difference between the efficiencies of hosts and hotels to reduce renters’ cost of effort. The aggregate of hosts, intermediary, and hotels benefits from (a variety in) these effort costs, which indicates that the intermediated sharing of goods is an economically viable, robust phenomenon.

JEL-Classification: C72, D43, D47, L13, L14, O18, R31.
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∗Chair of Operations, Economics and Strategy, École Polytechnique Fédérale de Lausanne, Station 5, CH-1015 Lausanne, Switzerland. Phone: +41 (21) 693 01 41. E-mail: thomas.weber@epfl.ch.
Introduction

A burgeoning sharing economy has brought collaborative consumption as a real possibility to individuals. Already in 2009, the *New York Times* noted that “[s]haring is clean, crisp, urbane, postmodern; owning is dull, selfish, timid, backward” [27]. The notion that an asset such as a car or a flat can be shared via intermediaries such as Zipcar in the US or Mobility in Europe has successfully passed the proof-of-concept stage. Founded in 2008, AirBnB has enabled the peer-to-peer sharing of housing units, boasting currently more than 600,000 listings in 192 countries [1]. The commercial success has attracted substantial investor interest, putting AirBnB’s current valuation at approximately $10 billion [16]. Since the rapid growth of the collaborative-housing market (exceeding 100% in 2013) cannot fundamentally be fuelled by changes in demand-side or supply-side fundamentals, the explosion of shared housing is likely due to an intermediary’s successful new business model, which has managed to break a pre-existing market failure, presumably caused by a lack of trust between renters and hosts as well as a lack of critical mass on both sides of the market. The situation for collaborative housing is exemplary for the sharing of durable goods in general. Homes being particularly durable and valuable only exacerbates the impediments to sharing, since the stakes are particularly high. Yet, if the sharing problem can be solved for housing, then it could be considered essentially solved for any durable good, *modulo* some minor ‘standard’ transaction costs, e.g., to match market participants, which are largely neglected here and have been discussed elsewhere [4, 6, 11, 48]. For concreteness, we develop a model for an intermediated collaborative-housing market with particular attention to the moral-hazard problem embedded in the underlying private peer-to-peer transactions.

The recent growth of collaborative housing suggests that intermediation has helped to overcome a persistent market failure stemming from moral-hazard issues in the shared use of private space. Other market imperfections, such as the cost of matching and coordination, may have contributed to offset any expected gains for hosts and thus to suppress the supply of private short-term rental units. However, the two latter imperfections cannot be considered decisive. The pervasiveness of trading platforms (such as eBay), social networks (such as Facebook), or travel sites (such as Booking) reduces matching to a minor hurdle in a networked society. The cost of coordination in private housing transactions (e.g., having to agree on details on where and when to pick up and drop off the keys, or to learn about idiosyncratic usage issues such as where to turn on the heat), in hindsight, also cannot be prohibitive, as those costs remain essentially unchanged by the presence of an intermediary, save for a codification of the process in the form of recommended rules and procedures. This suggests that the main reason why collaborative-housing markets have not existed until the recent emergence of specialized intermediaries (such as AirBnB, Wimdu or 9Flats) is a persistent agency problem. Using a simple model, this paper shows that a trusted online intermediary can enable short-term collaborative-housing transactions by implementing the first-best (i.e., *ex post* optimal) outcome of the underlying agency problem, thus effectively eliminating moral hazard.

In our model, heterogeneous renters (or “agents”) and vertically differentiated hosts can use an intermediary to increase the probability of matching. The intermediary offers an insurance contract which requires the renter to make a deposit and allows for the host to file claims in case a damage occurs that exceeds a predefined minimum threshold. The insurance terms also specify the settlement amounts to be paid to the host as a function of the claim amount, as well as the amount to be kept from the renter’s deposit, which in general includes a surcharge.
over and above the claim amount. For the matching, codification, and insurance services, the intermediary obtains a commission from both parties, as a portion of the posted price for the short-term accommodation. Agents, who vary in the utility they derive from extra housing quality, face a choice of staying at one of the available private hosts or else at a local hotel. The hosts offer vertically differentiated housing options and choose their prices in equilibrium, given the intermediary’s surcharges and a hotel price. We show that, provided all the parties are risk-neutral, the intermediary can fully cover the hosts’ claims (without a deductible) and also provide first-best (i.e., economically efficient) incentives for the renters to exert care, even if their costs of doing so is heterogeneous ex post (but homogeneous ex ante). In other words, the intermediary induces first-best renter behavior, which is comparable to the hosts’ own behavior in a continued use of their own space. The amount the intermediary can extract from the hosts’ surplus is limited only by the value of their outside option, for example, renting it to agents in a direct transaction (i.e., without the intermediary) or by doing nothing.

The remainder of this paper is organized as follows. After a brief review of the relevant literature, we introduce the elements of a model for an intermediated market for short-term collaborative housing in which the actions evolve over five time periods. The model is then solved period-by-period, starting with the last. This implies results on market structure and behavior. Extending the basic model, we then add a degree of realism by considering positive verification costs for the intermediary. A discussion of managerial implications and the role of information systems is followed by a conclusion.

**Literature**

A number of reasons why individuals may like to share their private property with others have been provided by Belk [8, 9]. Additionally, an increasing scarcity of natural resources and a growing awareness that favors use over possession contribute to an increase in collaborative consumption in a variety of domains, such as cars, tools, and housing [14, 15, 39]. From an economic viewpoint, and in the absence of reputational concerns, sharing a resource becomes attractive when the expected benefits of the transaction outweigh the expected costs. For a potential host in a collaborative-housing agreement, expected benefits include economic rents that can be extracted from agents; expected costs include upfront advertising expenditure to encourage matching with a renter, opportunity costs from foregone own use, and—most importantly—the cost of agency. The latter derives from the fact that the agent’s actions while enjoying the rented space are unobservable, and thus largely non-contractible. To the best of our knowledge intermediated sharing, with an emphasis on the economic incentives for collaborative usage, has not yet been discussed in the academic literature.

The standard business proposition of an intermediary consists in lowering the cost of interaction for the two parties on either side of a market [41]. Fundamentally, the intermediary is viable if its intermediation cost (per transaction) is below the cost of a direct transaction. Based on earlier literature, Bailey and Bakos [4] identify four important roles of market intermediaries: to aggregate demand and/or supply, to reduce the operating costs in markets, to match transacting parties, and to provide trust. By reducing informational asymmetries and the related agency problems of adverse selection and moral hazard, an intermediary can generate trust. Biglaiser and Friedman [13] show that middlemen are able to reduce moral hazard by providing information about the claimed quality of products. In finance, intermediation has
been considered with moral hazard on the part of the borrower who may be asked to place a bond. Nonfinancial intermediaries such as eBay establish trust by providing a public feedback mechanism, which shares concrete details about transactions. This also holds true for collaborative-housing intermediaries like AirBnB, where both renters and hosts can establish profiles and reputations based on the recorded performance evaluations of past transactions.

Our work is related to recent contributions on two-sided markets. Roger and Vasconcelos examine platform pricing and moral hazard in a dynamic setting with reputation, where sellers can be induced by a two-part tariff, including a fixed registration fee and a variable transaction fee, to take a favorable high-effort action. In that setting, the elimination of the moral-hazard problem depends on the availability of the fixed participation fee, which allows sellers effectively to place a bond. In our static model the intuition is similar, only that in the case of temporary housing the moral hazard lies with the buyer (renter), who is therefore asked to place a bond in the form of a deposit. Because of the embedded insurance problem in this setting, the novelty of our study lies not in confirming that it is effective to place a bond in order to eliminate moral hazard, but in solving the particulars of the business problem with an embedded actuarial insurance problem and in showing how one can effectively separate the agency problem in a budget-neutral way from the rent-extraction (or profit-maximization) problem. The paper is an extended version of, dropping most distributional assumptions made there, allowing for more general insurance terms, and solving the model for the entire range of outside hotel prices. The hotels’ pricing decision is endogenized for the benchmark case of revenue maximization. Lastly, in contrast to, verification of damage claims can be costly and hotels induce an agency cost which may or may not exceed the agency cost a renter expects when opting for collaborative housing.

Model

The model, in the form of a dynamic game, comprises renters of different types, hosts offering collaborative-housing arrangements, an intermediary, and one (or several) hotel(s) as outside option. All parties are assumed to be risk-neutral. The proofs of all results are contained in Appendix A; Table in Appendix B summarizes the notation.

Renters. A potential renter (henceforth also referred to as “agent”) has a marginal utility for staying at a place of perceived quality; any stay is for a time period of unit length (e.g., one day). For simplicity, we assume that all renters perceive quality in the same way (so we can restrict attention to vertical differentiation) and that their types are uniformly distributed on the interval . Given a matching probability for an intermediated private transaction, a renter of type has expected payoff

where and are the price-quality-tuples for stays at a private property or a hotel, respectively, and is a surcharge rate (fixed by the intermediary) over the posted price (so the renter’s total monetary transfer is , excluding a possible deposit). The non-negative constant denotes an expected agency cost for the renter in a collaborative-housing agreement (respectively, at a hotel). This cost is unaffected by an agents choice of host; yet, it may be lower when staying at a hotel. It includes both the expected monetary damage to
the agent and the (expected) cost of the effort \( e \) for his exerting care. The expected agency cost for the renter is determined by his cost of effort, the damage distribution, and the intermediary’s transaction terms.

**Hosts.** There are two potential hosts (1 and 2) who possess properties of the respective qualities

\[ q_1 = q_0 - \varepsilon \quad \text{and} \quad q_2 = q_0 + \varepsilon, \]

where \( 0 < \varepsilon < q_0 \). Thus, private properties are considered by the renter akin to a mean-preserving spread [37] of hotels: some private properties are better, some are worse, with a random selection of the two being of comparable quality. This assumption seeks to eliminate any intrinsic advantage or disadvantage of the collaborative-housing market; it can be easily relaxed.\(^5\) Given a demand \( D_i \in [0, 1] \), which may depend on the prices of all available housing options, host \( i \in \{1, 2\} \) obtains a profit of

\[ \pi_i = D_i((1 - h)p_i - \delta), \]

where \( h \in [0, 1] \) is a commission rate charged as a percentage of the host’s posted price \( p_i \), and the non-negative constant \( \delta \) is an expected agency cost which remains unaffected by the host’s pricing decision. The host’s profit depends on the terms set by the intermediary, the renter’s exerted effort, and the distribution of possible damage.

**Intermediary.** The intermediary provides matching between the two sides of the market, which includes a trusted insurance service to mitigate moral hazard. The intermediary’s main goals are to overcome the market failure created by the lack of trust between the two sides of the market and to extract the maximum possible rent from a collaborative-housing transaction between renter and host.

First, to provide appropriate incentives for the renter to exert an efficient amount of care and for the host not to distort prices in the market based on a positive expected agency cost, the intermediary sets the terms \((H, R)\) which imply an insurance agreement, subject to any damage claim having to exceed a fixed minimum amount \( m \geq 0 \). On the host side, \( H(x) \) defines the host’s deductible paid back to the intermediary, after the claimed damage \( x \geq m \) has been covered by the intermediary. The resulting net settlement is \( x - H(x) \), paid from the intermediary to the host. For example, the settlement function can be

\[ H(x) = \begin{cases} 0, & \text{if } x < m, \\ \min\{d, x\}, & \text{otherwise}, \end{cases} \]

where \( d \) denotes a deductible, i.e., an amount subtracted from any claim a host files. It turns out that in general the function \( H(\cdot) \) may need to be somewhat more complex to help implement ex post optimal, first-best incentives. On the renter side, \( R = (f, s) \) includes a fixed deposit \( f \geq 0 \) and a surcharge rate \( s \geq 0 \). The minimum damage claim \( m \) defines a lower bound for what constitutes a “significant” damage that can be claimed by the host. This limits the transaction cost for the intermediary, for example, caused by the necessity to verify the damage claims so as to prevent hosts’ cheating. The renter’s fixed deposit limits the maximum outlay. How much of the deposit is paid back depends on the surcharge rate \( s \): given a verified claim \( x \geq m \), the renter is refunded an amount \( \max\{0, f - (1 + s)x\} \).

Second, to extract rent from the transaction, the intermediary chooses a commission structure \((h, r)\), where \( h \in (0, 1] \) is a percentage of the posted price \( p_i \) charged to host \( i \) and \( r \geq 0 \) is the percentage of \( p_i \) that is added for the renter. Hence, for a collaborative-housing arrangement...
between a renter and host $i$, the payment from the renter to the intermediary is $(1 + r)p_i$ while the intermediary pays $(1 - h)p_i$ to host $i$.

**Outside option (hotels).** To simplify the analysis, we assume that the hotel market features a single quality $q_0 > 0$. The corresponding price $p_0$ is fixed before anything else happens under the benchmark assumption that the hotel sector behaves as if it were a single strategic monopolist. Alternatively, one can dispense with the price-setting period and assume that the hotel market is competitive and $p_0$ reflects a competitive price for that segment [46], which amounts to considering only a subset of the complete model solution. Unlike Weber [46], we include an baseline agency cost $\varphi_0$, which the agent expects to incur in a hotel environment. Because the hotel is more streamlined for receiving guests than private hosts, the expected agency cost for the former typically do not exceed the agency cost for the latter (i.e., $\varphi \geq \varphi_0$). In case they do, then not by much,$^6$ so we assume that

$$-\varepsilon/2 \leq \varphi - \varphi_0.$$  

**Timing.** The dynamic game takes place over five time periods (0–4). In period 0, the hotel chooses the price $p_0$. The reason for decoupling the pricing of the outside option from the rest of the game is that the market for collaborative housing is comparatively small. In period 1, the intermediary specifies the rate structure $(h, r)$ as well as the terms $(H, R)$ pertaining to the moral-hazard side of the rental agreement; the latter determine the expected agency costs $\delta$ and $\varphi$. In period 2, the hosts choose their prices $p_1$ and $p_2$. In period 3, each agent decides to stay at the hotel or with one of the two hosts. In period 4, each renter learns about his effort-cost type $\vartheta$ and then chooses his effort $e$ to mitigate any potential moral hazard. Conditional on the effort, zero damage occurs with probability $z(e)$. In case a monetary damage $x \geq m$ is incurred by the host, where $m \geq 0$ is the pre-agreed minimum, the host can reclaim it from the intermediary. In case of a damage at the hotel, the hotel keeps (a portion of) the renter’s deposit. All final payoffs (to the renters, the hosts, the intermediary, and the hotel) are realized at that point. Fig. 1 provides an overview.

**Equilibrium.** The solution of the model, yielding the actions of the different players (agents/renters, hosts, intermediary, and hotel) as part of a subgame-perfect Nash equilibrium, proceeds successively, by backward induction starting with the last period.

**Period 4: Moral Hazard and Incentives**

During the rental period, the agent’s effort $e$ to prevent damage is unobservable and therefore non-contractible. Since prevention efforts are costly in terms of attention and time spent for
exerting care, the renter is trading off the cost of effort against the costs resulting from a lack of effort. Too little effort on the part of the renter increases the likelihood of a random monetary damage $\tilde{x}$ for the host. Such (monetized) rental damage may include the breakage of items, degradation of the property through overuse, or excessive uncleanliness. As a consequence, the short-term rental contract is subject to moral hazard. Without loss of generality, the renter’s effort $e$ is scaled so as to be measured directly in the probability of no damage, $z(e)$. That is,

$$z(e) = e \in [0, 1]$$

measures the likelihood that the rental contract ends without incident. With probability $P = 1 - e$, the host observes a damage $x > 0$. An incident is significant if the corresponding damage $x$ exceeds the pre-agreed minimum damage $m$. For damage realizations below $m$, there is no remedial action. Let $F(x) = \text{Prob}(\tilde{x} \leq x)$ denote the cumulative distribution function for positive damage realizations $x$ in $(0, \infty)$. Then the expected value of a claim as a function of $m$ is

$$\mu(m) = E[\tilde{x}|\tilde{x} \geq m] \text{Prob}(\tilde{x} \geq m).$$

The renter’s cost of effort is $C(e, \vartheta) = \vartheta e^2 / 2$, where $\vartheta > 0$ is the (maximum) marginal cost of effort (at $e = 1$).\(^7\) In general, the renters are heterogeneous with respect to their effort-cost parameter $\vartheta$; the corresponding cumulative distribution function is $G(\vartheta)$.

Given a deposit $f$ and a surcharge rate $s$ in case of damage, the renter’s expected agency cost at the optimal effort $e^*(L, \vartheta) = \min\{1, L/\vartheta\}$ is

$$\varphi = \min_{e \in [0, 1]} \{(1 - e)L + \vartheta e^2 / 2\} = \begin{cases} \vartheta / 2, & \text{if } \vartheta \in (0, L), \\ L(1 - L/(2\vartheta)), & \text{otherwise}, \end{cases}$$

where the renter’s expected liability amounts to

$$L = E \left[ \min \{f, (1 + s)\tilde{x}\} | \tilde{x} \geq m \right] \text{Prob}(\tilde{x} \geq m).$$

The renter’s optimal effort does not depend on the price paid for the accommodation. Indeed, by the time the decision about the effort is made, the cost for the accommodation is sunk and the renter’s only concern is about the security deposit and about the expected damage as a function of the exerted care. The renter’s expected agency cost is therefore

$$\varphi = L(1 - G(L)) + \int_0^L \frac{\vartheta \, dG(\vartheta)}{2} - L^2 \int_L^\infty \frac{dG(\vartheta)}{2\vartheta}.$$

The renter’s effort results in the expected damage probability,

$$P = 1 - E \left[ e^*(L, \vartheta) \right].$$

A host’s expected agency cost $\delta$ depends on the probability of damage $P$ and the intermediary’s given settlement function $H$ for the host,

$$\delta = P \cdot (\mu_0 - E \left[ \tilde{x} - H(\tilde{x}) | \tilde{x} \geq m \right] \text{Prob}(\tilde{x} \geq m));$$

by default: $H(x) = \min\{d, x\}$ for $x \geq m$, where $d$ is a deductible.
Period 3: Market Segmentation

All renters face the choice among different accommodations, each of which is characterized by a price, a quality, and an expected agency cost. Which of the three options, \((p_0, q_0, \varphi_0)\) (for the hotel) or \((p_i, q_i, \varphi)\) (for host \(i \in \{1, 2\}\)), a renter prefers, depends on his type \(\theta \in \Theta\), his marginal utility for accommodation quality. The resulting demands for the three housing options split the market into segments, \(\Theta_0, \Theta_1, \text{ and } \Theta_2\). The number of active (i.e., non-empty) segments depends on the relative attractiveness of the options. For example, if both hosts charge exorbitant prices, then all renters would choose the outside option, namely to stay at a hotel.\(^8\)

If we ignore for a moment the fact that \(\theta\) is by assumption limited to the set \(\Theta = [0, 1]\), then a renter indifferent between staying at property 1 and staying at a hotel would be of type
\[
\theta_{10} = \frac{p_0 - (1 + r)p_1 - (\varphi - \varphi_0)}{\varepsilon},
\]
and a renter indifferent between staying at property 2 and staying at a hotel would be of type
\[
\theta_{20} = \frac{(1 + r)p_2 - p_0 + (\varphi - \varphi_0)}{\varepsilon}.
\]
Lastly, to be indifferent between the two collaborative-housing options a renter would need to be of type
\[
\theta_{12} = \frac{(1 + r)(p_2 - p_1)}{2\varepsilon}.
\]
If any of the three critical types \(\theta_{10}, \theta_{20}, \theta_{12}\) lies outside the type space \(\Theta = [0, 1]\), then effectively one market segment must vanish. For example, if \(\theta_{12} > 1\), then all of the available renters of types \(\theta \in [0, 1]\) would prefer staying with host 1 over staying with host 2, implying that the demand for host 2 vanishes, \(D_2 = 0\). The market segments for the two collaborative-housing options are \(\Theta_1 = \{\theta \in \Theta : \theta \leq \min\{\theta_{10}, \theta_{12}\}\}\) and \(\Theta_2 = \{\theta \in \Theta : \theta \geq \max\{\theta_{20}, \theta_{12}\}\}\). Conversely, the segment of renters preferring the outside option of staying at a hotel consists of the remaining types in \(\Theta_0 = \Theta \setminus (\Theta_1 \cup \Theta_2)\). For the collaborative-housing market to stay active (i.e., \(\Theta_1 \cup \Theta_2 \neq \emptyset\)), the outside option cannot be too cheap.

Remark 1 (Viability of Collaborative Housing). The collaborative-housing market exists if and only if
\[
p_0 \geq \min\{(1 + r)p_1, (1 + r)p_2 - \varepsilon\} + \varphi - \varphi_0.
\]
Indeed, the demand for both hosts disappears, \(D_1 = D_2 = 0\), if and only if \(\theta_{10} \leq 0\) and \(\theta_{20} \geq 1\). These two inequalities taken together are equivalent to the stated inequality. All else equal, a high expected agency cost \(\varphi\) for the renter can cause the collaborative-housing market to fail. On the other hand, more quality dispersion (i.e., a larger \(\varepsilon\)) in the rental qualities provided by the hosts can overcome market failure at the high end of the market.

Period 2: The Hosts’ Pricing Game

Each host \(i\) tries to maximize his profits \(\pi_i\) given the terms set by the intermediary. Since the intermediary claims the percentage \(h \in [0, 1]\) of the posted price \(p_i\), the hosts’ expected payoffs (conditional on matching with a renter) become
\[
\pi_1 = \max\{0, \theta_{10}\} ((1 - h)p_1 - \delta) \quad \text{and} \quad \pi_2 = \max\{0, 1 - \theta_{20}\} ((1 - h)p_2 - \delta).
\]
The non-negative constant \( \delta \) denotes an expected agency cost for each host in a collaborative-housing agreement. It consists of the expected monetary damage net of any damage compensation by the intermediary. The intermediary’s commission structure \((h, r)\) is relevant for the hosts’ pricing decisions. As will become clear below, of particular relevance is the “commission ratio”

\[
\rho = \frac{1 + r}{1 - h} \geq 1,
\]

which describes the ratio of intermediated prices between the two sides of the market: for any given posted price \( p \), the renter pays \((1 + r)p\) while the host receives \((1 - h)p\).

The distortion introduced by the moral-hazard portion of the renter-host interactions are captured \textit{ex ante} by the “effective agency cost”

\[
\alpha = (\varphi - \varphi_0) + \rho \delta.
\]

The effective agency cost \( \alpha \) describes the sum of the excess agency cost over what the renter would expect at a hotel (i.e., \( \varphi - \varphi_0 \)) and the product of the commission ratio with the expected agency cost for a host (i.e., \( \rho \delta \)). It can be interpreted as the marginal cost for a host (and therefore \( p - \alpha \) can be viewed as his gross margin).\(^9\)

\textbf{Theorem 1 (Host Economics).} For any given price of the outside option \((p_0 \geq 0)\), the hosts’ respective equilibrium prices \((p_1^*, p_2^*)\), equilibrium demands \((D_1^*, D_2^*)\), and equilibrium profits \((\pi_1^*, \pi_2^*)\) exist, and their unique values are provided in Table\[7\] .

The price \( p_0 \) of the outside option is critical for the hosts’ economics. Fig.\[2\] shows the equilibrium prices. For host \( i \) to be viable, she has to charge a nominal price \( p_i \) that at the very least overcomes \( \delta/(1 - h) \), given the intermediary’s retaining the percentage \( h \) and that the expected agency cost is \( \delta \). As the price for hotels increases, the hosts are able to also increase their prices and obtain a positive rent from offering collaborative-housing options. Interestingly, what matters for the hosts’ economics in Table\[1\] is the “excess price,”

\[
p_0 - \alpha,
\]

that the hotels charge above and beyond the effective agency cost \( \alpha \). The excess price can be positive or negative.

\textbf{Corollary 1 (Comparative Statics).} For any host \( i \in \{1, 2\} \), the equilibrium price \( p_i^* \) and profit \( \pi_i^* \) are both non-decreasing in \( p_0 - \alpha \). The demand \( D_1^* \) for host 1 is non-decreasing in \( p_0 - \alpha \) for \( p_0 - \alpha < \varepsilon/2 \) and non-increasing otherwise.
Figure 2: Equilibrium price, $p_i^* = \frac{k_i \varepsilon}{1 + \frac{\delta}{1 - \eta}} + \frac{\delta}{1 - \eta}$, for host $i \in \{1, 2\}$ as a function of $p_0$.

Figure 3: Market segments in the type space $\Theta$ as a function of $p_0$. 
The hosts’ payoffs are decreasing in the effective agency cost; even if their own agency cost \( \delta \) vanishes (which happens when the intermediary eliminates moral hazard), the hosts’ payoffs are also decreasing in the commission ratio \( \rho \). As hotels in the area become more expensive (i.e., when \( p_0 \) increases) the rents that hosts can extract from renters, all else equal, also increase. The evolution of the market shares as a function of \( p_0 \) is non-trivial. While the low-quality host 1’s demand never decreases as the outside option becomes more expensive, the demand for the high-quality host 2 is non-monotonic. As \( p_0 \) exceeds \( \alpha - \varepsilon \), host 2 becomes economically viable and can charge a positive margin (above \( \delta/(1-h) \)) which increases as long as \( p_0 \) stays below \( \alpha + \varepsilon \). In that range some renters opt out of the collaborative-housing market and stay at hotels. As \( p_0 \) moves beyond \( \alpha + (\varepsilon/2) \) host 2’s demand decreases. The reason is that instead of reacting strategically to host 1’s pricing decision, it is better to simply choke off the demand for hotels.\(^{10}\) As soon as \( p_0 \geq \alpha + \varepsilon \), those concerns vanish, and the two hosts compete on price with their vertically differentiated good as (intermediated) duopolists. The nature of the competition is “local” in the sense that only adjacent accommodation providers are in actual competition with each other.\(^{11}\) Fig. 3 shows the evolution of the market segments as a function of \( p_0 \).

**Corollary 2 (Market for Collaborative Housing).** The high-quality host (host 2) joins the market if and only if \( p_0 > \alpha - \varepsilon \). The low-quality host (host 1) joins the market if and only if \( p_0 > \alpha \).

The full collaborative-housing market is available if the price for hotels exceeds the effective agency cost. This corresponds to the inequality in Remark 1 earlier. The hosts are endowed with the accommodations they offer. The construction of their respective houses or, alternatively, the signing of their various lease agreements occurred some time in the past and constitute therefore fixed boundary conditions. Hence, both hosts’ costs are the same, only that the high-quality host must, all else equal, be economically more viable than the low-quality host. This is reflected in the last result, since for host 2 to offer collaborative-housing arrangements, it is enough that the hotel price exceeds the effective agency cost \( \alpha \), diminished by the quality premium \( \varepsilon \).

### Period 1: The Intermediary’s Design Problem

To maximize profits the intermediary faces two problems, both distributed across the two sides of the markets. First, it needs to solve the moral-hazard problem by setting incentives through the insurance terms \((H,R)\) for host and renter. Second, the intermediary chooses a commission structure \((h,r)\) to extract as much rent as possible. The solutions to the two problems are examined in turn.

#### Eliminating Moral Hazard

Consider now the intermediary’s expected cost of insuring the transaction,

\[
\Delta = P \cdot K,
\]

where, given the insurance terms \((H,R)\) and the minimum damage-claim size \( m \), the expected capital at risk is

\[
K = \mu(m) - \int_m^\infty \min\{f, (1+s)x\} \, dF(x) - \int_m^\infty H(x) \, dF(x).
\]

\( P \) denotes the expected value of the claim size, \( \mu(m) \) the coverage, \( f \) the distribution function of the claim size, and \( H(x) \) the distribution function of the expected claim size given that the claim exceeds \( m \).
The capital at risk $K$ is composed of three positions: first, the expected damage conditional on the fact that only claims above the minimum amount $m$ are processed; second, the payments from the renter who overpays $(1 + s)x$ for any damage $x$ above $m$ at the penalty rate $s$, up to a total of the deposited amount $f$; third, the contribution from the host who makes a payment of $H(x)$ after full compensation of $x$ for claims above $m$.

Eliminating the effects of moral hazard in the transactions between renters and hosts, and thus in the intermediary’s rent-extraction problem, implies the following three requirements (R1–R3) for the insurance terms:

**R1.** First-best incentives for any renter: $L = E[\hat{x}] = \mu_0$.

**R2.** No agency cost for the hosts: $\delta = 0$.

**R3.** No agency cost for the intermediary: $\Delta = 0$.

An insurance contract satisfying these requirements is free of moral hazard. The first requirement (R1) ensures that the expected damage a renter sees is the same as if he were fully liable, which then results in the first-best effort $e'(\mu_0, \vartheta)$, for any cost-of-effort type $\vartheta$. The second requirement (R2) insulates the host from any expected cost of the agency relationship and, by Theorem 1, also decreases prices. Furthermore, the effective agency cost $\alpha$ drops to $\varphi - \varphi_0$ and becomes independent of the intermediary’s commission ratio $\rho$. The last requirement (R3) ensures that the intermediary does not incur any expected agency cost and thus acts as a mere conduit for the agency relationship between host and renter.

**Lemma 1.** Let $m > 0$ be a given minimum-damage claim threshold.

(i) Given a non-negative deposit $f$, the renter’s liability is maximized for $s \geq s^*$, where

$$s^* = \max\{0, (f/m) - 1\},$$

and its maximum value is $L = f$ (independent of $m$).

(ii) The minimum deposit satisfying R1 is $f = \mu_0$ when used together with a surcharge rate $s \geq s^*$.

The renter’s liability is increasing and concave for $s \in [0, s^*]$. For surcharge rates $s$ that exceed $s^*$, the renter’s expected liability remains at $f$, which in turn renders $f = \mu_0$ the smallest possible deposit which implements first-best incentives for the renter (requirement R1).

**Lemma 2.** Requirements R2–R3 are satisfied if

$$E[H(\hat{x})|\hat{x} \geq m] \text{Prob}(\hat{x} \geq m) = \mu(m) - \mu_0.$$ 

The last result implies that for the agency problem to vanish the contribution by the host must be equal to the difference between the expected significant damage $\mu(m)$ (above the threshold $m$) and the expected overall damage $\mu_0$. Since $\mu_0 \geq \mu(m)$ for any $m \geq 0$, the host’s expected contribution can therefore never be positive. Moreover, if $m > 0$, then the host needs to be compensated for the fact that some of his claims may not be processed at all, so that $H(x) < 0$ for some $x \geq m$. 

11
**Theorem 2** (First-Best Incentive Contracts). Given a minimum damage $m > 0$, any insurance terms $(H, R)$, with $R = (f, s)$ such that $f = \mu_0$, $s \geq s^* = \max\{0, (f/m) - 1\}$, and

$$E[H(\tilde{x})|\tilde{x} \geq m] = \frac{\mu(m) - \mu_0}{1 - F(m)} \leq 0,$$

eliminate moral hazard in renter-host transactions (i.e., satisfy requirements R1–R3).

To eliminate moral hazard in renter-host transactions, the settlement function $H$ must be such that it compensates the host for his expected outlays due to the lower bound $m$ on claims. From the renter’s perspective, the resulting (first-best) agency cost $\varphi^*$ results from his effort when facing a liability which directly corresponds to the expected value $\mu_0$ of the damage distribution.

**Corollary 3.** Any moral-hazard-free contract satisfying requirements R1–R3 induces an expected agency cost of

$$\varphi^* = \varphi|_{L=\mu_0} = \mu_0(1 - G(\mu_0)) + \int_0^{\mu_0} \frac{\vartheta}{2} dG(\vartheta) - \mu_0^2 \int_0^{\infty} \frac{dG(\vartheta)}{2\vartheta}$$

for the renter.

The first-best effort $e^{\varphi_0}(\vartheta) = \min\{1, \mu_0/\vartheta\}$ is induced for all cost types $\vartheta$. By eliminating the moral-hazard issue, the intermediary effectively decouples its profit-maximization problem from the rent-extraction problem. Although it is conceivable that a small profit could be made from providing an insurance service against moral hazard (e.g., by applying a deductible, so $H \geq 0$), this effectively reduces the hosts’ surplus and thus allows for less rent extraction (see below).

**Remark 2.** Theorem 2 implies that in equilibrium the intermediary’s fee structure can be linear on both sides (e.g., if $H(x) \equiv \mu(m) - \mu_0$). This meets with standard contractual arrangements involving moral hazard [12]. It also contains a penalty clause for the renter, which is common to limit abuse of leased real-estate assets [10].

**Remark 3.** Another optimum is obtained by extending the space of feasible policies, allowing for an unbounded penalty rate. In that case, a policy with $H$ as in Theorem 2 and $R^* = (f^*, s^*) = (\mu_0, \infty)$ is also optimal, implementing first-best effort with $L = \mu_0$ and removing all capital-at-risk for the intermediary (i.e., $K = 0$). The advantage of this “zero-tolerance” policy is robustness in the sense that this solution does not require any knowledge about the actual damage other than that it exceeds $m$. The problem is that it lacks any kind of proportionality, in the sense that even small infractions require the forfeit of the renter’s full deposit. In addition, from the renter’s perspective the resulting penalty distribution is second-order stochastically dominated [37] (as a mean-preserving spread) by the penalty distribution of any other policy $R = (f, s)$ with $L = \mu_0$. Hence, it will perform strictly worse as soon as renters exhibit risk aversion. The distributional robustness is therefore offset by the fragility with respect to renters’ risk attitude. The lack of proportionality also increases the potential for disagreement and therefore the need for legal recourse with its (unmodelled) additional frictional expense.
Extracting Rent

The intermediary’s profit depends on the hosts’ profits and their respective demands.

**Lemma 3.** The intermediary’s profit is

\[ \pi_I = (\rho - 1) (\pi^*_1 + \pi^*_2 + \delta(D^*_1 + D^*_2)) - \Delta, \]

where \( \Delta \) is the intermediary’s agency-related cost of guaranteeing the transaction.

By Lemma 2 the requirements R2 and R3 of a moral-hazard-free contract are equivalent; each implies that the hosts’ and the intermediary’s expected agency costs vanish. Before seeking to optimize the intermediary’s commission structure \((h, r)\), it is useful to establish the invariance of demand in equilibrium.

**Lemma 4.** For insurance terms satisfying requirement R2, the hosts’ equilibrium prices and equilibrium demands become independent of the commission ratio \( \rho \). The hosts’ equilibrium profits become proportional to \( 1/\rho \).

Even though demands are invariant with respect to changes in the commission structure, the posted prices \( p_i \) for collaborative housing decrease when the intermediary’s commission rate \( r \) for renters is increased, so that the intermediated end prices \((1 + r)p_i\) stay constant. An analogous finding about the neutrality of the price structure with respect to the intermediary’s commission structure was obtained by Caillaud and Jullien [17], in the context of competition between intermediaries.

**Lemma 5.** For insurance terms satisfying requirement R2, the amounts paid by consumers to the intermediary do not depend on the commission structure.

**Lemma 6.** For insurance terms satisfying requirement R2 and provided that there is a market for collaborative housing (i.e., given that \( p_0 + \varepsilon > \varphi - \varphi_0 \)) in which renters and hosts cannot transact directly, the intermediary’s profit \( \pi_I \) is increasing in the commission ratio \( \rho \). Furthermore, the intermediary can extract all surplus from the hosts in the limit, \( \lim_{\rho \to \infty} \pi_I = [\pi^*_1 + \pi^*_2]_{\rho=1} \).

In actuality, the hosts’ outside option consists in ignoring moral hazard and transacting directly with renters; this limits the commission ratio \( \rho = (1 + r)/(1 - h) \) the intermediary can ask for. In a direct transaction, the host cannot charge a deposit and the renter’s expected agency cost vanishes and the host’s expected agency cost becomes \( \delta = \mu_0 \). Hence, the overall effective agency cost in a direct transaction is

\[ \alpha_d = \mu_0 - \varphi_0. \]
Table 2: Optimal commission ratio as a function of $p_0$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$-\varepsilon \leq p_0 - (\varphi^* - \varphi_0) &lt; \varepsilon/2$</th>
<th>$\varepsilon/2 \leq p_0 - (\varphi^* - \varphi_0) &lt; \varepsilon$</th>
<th>$\varepsilon \leq p_0 - (\varphi^* - \varphi_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^*$</td>
<td>$(1 + \frac{\mu_0 - \varphi^*}{p_0 + \varepsilon - (\mu_0 - \varphi_0)})^2$</td>
<td>$\frac{7(p_0 - \alpha) + \varepsilon}{6(1+r)} + \frac{\delta}{1-h}$</td>
<td>1</td>
</tr>
</tbody>
</table>

**Lemma 7.** For insurance terms satisfying requirement R2, the effective agency cost $\alpha_d$ of a direct transaction is higher than the effective agency cost $\alpha^*$ with intermediation, and

$$\left(\frac{\mu_0}{2}\right) G(\mu_0) \leq \alpha_d - \alpha^* \leq \left(\frac{\mu_0}{2}\right) (1 + G(\mu_0)) \left(\leq \mu_0\right);$$

the bounds for $\alpha_d - \alpha^*$ are tight.

The preceding result brackets the improvement in agency cost that intermediation can achieve in a collaborative-housing market. Since by Corollary 1 the comparative statics of the hosts economics depend on the excess price $p_0 - \alpha$, the improvement of the effective agency cost (from $\alpha_d$ to $\alpha^*$) is a perfect substitute to an increase in the price of hotels. For a fixed price $p_0$ of the hotels, the intermediary’s presence acts as if that price is in fact higher (by $\alpha_d - \alpha$), which by Corollary 2 can render the collaborative-housing market viable.

In order to leave the market structure unchanged, the intermediary cannot increase the commission ratio beyond what would decrease the hosts’ payoffs to their direct-transaction payoffs, assuming for now that the matching probability remains constant; this assumption is relaxed in Corollary 4. A commission structure $(h, r)$ is called symmetric if $h = r$.

**Theorem 3 (Optimal Symmetric Commission Structure).** For insurance terms satisfying requirement R2, the intermediary’s optimal commission ratio is

$$\rho^* = \min \left\{ \frac{\pi^*_i}{\pi^*_i\mid(\alpha,\rho)=(\varphi^* - \varphi_0,1)} : \pi^*_i\mid(\alpha,\rho)=(\mu_0 - \varphi_0,1) > 0, i \in \{1, 2\} \right\} \geq 1,$$

corresponding to a symmetric commission structure $(h^*, r^*)$ with

$$h^* = r^* = \rho^* - 1 = \frac{2}{\rho^* + 1} = 1 - \frac{2}{1 + \rho^*} \in [0, 1).$$

Table 2 provides the values for $\rho^*$ as a function of $p_0$.

It is interesting to note that for large hotel prices $p_0$, the intermediary looses all pricing power, and $\rho^* = 1$ (and, by Lemma 3, therefore $\pi^*_I = 0$ in equilibrium). This comes as no surprise, since—as noted earlier—the presence of the intermediary is a perfect substitute increasing $p_0$, all else equal. In other words, when hotels are very expensive, then because the consumers are captive (i.e., they have to stay somewhere), the hosts are in direct competition with each other and cannot do worse even when transacting directly with the consumers.

When accounting for a contribution to increasing the matching probability (in addition to the value generated by eliminating moral hazard), say, from $\beta_d \in (0, 1)$ for direct matching to $\beta$ on the collaborative-housing platform, the intermediary’s rent-extraction capability increases proportionally over the baseline result in Theorem 3.
Corollary 4. Given a matching enhancement from $\beta_d$ to $\beta$, with $0 < \beta_d \leq \beta \leq 1$, the intermediary’s optimal commission ratio increases from $\rho^*$ to $\hat{\rho}^* = (\beta / \beta_d)^2 \rho^*$, where $\rho^*$ is as in Table 2 for any given price $p_0$.

As the direct-matching probability $\beta_d$ approaches zero, the intermediary can extract an increasing share of the hosts’ surplus, for their outside opportunities vanish. More specifically, if $\beta_d \to 0^+$, then $\hat{\rho}^* \to \infty$.

**Period 0: Pricing the Outside Option**

Hotels constitute the outside option for renters. To simplify the analysis, we assume the hotels use their excess capacity to compete with collaborative housing and that the marginal cost for allocating this capacity is effectively zero. To optimize the contribution margin from this capacity, hotels can restrict attention to maximizing revenue. As a benchmark, we assume that hotels pool their capacity and do not compete on price.15 This market-power advantage is partially offset by the fact that hotels move first, which in a sequential pricing game amounts to a strategic disadvantage [19].16

**Theorem 4 (Hotel Economics).** The equilibrium price for hotels is

$$p_0^* = \max\{\alpha, (\alpha/2) + (\varepsilon/4)\}.$$ 

At that price the market share for hotels is $D_0^* = \min\{1, (1/2) + (\alpha/\varepsilon)\} / 2$, and their optimized profit (i.e., revenue) amounts to $\pi_0^* = \min\{\alpha/2, ((\alpha/2) + (\varepsilon/4))^2 / \varepsilon\}$.

The last result shows that a key determinant of the hotel price is the effective agency cost, $\alpha = (\varphi - \varphi_0) + \rho \delta$. As shown in the preceding section, insurance terms can be set such as to eliminate any host’s expected agency cost $\delta$ (by requirement R2) which also decouples the agency problem from the rent-extraction problem. This in turn forces the hotels to decrease their price. Theorem 4 also implies that, all else equal, hotels have an incentive to structure their operations so as to minimize the agency cost $\varphi_0$ their guests expect. The less effort a guest needs to exert in a hotel or the less liable he is for any damage there, the lower the agency cost $\varphi_0$, which in turn allows hotels to increase the price $p_0^*$.

**Market Structure in a Shared-Housing Economy**

The equilibrium actions for the hosts and the renters are obtained by substituting the revenue-maximizing price $p_0^*$ in the earlier results. The resulting prices, profits, and market shares solely depend on the quality dispersion $\varepsilon$ and on the effective agency cost $\alpha^* = \varphi^* - \varphi_0$ under a moral-hazard free contract.

**Theorem 5 (Equilibrium in Collaborative-Housing Economy).** Given a first-best incentive contract (satisfying the requirements R1–R3) with $\alpha^* = \varphi^* - \varphi_0$, the respective equilibrium prices $(p_0^*, p_1^*, p_2^*)$, equilibrium demands $(D_0^*, D_1^*, D_2^*)$, and equilibrium profits $(\pi_0^*, \pi_1^*, \pi_2^*)$ exist, and their unique values are provided in Table 3.
Figure 4: Equilibrium prices \( (p_0^*, p_1^*, p_2^*) \) as a function of \( \alpha^* \).

Table 3: Industry prices, profits, and demands as a function of \( \alpha^* = \varphi^* - \varphi_0 \).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>(-\varepsilon/2 \leq \alpha^* &lt; \varepsilon/2)</th>
<th>(\varepsilon/2 \leq \alpha^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_2^* )</td>
<td>( \frac{(5\varepsilon/2) - \alpha^*}{4(1+r)} )</td>
<td>( \frac{\varepsilon}{2(1+r)} )</td>
</tr>
<tr>
<td>( p_1^* )</td>
<td>( \frac{\varepsilon}{4(1+r)} )</td>
<td>0</td>
</tr>
<tr>
<td>( p_0^* )</td>
<td>( \frac{\varepsilon}{2} + \frac{\varepsilon}{3} )</td>
<td>( \alpha^* )</td>
</tr>
<tr>
<td>( D_2^* )</td>
<td>( \frac{1}{2} + \frac{(\varepsilon/2 - \alpha^*)}{4\varepsilon} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( D_1^* )</td>
<td>( \frac{4\varepsilon}{(\varepsilon/2 - \alpha^*)} )</td>
<td>0</td>
</tr>
<tr>
<td>( D_0^* )</td>
<td>( \frac{1}{2} - \frac{(\varepsilon/2 - \alpha^*)}{2\varepsilon} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \pi_2^* )</td>
<td>( \frac{16\rho\varepsilon}{((5\varepsilon/2 - \alpha^*)^2} )</td>
<td>( \frac{\varepsilon}{4\rho} )</td>
</tr>
<tr>
<td>( \pi_1^* )</td>
<td>( \frac{16\rho\varepsilon}{((\varepsilon/2 - \alpha^*)^2} )</td>
<td>0</td>
</tr>
<tr>
<td>( \pi_0^* )</td>
<td>( \frac{16\rho\varepsilon}{(\varepsilon + 2\alpha^*)^2} )</td>
<td>( \frac{\alpha^*}{2} )</td>
</tr>
</tbody>
</table>
Having derived the equilibrium actions for the renters, the hosts, the intermediary, and the hotels, it is now possible to draw some interesting conclusions about the likely market structure in a sharing economy: coexistence of options, viability of a sharing intermediary, and Pareto-improvement for all parties.

1. **Coexistence.** By Theorem 4, hotels set their price $p_0^*$ so as to allow for the existence of collaborative housing. Their prices undercut the high end of the collaborative-housing market. If the effective agency cost $\alpha^* = \varphi^* - \varphi_0$ is large compared to the dispersion of quality values in the hosts' qualities (i.e., if $\alpha^* > \varepsilon/2$), then hotels price so as to shut down the low-quality hosts ($D_1^* = 0$ and $p_0^* = \alpha^*$); see Fig. 4. If hosts manage to make their properties more robust and hotel-like, so $\varphi^* \to \varphi_0$, then hotels will allow their own offering to coexist with the collaborative-housing segments. In equilibrium, collaborative housing has a market share between 50% and 75%; correspondingly, the market demand for hotels $D_0^*$ varies between 50% and 25%. Fig. 5 shows the evolution of the market shares as a function of the effective agency cost $\alpha^*$ in equilibrium.\(^{17}\) Coexistence amounts to accommodation of collaborative housing by the incumbent hotels instead of using a limit-pricing strategy. In spirit, this outcome is similar to the result of “judo economics”\(^{20}\), where an incumbent effectively ignores an entrant because of low initial sales that may be due to some capacity constraint. As the supply of collaborative housing grows, the number of consumers who consider collaborative housing is likely to increase, eventually driving hotels to compete more directly, for example, if they set prices at the same time as the hosts (see notes 15 and 16).

2. **Viability of the Intermediary.** The intermediary creates value by guaranteeing housing transactions that mitigate moral hazard. Insurance terms that satisfy requirements R1 through R3 motivate the renter to exert first-best effort, no matter what his cost type. At the same time, the hosts’ expected damage is eliminated, even if a positive minimum transaction amount $m$ imposes a lower bound on their possible damage claims. The

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\(^{17}\) **Figure 5:** Market segments $(D_0^*, D_1^*, D_2^*)$ as a function of $\alpha^*$. 

\(^{20}\)
intermediary is viable in the model because all the costs of insuring the collaborative-housing arrangements are covered by payments from the transacting parties: the renter pays for damage at a surcharge rate \( s \), which for low damage realizations offsets the tail of high damage realizations (beyond the renter’s deposit \( f \)) covered by the intermediary. In reality, a collaborative-housing intermediary typically uses a reinsurance in order to ensure liquidity.\(^{18}\)

3. **Pareto-improvement.** In our model, the renters are captive consumers in the sense that they must find a place to stay. Hence, if agency cost is very large, the price for an accommodation becomes high because hotels set their prices relative to this intrinsic cost of using collaborative housing. Naturally, in real life the price for hotels is limited by competition between hotels. Having hotels act as a single revenue-maximizing monopolist imposes the worst possible boundary conditions on the market of collaborative housing. Still, the model indicates that collaborative housing is present in equilibrium. Consumers benefit from the collaborative-housing because prices decrease overall.\(^{19}\) Lastly, the hosts gain positive profits, which are bounded from below by what they could earn from matching with renters directly, without the help and guarantees of the intermediary. Hence, they are able to earn rents from the property which without participation in the sharing economy would remain unused.

It is interesting to note that the industry as a whole is better off when the difference \( \alpha^* = \varphi^* - \varphi_0 \) between the renters’ agency costs for hosts and their agency costs for hotels is sufficiently large.

**Corollary 5.** The aggregate industry profits, \( \Pi = \pi_I^* + \sum_{i=0}^{2} \pi_i^* \), is convex in \( \alpha^* = \varphi^* - \varphi_0 \); it is minimized at \( \alpha^* = \epsilon/6 \).

The intuition is that differentiation (in terms of agency costs) relaxes the competition between hosts and hotels. This is similar to a well-known result by Shaked and Sutton \[40\]: product differentiation softens price competition. Fig. \ref{fig:eqlProf} shows the equilibrium profits and hosts (under the assumption that \( \rho = 1 \), for simplicity) as well as the aggregate industry profit.

### Extension: Verification Cost

The model assumed that the intermediary has no cost of verifying that a given damage claim is justified. Verification is in general necessary in order to eliminate cheating by hosts. In its absence, hosts could always file a damage claim, which in equilibrium would drive up the expected agency cost from a renter’s perspective leading to deposits so large that no renter would be able to afford, effectively leading to market failure. If the cost of verifying a claim is \( v > 0 \), then the expected effective damage, given a minimum damage threshold \( m \), is

\[
\hat{\mu}_v(m) = \mu(m) + v.
\]

To cover this cost and adjust the renter’s incentives, the intermediary can simply include the expense in a modified security deposit \( \hat{f} = \hat{\mu}_v(0) = \mu_0 + v \). The renter would adjust his effort so as to prevent not only damage but also verification. The resulting ‘modified’ requirement R1 becomes
Figure 6: Industry profits ($\pi_0^*, \pi_1^*, \pi_2^*$ and aggregate) for $\rho = 1$ as a function of $\alpha^*$.

R1’. Intermediated first-best incentives for any renter with verification cost: $\hat{L} = \mu_0 + v$.

The requirements R2 and R3 remain unchanged. The overall effect of verification cost is that the effective agency cost under a first-best incentive contract becomes $\hat{\alpha}^* = \hat{\varphi}^* - \varphi_0$, where

$$\hat{\varphi}^* = (\mu_0 + v)(1 - G(\mu_0 + v)) + \int_0^{\mu_0+v} \frac{\vartheta dG(\vartheta)}{2} - (\mu_0 + v)^2 \int_{\mu_0+v}^\infty \frac{dG(\vartheta)}{2\vartheta} \geq \varphi^*.$$  

Hence, the effective agency cost increases with verification cost, so $\hat{\alpha}^* \geq \alpha^*$. The hotels’ “excess price” decreases, since

$$p_0^* - \alpha^* = \left(\frac{1}{2}\right) \max \left\{ 0, \frac{\varepsilon}{2} - \alpha^* \right\} \geq \left(\frac{1}{2}\right) \max \left\{ 0, \frac{\varepsilon}{2} - \hat{\alpha}^* \right\} = \hat{p}_0^* - \hat{\alpha}^*,$$

where $\hat{p}_0^*$ denotes their equilibrium price when the intermediary faces a positive verification cost. By Corollary 1, the prices for collaborative housing increase and the demand for all active segments of collaborative housing decreases. On the other hand, Theorem 4 implies that hotels can charge a higher price, even though their equilibrium market share increases (for the demand for the more expensive collaborative-housing contracts decreases).

**Managerial Implications and Limitations**

**Implications.** The intermediation of moral hazard in a sharing economy is enabled by information systems. An intermediary aggregates information of renters and hosts, providing a platform where network externalities on both sides of the market are likely to increase the attractiveness of the intermediary’s service. In the context of the model, the intermediary has an interest to design an insurance scheme that effectively eliminates the cost of agency to the host and to set
the cost of agency for the renter to a first-best level so as to encourage an efficient amount of effort. The latter is subject to the fact that the renter needs to exert first-best effort in order to effectively decouple the intermediary’s contract-design and rent-extraction problems. The role of information systems is to process the data and provide smart aggregates. For example, the damage distribution $F$ needs to be estimated empirically. Any given host is unlikely to have sufficient data, especially before having had experience with the collaborative-housing market. By aggregating observations over a large number of rental transactions, the intermediary can set incentives correctly.

Let us now interpret the different components of the multipart insurance scheme $(H, R)$. For a positive minimum claim $m > 0$, by requirement R2 the function of $H(\cdot)$ is to provide some payout to the host in case of a realized damage $x \geq m$, over and above a full reimbursement of that claim. The reason is that at least in expectation, the host needs to be compensated for the fact that a small damage (less than $m$) cannot be claimed. From a practical viewpoint, the intermediary may want to set the minimum damage claim so as to minimize transaction cost (i.e., $m$ should not be too low). At the same time, the cutoff value $m$ should not be too large in order to not encourage cheating by the hosts, since their overcompensation by $H(\cdot)$ (yielding on average a negative deductible) must increase in magnitude with $m$ (to satisfy requirement R2).

On the renter’s side, optimal incentives are induced by internalizing the expected damage in the renter’s payoff by requiring him to post a bond (i.e., deposit). The surcharge rate $s \geq s^*$ ensures that the renter overpays for low damage realizations, compensating the intermediary (or the reinsurance provider), at least in expectation, for large payouts in case of a large damage. The surcharge rate $s^*$ for a moral-hazard-free contract in Theorem 2 is a minimum value and higher surcharge rates also work. However, as pointed out in Remark 3, a larger surcharge rate tends to increase the risk for the renter, resulting in a second-order stochastically dominated lottery. The latter would be less appreciated by risk-averse renters. Lastly, it is possible to incorporate a verification cost $v$ into the renter’s insurance terms $R = (f, s)$, namely by increasing the deposit $f$ accordingly to $\hat{f} = f + v$.

**Limitations.** The intermediary is able to completely solve the moral-hazard problem, because both the renters and the hosts are risk-neutral in the model, which goes back to a classical result by Stiglitz [43]. The insurance terms specified in Theorem 2 lead to a first-best effort in equilibrium, provided the renter can earn all the marginal value of his extra effort (the prevention of damage). For this, the intermediary effectively sells all the rights to do damage to the renter at the price of the fixed deposit $f^* = \mu_0$. It is important to note that with a small change in timing our model also has an adverse-selection flavour (in the sense of Akerlof [2]). Indeed, if renters learn their cost types $\vartheta$ at the outset of the game, then chances are that some renters would not want to participate, which then worsens the risk for the remaining population and so forth, until the market potentially unravels.

A real-world intermediary may not hold enough capital to buffer large damage claims. This is especially true in the expansion phase of the collaborative-housing business. Intermediaries make therefore use of reinsurance services to mitigate the actuarial risk of large payouts (see note 18). In the long run, an intermediary may be able to build up enough buffer to bypass reinsurance agreements altogether. In terms of incentives, the (unmodelled) feedback provided by trust mechanisms (e.g., user profiles, host ratings, public comments), resulting in reputations for renters and hosts, respectively, contributes to inducing good behavior and thus increases the quality of the transactions. Thus, the annual total number of reported claims at collaborative-housing intermediaries tends to be small (see note 18). While moral hazard can
be the key obstacle to getting the market started, its significance is fading as the platform’s
user base reaches critical mass and other trust mechanisms, which are not part of our model,
significantly contribute to reducing the inefficiencies of asymmetric information in peer-to-peer
housing transactions.

Finally, it is stressed that, much for analytical convenience, the customer base in our model
was captive, in the sense that every agent needs to stay either with one of the two hosts or in a
hotel. Despite this pressure on consumers to deal with an outside option which endogenously
determines its price, they are able to obtain very reasonable deals because there is always a
choice between at least two options, industry profits are constrained in equilibrium by the
effective agency cost $\alpha^\ast$. In reality, the outside options are themselves dispersed (e.g., vertically
differentiated hotels), which is likely to further improve the renters’ situation, similar to the
logic in Shaked and Sutton [40] that we alluded to earlier.

Conclusion

We have shown that a trusted online intermediary can enable collaborative-housing transactions
in environments with moral hazard, implementing first-best actions by renters and fully insuring
hosts (with, on average, better-than-zero deductible) at a balanced budget. The main idea is to
surcharge renters in case of damage, up to their full deposit in the amount of the expected
damage cost, should a mishap occur. In our model, the intermediary’s ability to extract rents
is limited only by the hosts’ ability to transact directly with renters. It is important to note that
direct transactions are further curtailed by moral hazard on the host’s side. Indeed, if a host
were to ask the renter directly for a deposit, then because of the difficulties to \textit{ex post} verify
and agree on the value of a given damage and an appropriate compensation, the host may have
an incentive to keep the deposit (or portions thereof). In any case, possible conflicts about the
return of host-held deposit would be difficult to handle from the renter’s perspective who, by
hypothesis, is not local and whose costs of legal recourse are therefore elevated. This is in
contrast to the ubiquitous longer-term rental contracts, where direct deposits with the landlord
are common. Yet, even for such contracts the security deposit is often held by a bank in an
escrow account. The latter effectively takes on the intermediary’s function of observing and
verifying claims by either side, possibly initiating third-party arbitration in case of persistent
disagreement.

The optimal symmetric commission structure is such that hosts cannot improve their payoffs
by transacting directly with renters, even if matching is not an issue. In practice, however,
repeat transactions via the intermediary (and the resulting trust and good behavior via self-
enforcing relational contracts [26]) may encourage parties to circumvent the fee structure, which
would prompt the intermediary to design an ‘augmented’ commission structure that takes the
parties’ reputations and bilateral familiarity into account. A formal treatment of such reputation-
dependent pricing in a dynamic setting is left as an interesting open problem for future research.
Another important point not touched upon by our analysis is that the intermediary’s capacity of
rent extraction is limited in general by other intermediaries. The effects of competition between
several intermediaries depend on the degree of similarity between them; their respective rents
will be reduced unless they are able to collude.

Collaborative housing has public-policy implications because enabling private transactions
of this type leads to a more efficient use of existing assets, reducing the frictional costs of travel
on society. At the very least, collaborative housing can serve as a buffer in times of peak demand for temporary accommodations (e.g., during holiday seasons), thus discouraging the creation and maintenance of excess hotel space, resulting in a more efficient use of economic resources overall. By eliminating moral hazard, the intermediary enables the realization of gains from trade in the economy, which is welfare-improving from a social point of view.

The model and results developed in this paper, although constructed for the case of shared housing, should in principle be applicable to a broad range of collaborative-consumption domains, involving the sharing of any durable asset. Future research can include models with risk-averse agents and multiple rounds of interaction between renters and hosts.

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Notes

1 Verification costs are considered as an extension to the basic model.

2 Aggregation benefits result from network effects and economies of scale (or scope) when the intermediary is dealing with a variety of buyers and sellers on both sides of the market, which may lead it to offer differentiated matching services (Barghava and Choudhary [11]). Closely related to the former are benefits from a reduction in operating costs which often result from realizing economies of scale, including risk pooling and diversification. Lee and Clark [25] were among the first to discuss operational efficiencies through process reengineering when using online markets. Standing et al. [42] survey the information-systems literature on electronic marketplaces. By pooling information, intermediaries are often able to decrease the parties’ search costs and thus increase the matching probability [38]. Interestingly, when considering the intermediary’s self-interest, a significant price dispersion (and thus residual search) is bound to remain [13], even if the product market is homogeneous and firms are allowed to advertise [6]. Such inefficiencies are created by an endogenous obfuscation, driven by the intermediary’s fundamental concern to maintain a high level of search activity (Weber and Zheng [48]).

3 Xiao and Benbasat [49] provide a classification of the wide range of deception possibilities in online purchase transactions. Beldad et al. [7] add a review on the behavioral foundations of trust in online transactions. Pavlou and Gefen [30] examine what constitutes a psychological contract violation [34] in online transactions. The perceived risk is commensurate with the degree to which the outcome of the transaction cannot be predicted. Pavlou et al. [31] connect the perceived risk conceptually to standard agency problems that arise whenever substantive pieces of information and/or actions are hidden from some of the transacting parties. Kim and Benbasat [23] show empirically that trust is more easily established when using trust-assuring arguments à la Toulmin [45], involving claim, backing, and data. While a lack of trust can be compensated, for example, by a better price, but it tends to emerge empirically as a critical
factor determining online purchasing decisions [24].

4 As long as the intermediated matching probability \( \beta \) does not vanish, its precise value is irrelevant for our main results. We generally assume a sellers’ market where prospective renters contact hosts, who in turn approve a transaction, as it is common practice for current collaborative-housing intermediaries. We show that the intermediary can capitalize on the fact that its platform increases the matching probability (see Corollary 4). For our main analysis, we neglect this effect.

5 More precisely, we assume that there are two types of hosts, or alternatively, that either host can accommodate all of the prevailing demand. For the case where the hosts have capacity constraints, equilibria are more difficult to determine, so we leave this as a topic for further research. See Osborne and Pitchik [29] for details on capacity-constrained competition in an oligopoly setting.

6 Hotels usually deal with the moral-hazard problem by charging a security deposit upfront. The renter accepts those charges, since (based on the hotel’s reputation and on his agreement with the credit-card company) there is a relatively simple ex-post recourse procedure for the (therefore unlikely) event that a portion of the deposit is kept to cover a significant damage.

7 To be clear, \( \vartheta = \frac{\partial}{\partial e} \bigg|_{e=1} C(e, \vartheta) = \max \left\{ \frac{\partial C(e, \vartheta)}{\partial e} : e \in [0, 1] \right\} = \max \{ \vartheta e : e \in [0, 1] \} \).

8 For given posted prices \( p_1 \) and \( p_2 \), the demand segments also depend on the intermediary’s commission rate \( r \) charged to the renter. However, as we show below (see Lemma 5), this dependency vanishes in equilibrium because the hosts strategically decrease their prices by the surcharge factor \((1 + r)\). At the same time, the intermediary has an interest to decouple incentive contracting from rent extraction which eliminates the hosts’ expected agency cost. Together these actions render the market segments independent of the intermediary’s commission structure \((h, r)\) altogether.

9 As long as the demand for hotels is positive, the solution to host i’s profit-maximization is isomorphic to the solution of the standard monopoly pricing problem (Tirole [44]) of finding the best price \( p \), given a linear demand curve, \( a_i - bp \), and marginal cost \( c \), where \( a_i = (p_0 + (i - 1)e)/(\rho e) \), \( b = 1/(\rho e) \), and \( c = \alpha \); see Theorem 1.

10 The idea is somewhat related to the contestable-markets hypothesis by Baumol [5]: The hosts in the market set their prices so as to (ex post) prevent entry by a competitive fringe (consisting of the hotels in this case). This tactic becomes ineffective when \( p_0 < \alpha + (\varepsilon/2) \).

11 This is analogous to the finding by Xu et al. [50] for an oligopolistic pricing game where consumers face search costs.

12 However, an increase of \( s \) is perceived worse by any risk-averse renter. In our model, the renters are assumed to be risk-neutral, so that any surcharge rate \( s \) beyond \( s^* \) (including \( +\infty \) as the “zero-tolerance” option) in Theorem 2 is equally optimal.

13 Requirement R1 can be relaxed; see the section about verification cost.

14 In other words, the hosts have no outside option as to what to do with their property, at least in the short run.

15 Hotel capacity can be aggregated via intermediaries. Online booking agents such as Expedia or Hotwire routinely offer hotel capacity in a defined area of a city and for a given quality
category (star rating) at deep discounts without disclosing the name of a hotel until after the (irreversible) transaction has been completed. This corresponds to a pooled hotel capacity without branding effects which we think of as the outside option in the collaborative-housing market.

The hotels’ situation can be improved by reducing their commitment to the preset price; note that commitment can generally be viewed as a continuum [47]. In practice, this happens when hotels sell excess capacity through intermediaries such as Hotwire at a discount.

Weber [46] analyzes the special case where all housing providers coexist and the price of the outside option is exogenous.

Without private hosts, hotels would be able to charge an arbitrarily high amount due to the lack of outside options for renters, which is a stylized assumption. We also note that consumers are generically worse off with intermediary than when transacting directly: for them comparing intermediated against direct collaborative-housing transactions is isomorphic to comparing intermediated housing against hotels with a discounted price (below $p_0$).

The inequality follows from the envelope theorem. For $e^*(L, \vartheta) \in (0, 1)$, it is $\frac{de^*}{dL} = 1 - e^*(L, \vartheta) > 0$.

According to its public information as of April 2014, AirBnB has served more than 11 million guests [1].

See our earlier “coexistence” result in the section on “Market Structure in a Shared-Housing Economy.”

An AirBnB-commissioned study by HR&A [21] shows a significant impact of intermediated collaborative housing in the San Francisco Bay Area: for example, 72% of AirBnB-listed properties in San Francisco (exponentially growing from about 20 in 2008 to about 40,000 in 2012) are located outside the 6 main hotel zip codes of the city. A personal communication with the COO of 9Flats in Berlin in 2013 revealed that it is possible to trace an increase in the prices of flats to collaborative housing. Moreover, ancillary service providers are proliferating to facilitate key transfer and laundry services, effectively transforming a residential property into a commercial property.

References


Appendix A: Proofs

Proof of Theorem 1. The five subsets for $p_0$ indicated in Table I are examined in turn. For clarity, we restrict attention to the interior of each subset. Showing the continuity of the solutions by checking the one-sided limits at the boundaries of each segment is elementary and therefore omitted.

1. If $p_0 > \alpha + \epsilon$, then the hosts’ respective profits are $\pi_1 = \theta_{12}((1-h)p_1 - \delta)$ and $\pi_2 = (1-\theta_{12})((1-h)p_2 - \delta)$, where $\theta_{12} = (1+r)(p_2-p_1)/(2\epsilon)$ is the consumer type indifferent between the two hosts. The hosts’ best-response functions are

\[
BR_1(p_2) = \frac{1}{2} \left( p_2 + \frac{\delta}{1-h} \right) \quad \text{and} \quad BR_2(p_1) = \frac{1}{2} \left( p_1 + \frac{\delta}{1-h} \right) + \frac{\epsilon}{1+r}.
\]

Intersecting the hosts’ best responses yields the unique Nash equilibrium prices

\[
p_1^* = \frac{2\epsilon}{3(1+r)} + \frac{\delta}{1-h} \quad \text{and} \quad p_2^* = \frac{4\epsilon}{3(1+r)} + \frac{\delta}{1-h}.
\]

At these prices, the type indifferent between the two hosts is $\theta_{10}^* = 1/3$. Furthermore, the demand $D_0^*$ for the outside option vanishes because

\[
0 < \theta_{10}^* = \frac{1}{3} + \frac{p_0 - \alpha - \epsilon}{\epsilon} > \frac{1}{3} - \frac{p_0 - \alpha - \epsilon}{\epsilon} = \theta_{20}^*,
\]

as long as $p_0 > \alpha + \epsilon$.

2. If $\alpha < p_0 < \alpha + (\epsilon/2)$, then the hosts’ profit-maximizing prices are

\[
p_1^* = \frac{p_0 - \alpha}{2(1+r)} + \frac{\delta}{1-h} \quad \text{and} \quad p_2^* = \frac{p_0 + \epsilon - \alpha}{2(1+r)} + \frac{\delta}{1-h}.
\]

At these prices, the indifferent types $\theta_{10}^*, \theta_{12}^*, \theta_{20}^*$ are such that

\[
0 < \theta_{10}^* = \frac{p_0 - \alpha}{2\epsilon} < \theta_{12}^* = 1/4 < \frac{1}{4} + \frac{\alpha + (\epsilon/2) - p_0}{2\epsilon} = \theta_{20}^* < 1.
\]

Consequently, the hosts’ (positive) equilibrium demands are $D_1^* = \theta_{12}^*$ and $D_2^* = 1 - \theta_{12}^*$, respectively.

3. If $\alpha - \epsilon < p_0 < \alpha$, then host 1 offering the low-quality accommodation cannot break even. His market share therefore vanishes ($D_1^* = 0$). By offering a superior-quality accommodation, host 2 manages to stay in the market. The expressions for his optimal price $p_2^*$ and his equilibrium demand $D_2^*$ correspond to those given under 2.

4. If $0 \leq p_0 < \alpha - \epsilon$, then collaborative housing is not viable ($D_1^* = D_2^* = 0$). The outside option is so cheap that it effectively undercuts any price at which the hosts would obtain a positive gain.
5. If \( \alpha + (\varepsilon/2) < p_0 < \alpha + \varepsilon \), then at the Nash-equilibrium prices determined under 1. the demand for hotels would become positive. On the other hand, the presence of the outside option effectively decouples the competition among the hosts, whose respective demands are \( D_1 = \min\{\theta_{10}, \theta_{12}\} \) and \( D_2 = \min\{1-\theta_{20}, 1-\theta_{12}\} = 1-\max\{\theta_{20}, \theta_{12}\} \). Since \( D_0 = 0 \), it is necessarily \( \theta_{10}^* = \theta_{20}^* = \theta_{12}^* \). Hence,

\[
p_1^* + p_2^* = \frac{p_0 - (\varphi - \varphi_0)}{1 + r} = \frac{\alpha + (\varepsilon/2) + \lambda(\varepsilon/2) - (\varphi - \varphi_0)}{1 + r},
\]

for \( \lambda \in (0, 1) \), corresponding to \( p_0 \in (\alpha + (\varepsilon/2), \alpha + \varepsilon) \). As a result,

\[
\frac{d}{d\lambda}(p_1^* + p_2^*) = \frac{\varepsilon}{1 + r}.
\]

Because of the linearity of the model, the slope of \( p_1^* \) and \( p_2^* \) is constant in \( \lambda \). Using the results obtained under 1. and 2., continuity at the interval boundaries implies that

\[
\frac{dp_1^*}{d\lambda} = p_1^*|_{\lambda=1^-} - p_1^*|_{\lambda=0^+} = \frac{2\varepsilon}{3(1 + r)} - \frac{\varepsilon}{4(1 + r)} = \frac{5\varepsilon}{12(1 + r)}
\]

and

\[
\frac{dp_2^*}{d\lambda} = p_2^*|_{\lambda=1^-} - p_2^*|_{\lambda=0^+} = \frac{4\varepsilon}{3(1 + r)} - \frac{3\varepsilon}{4(1 + r)} = \frac{7\varepsilon}{12(1 + r)}.
\]

The boundary conditions for \( p_1^*, p_2^* \) for \( p_0 \in \{\alpha + (\varepsilon/2), \alpha + \varepsilon\} \) yield

\[
p_1^* = \frac{5(p_0 - \alpha) - \varepsilon}{6(1 + r)} + \frac{\delta}{1 - h} \quad \text{and} \quad p_2^* = \frac{7(p_0 - \alpha) + \varepsilon}{6(1 + r)} + \frac{\delta}{1 - h},
\]

and consequently,

\[
\theta_{10}^* = \theta_{20}^* = \theta_{12}^* = \frac{p_0 - \alpha + \varepsilon}{6\varepsilon} = D_1^* = 1 - D_2^*,
\]

for all \( p_0 \in (\alpha + (\varepsilon/2), \alpha + \varepsilon) \).

By continuity, 1. through 5. together establish the results for the prices and the demands in Table 1 for all \( p_0 \geq 0 \). The hosts’ respective profits are obtained from these results by computing

\[
\pi_i^* = ((1 - h)p_i^* - \delta)D_i^*,
\]

for \( i \in \{1, 2\} \).

\[\square\]

**Proof of Corollary 1.** The comparative statics for host 1 obtain via direct differentiation of the results in Table 1 with respect to the excess price \( \bar{p}_0 = p_0 - \alpha \). For host 2, the comparative statics obtain directly for the price \( p_2^* \). For \( p_0 \in [\alpha + (\varepsilon/2), \alpha + \varepsilon] \) it is \( d\pi_2^*/d\bar{p}_0 \geq 0 \) if and only if \( \bar{p}_0 \leq 17\varepsilon/7 \), which on that segment is always satisfied. For all other segments, \( \pi_2^* \) is trivially non-decreasing in \( \bar{p}_0 \). Lastly, host 2’s demand \( D_2^* \) is first non-decreasing (for \( \bar{p}_0 < \varepsilon/2 \)) and then non-increasing (for \( \bar{p}_0 > \varepsilon/2 \)), which follows directly via differentiation of the corresponding expression in Table 1 with respect to \( \bar{p}_0 \).

\[\square\]
Proof of Corollary 2. From Theorem 1 it follows that $D^*_2 > 0$ if and only if $p_0 > \alpha - \varepsilon$, and $D^*_1 > 0$ if and only if $p_0 > \alpha$. □

Proof of Lemma 1. (i) Consider first the case where $f, m$ are fixed with $f > m > 0$ are fixed, and let $s$ be in the interval $[0, s^*]$. Then

$$L = f + \int_m^{f/(1+s)} ((1+s)x - f) \, dF(x),$$

and, using the Leibniz rule,

$$\frac{dL}{ds} = \int_m^{f/(1+s)} x \, dF(x) = \mu(m) - \mu(f/(1+s)) \geq 0.$$

This derivative vanishes if $s = s^* = (f/m) - 1$, which is a maximizer (yielding $L = f$), since $L$ is strictly concave in $s$ (i.e., $d^2L/ds^2 < 0$); this can be seen by differentiating the last expression using again the Leibniz rule. In the case where $m \geq f > 0$, the renter’s liability is maximized for $s = 0$ and

$$L = f + \int_m^{\max\{m,f/(1+s)\}} ((1+s)x - f) \, dF(x) = f.$$

(ii) For $s \geq s^*$, the renter’s liability is by part (i) equal to $L = f$, and therefore increasing in $f$. Thus, the smallest deposit for which $L = \mu_0$ must be $f = \mu_0$. □

Proof of Lemma 2. Since the renters’ cost types $\vartheta$ are heterogeneous, at an expected liability of $L = \mu_0$ (by R1) it is not guaranteed that the probability of damage $P = 1 - e^*(\mu_0, \vartheta)$ vanishes. Hence, R3 can in general only be satisfied if the expected capital-at-risk vanishes,

$$K = \mu(m) - L - \int_m^{\infty} H(x) \, dF(x) = \mu(m) - \mu_0 - \int_m^{\infty} H(x) \, dF(x) = 0.$$

The host’s expected agency cost $\delta$ consists of his expected contribution conditional on damage times the probability $P$ of damage. As discussed earlier, $P$ does not generally vanish. Hence, to ensure that R2 ($\delta = 0$) is always satisfied, necessarily

$$\int_m^{\infty} x \, dF(x) + \int_m^{\infty} H(x) \, dF(x) = \mu_0 - \mu(m) + (1 - F(m))E[H(\bar{x})|\bar{x} \geq m] = 0.$$

Hence R2 and R3 lead to the same restriction, namely that $(1 - F(m))E[H(\bar{x})|\bar{x} \geq m] = \mu(m) - \mu_0$. □

Proof of Theorem 2. Combining the conditions in Lemma 1 and Lemma 2 for satisfying the requirements R1–R3 leads to the conditions in Theorem 2. □

Proof of Lemma 3. Let the intermediary’s cost for enabling the housing transactions be $\Delta$. Given the commission structure $(h, r)$, the intermediary obtains the fraction $h$ from the host’s revenue and (roughly speaking) the fraction $r$ of the renter’s payment at nominal prices, resulting in the profit

$$\pi_I = (h + r)(p_1^* D_1^* + p_2^* D_2^*) - \Delta.$$
Since \((h + r) = (\rho - 1)(1 - h)\) and host \(i\)'s profit can be written in the form
\[
\pi_i^* = ((1 - h)p_i^* - \delta)D_i^* = (1 - h)p_i^*D_i^* - \delta D_i^* ,
\]
one obtains
\[
\pi_I = (1 - \rho) \sum_{i=1}^{2} ((1 - h)p_i^*D_i^* - \delta D_i^* + \delta D_i^*) - \Delta
\]
\[
= (1 - \rho) \sum_{i=1}^{2} (\pi_i^* + \delta D_i^*) - \Delta ,
\]
which completes the proof.

**Proof of Lemma 4.** By R2, it is \(\delta = 0\). Hence, the effective agency cost is \(\alpha = \varphi - \varphi_0\) and thus independent of \(\rho\). Table 1 then implies all of the conclusions.

**Proof of Lemma 5.** This result follows from Table 1 when setting \(\alpha = \varphi - \varphi_0\).

**Proof of Lemma 6.** The assertions follow directly from Theorem 1 and Lemma 4.

**Proof of Lemma 7.** Given that the intermediary’s insurance terms satisfy requirement R2, transacting directly increases the effective agency costs by
\[
\alpha_d - \alpha^* = \mu_0 - \varphi^* = \mu_0G(\mu_0) - \int_{0}^{\mu_0} \frac{\theta dG(\theta)}{2} + \mu_0^2 \int_{\mu_0}^{\infty} \frac{dG(\theta)}{2\theta} .
\]
Thus, on the one hand one obtains a lower bound,
\[
\alpha_d - \alpha^* \geq \left( \frac{\mu_0}{2} \right) G(\mu_0) + \int_{0}^{\mu_0} \frac{(\mu_0 - \vartheta) dG(\vartheta)}{2} \geq \left( \frac{\mu_0}{2} \right) G(\mu_0) ,
\]
and on the other hand,
\[
\alpha_d - \alpha^* = \mu_0 \left[ \int_{0}^{\mu_0} \left( 1 - \frac{\vartheta}{2} \right) dG(\vartheta) + \frac{1}{2} \int_{\mu_0}^{\infty} \left( \frac{\mu_0}{\vartheta} \right) dG(\vartheta) \right] \leq \mu_0 \left[ G(\mu_0) + \frac{1 - G(\mu_0)}{2} \right] ,
\]
which yields an upper bound. The lower bound is achieved if the cost-type distribution has all its mass at \(\vartheta = \mu_0\). The upper bound becomes arbitrarily close to binding when the cost type distribution is concentrated with equal point mass (of \(1/2\)) at the two points \(\vartheta_0 = \xi > 0\) and \(\vartheta_1 = \mu_0 + \xi\), and \(\xi \to 0^+\).

**Proof of Theorem 3.** As noted in Lemma 6 the intermediary’s profit is increasing in \(\rho\) when the hosts do not have any outside options to earn economic rents from (sub-)letting their private space. Let \(p_0 \geq 0\) be fixed. By Theorem 1 (and Table 1) host \(i\)'s equilibrium profit is such that
\[
\rho \pi_i^*|_{(\alpha^*, \rho)} = \pi_i^*|_{(\alpha^*, 1)} ,
\]
for all admissible \((\alpha, \rho)\). For both hosts’ direct-transaction profits to be smaller than under an intermediated transaction, the commission ratio needs to stay finite. More specifically,
\[
\pi_i^*|_{(\alpha^*, \rho)} \geq \pi_i^*|_{(\alpha_d, 1)} ,
\]
so that

\[ \rho \leq \frac{\pi^*_i(\alpha^*, 1)}{\pi^*_i(\alpha_d, 1)} = \frac{\pi^*_i(\varphi^* - \varphi_0, 1)}{\pi^*_i(\mu_0 - \varphi_0, 1)}, \]

for \( i \in \{1, 2\} \). The formula for the optimal commission ratio in \( \rho^* \) in the statement of the proposition takes the smaller of the two constraints (for \( i \in \{1, 2\} \)) as binding.\(^{25}\) The symmetric commission rates \( h^* = r^* \) follow immediately from the relation \( \rho^* = (1 + r^*)/(1 - h^*) \). \( \Box \)

**Proof of Corollary 4.** The matching-enhanced optimal commission ratio obtains from the conditions \( \hat{\beta} \hat{\pi}_1 \leq \beta^* \pi^*_i \) for both hosts \( i \in \{1, 2\} \). Note that for \( \hat{\beta} = \beta^* \) these conditions specialize to what was discussed at the beginning of the proof of Theorem 2. \( \Box \)

**Proof of Theorem 4.** The demand for hotels (as a function of \( p_0 \)) is \( D_0 = 1 - D^*_1 - D^*_2 \), where \( D^*_1 \) and \( D^*_2 \) are given in Theorem 1. In order for demand \( D_0 \) to remain positive, Table 1 implies that the revenue-maximizing price \( p^*_0 \) cannot exceed \( \alpha + (\varepsilon/2) \). On the other hand, \( p_0 \) cannot be smaller than \( \alpha - \varepsilon \), since otherwise a market for collaborative housing would not exist and therefore charging a smaller price could not maximize the hotels’ revenues. If \( \alpha - \varepsilon \leq p_0 < \alpha \), then by Table 1 hotel revenues,

\[ \pi_0 = \left( \frac{1}{2} + \frac{\alpha - p_0}{2\varepsilon} \right) p_0, \]

are increasing and strictly convex in \( p_0 \). Hence, we obtain that necessarily

\[ \alpha \leq p^*_0 < \alpha + (\varepsilon/2). \]

To determine the optimal price, note first that hotel revenues are

\[ \pi_0 = \left( \frac{1}{2} - \frac{p_0 - \alpha}{\varepsilon} \right) p_0, \]

for all \( p_0 \in [\alpha, \alpha + (\varepsilon/2)) \), which is strictly concave on that interval. The slope of the revenues,

\[ \frac{d\pi_0}{dp_0} = \frac{1}{2} + \frac{\alpha}{\varepsilon} - \frac{2p_0}{\varepsilon}, \]

is negative if and only if \( p_0 > (\alpha/2) + (\varepsilon/4) \). Hence whenever \( \alpha > \varepsilon/2 \), because of the concavity of \( \pi_0 \), for all \( p_0 \in [\alpha, \alpha + (\varepsilon/2)) \), it is \( p^*_0 = \alpha \). If \( \alpha \leq \varepsilon/2 \), the first-order condition \( d\pi_0/dp_0 = 0 \) yields that \( p^*_0 = (\alpha/2) + (\varepsilon/4) \). Combining the last two cases, we obtain that \( p^*_0 = \max\{\alpha, (\alpha/2) + (\varepsilon/4)\} \). Substituting this price into the expression for hotel demand yields

\[ D^*_0 = [1 - D^*_0 - D^*_2]|_{p_0 = p^*_0} = \min\{1, (1/2) + (\alpha/\varepsilon)\} = \min\left\{\frac{1}{2}, \frac{1}{4} + \frac{\alpha}{2\varepsilon}\right\}. \]

As a result, \( \pi^*_0 = p^*_0 D^*_0 = \min\{\alpha/2, ((\alpha/2) + (\varepsilon/4)^2)/\varepsilon\} \), concluding the proof. \( \Box \)

**Proof of Theorem 5.** The result follows by combining the conclusions from Theorem 1 and Theorem 4. \( \Box \)
Proof of Corollary 5. By Theorem 5, the aggregate industry profits, for $-\varepsilon/2 < \alpha < \varepsilon/2$, are

$$\Pi = \pi^*_I + \sum_{i=0}^{2} \pi^*_i = \frac{1}{16\varepsilon} \left[ (\varepsilon + 2\alpha^*)^2 + \left(\frac{\varepsilon}{2} - \alpha^*\right)^2 + \left(\frac{5\varepsilon}{2} - \alpha^*\right)^2 \right].$$

Hence, $\Pi$ is strictly convex there, and attains its (global) minimum at $\alpha^* = \varepsilon/6$. □
## Appendix B: Notation

Table 4: Summary of notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Domain/Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Renter’s cost of effort</td>
<td>$C(e, \vartheta) = \vartheta e^2/2$</td>
</tr>
<tr>
<td>$d$</td>
<td>Deductible</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Demand for housing type $i$</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$e$</td>
<td>Renter’s effort</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$f$</td>
<td>Fixed deposit</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$F$</td>
<td>Cumulative distribution function for types $\theta \in \Theta$</td>
<td>$F : \Theta \to [0, 1]$</td>
</tr>
<tr>
<td>$G$</td>
<td>Cumulative distribution function for types $\vartheta &gt; 0$</td>
<td>$G : \mathbb{R}_{++} \to [0, 1]$</td>
</tr>
<tr>
<td>$h$</td>
<td>Intermediary’s commission rate for host</td>
<td>$[0, 1)$</td>
</tr>
<tr>
<td>$H$</td>
<td>Host’s settlement as a function of the damage realization</td>
<td>$H : \mathbb{R}_{+} \to \mathbb{R}$</td>
</tr>
<tr>
<td>$i$</td>
<td>Housing type</td>
<td>${0, 1, 2}$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Affine transformation of the equilibrium price, $p_i^* = \frac{k_i e}{1+r} - \frac{\delta}{1-h}$</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$K$</td>
<td>Expected capital at risk for intermediary</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Expected liability for renter</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Minimum damage claim</td>
<td>$[0, \mu(m)]$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Price of housing type $i$</td>
<td>$\mathbb{R}_{+}$</td>
</tr>
<tr>
<td>$P$</td>
<td>Probability of positive damage, $P = 1 - z$</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Quality of housing type $i$</td>
<td>$\mathbb{R}_{++}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Intermediary’s commission rate for renter</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$R$</td>
<td>Insurance terms for renter, $R = (f, s)$</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$s$</td>
<td>Surcharge rate</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$u_R$</td>
<td>Renter’s utility</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$v$</td>
<td>Verification cost</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$x$</td>
<td>Damage realization</td>
<td>$\mathbb{R}_+$</td>
</tr>
<tr>
<td>$z$</td>
<td>Probability of zero damage, $z = 1 - P$</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Effective agency cost, $\alpha = (\varphi - \varphi_0) + \rho \delta$</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Matching probability with intermediary</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Expected agency cost for host</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Expected agency cost for intermediary, $\Delta = P \cdot K$</td>
<td>$\mathbb{R}$</td>
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<tr>
<td>$\varepsilon$</td>
<td>Quality dispersion</td>
<td>$(0, q_0)$</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Renter type (marginal cost of effort)</td>
<td>$\mathbb{R}_{++} = (0, \infty)$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Renter type (marginal utility for quality)</td>
<td>$\Theta$</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Type space for $\theta$</td>
<td>$\Theta = [0, 1]$</td>
</tr>
<tr>
<td>$\mu(m)$</td>
<td>$E[\tilde{x}</td>
<td>\tilde{x} \geq m] \cdot \text{Prob}(\tilde{x} \geq m) = \int_m^{\infty} x , dF(x)$</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Expected damage, $\mu_0 = \mu(0) = E[\tilde{x}] = \int_0^{\infty} x , dF(x)$</td>
<td>$\mathbb{R}_{+}$</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>Profit for housing type $i$ ($\pi_i$: for intermediary)</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Commission ratio, $\rho = (1 + r)/(1 - h)$</td>
<td>$[1, \infty)$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Renter’s expected agency cost ($\varphi_0$ when staying at a hotel)</td>
<td>$\mathbb{R}_{+}$</td>
</tr>
</tbody>
</table>