December 2002

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Recommended Citation

Weber, Thomas, "The Value of Shared Information Services" (2002). *ICIS 2002 Proceedings*. 19.
http://aisel.aisnet.org/icis2002/19

THE VALUE OF SHARED INFORMATION SERVICES

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Abstract

In this paper, we analyze the value of shared information services, both when they are operated by their members and when they are implemented by a monopoly provider. The value of information is defined as the compensating variation in price that makes a risk-averse agent indifferent between procuring an informative signal or not. We provide investment sharing rules that implement an individually rational Nash bargaining solution and compare this to the situation in which a nonscreening monopolist maximizes profits. We find that any efficient price schedule for information should take into account (1) the agent's confidence in the signal, (2) the project risk, (3) the agent's risk aversion, as well as (4) her wealth and the mean return if at least one of them is quite small. Interestingly, in a cooperative bargaining situation an agent's investment share may either increase or decrease when risk aversion goes up, depending on whether demand for information decreases faster than implicit bargaining power relative to the other agents or vice versa. We further show that even for CARA utilities, there are important wealth effects. Our results, including the definition of a critical Nash network size, *provide a benchmark for the value of information that is shared by a group of agents for use in their respective projects and not employed strategically against each other.*

Keywords: Information networks, shared investment, bargaining, value of information

1   INTRODUCTION

In the current global networked economy, information sharing has become an imperative. Still, the *value of the information* to be shared is often poorly defined and ill-assessed. Much less clear is *how much each beneficiary of such shared information ought to contribute* financially in order to at least offset the expenses for the creation and dissemination of the information. A widespread inability of dotcom's to profitably offer purely informational content over the Internet speaks for itself. This is particularly striking in the light of the often-repeated argument that "information is nonrival," i.e., due to its generally very low cost of reproduction, more than one agent is able to observe at the same time essentially the same realization of the same informative signal (e.g., the report of a news event or a current stockmarket price). From such an observation, each agent then draws conclusions relating to her respective project and adjusts her actions accordingly. Hence, depending on the differences in the projects the agents pursue, their levels of confidence in the observed signal in terms of providing decision-relevant information about an uncertain future payoff are generally varied.

One can think of a *shared information service* as an organization that is run by its members for the sole purpose of producing information that is being used in a *nonstrategic* but *excludable* way.[1] In particular, we assume that the individual members of

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[1]This means that the group using the information can prevent others from using this same information source, but each member does not use the information in a strategic way against another member. In the course of the paper, we relax the assumption of nonstrategic agent interaction somewhat by explicitly considering positive and negative network effects (cf. section 3.4).

the shared information service pursue *independent* projects; for instance, the involved agents comprise only a small portion of a common market, in which they do not have market power or they act in different fields. There are many concrete examples of nonstrategic shared information services, such as the Aviation Weather Forecast, the Insurance Services Office,[2] networked medical expert and knowledge-base systems, any type of member-operated information- or news-aggregation networks, and national agencies with a common set of objectives.[3] We allow for the possibility of customization or versioning of the information (Shapiro and Varian 1998): the precise observation of the signal may be different from agent to agent; moreover, even the signal itself may be different across users. For instance, queries to a common database input by different users corresponding to their individual needs yield different informative signals, even though the underlying information resource is shared.

The *economic incentives* for setting up and using shared information services stem from economies of scale and scope in assembling the raw information as well as economies of specialization in generating the informative signal. The fixed cost for the formation of an information service capable of producing signals of acceptable reliability (confidence) is often too high for any single firm, which calls for group action and a contractual bargaining solution to avoid the tragedy of the commons (Liebowitz and Margolis 1994). We thereby assume that the firms, once they decide to enter the bargaining negotiations, act rational in the sense of Harsanyi (1966) and that the coordination of the negotiations does not pose particular problems.

Pioneering contributions in strategic information sharing have been made by Clarke (1983) and Novshek and Sonnenschein (1982) for oligopolistic firms producing homogeneous goods (no incentive for information sharing), as well as Vives (1984) for a differentiated Cournot duopoly (incentive for information sharing exists if the goods are complements or weak substitutes). Raith (1996) unifies and generalizes this and most subsequent work on strategic information sharing with market interaction. As far as we know, there exists no prior treatment of information sharing in a *cooperative* context in the Economics literature. In the literature on information systems, some attention is paid to investments in cooperative interorganizational information systems. For instance Wang and Seidmann (1995) consider the investment in an electronic data interchange (EDI) system and show that a "supplier's adoption of EDI can generate a positive externality for the buyer and negative externalities for other suppliers" (p. 401). These externalities play a role in the adoption dynamics of interorganizational systems (Riggins et al. 1994). A question untouched by Wang and Seidman and other authors in this area is precisely *how* the investment for shared information systems should be distributed between the different users, based on the *decision value* of the information. Our goal here is to derive *efficient* investment sharing rules, based on cooperative bargaining. We thereby model explicitly the information to be shared as a signal that is imperfectly correlated with a decision maker's uncertain investment payoff. Nash (1953) founded the field of "cooperative bargaining theory," showing that, based on a number of axioms, a unique solution can be obtained that implements a Pareto-optimal allocation. A good overview is provided by Roth (1979). In particular, we focus on the following two research questions:

1. Given that a number of economic agents (e.g., firms, federal agencies, or individuals) may experience a benefit from setting up a shared information service, how much should each one contribute financially, and how much is the overall venture worth? An answer to this question can serve as an upper bound for what could be charged by, say, a monopolist information provider to that same group of agents, and we will compare both cases. In the first case, firms are assumed to cooperatively bargain about the share of the implementation cost; in the second case, a monopolist information provider maximizes his profits if he decides to enter. Naturally, the cooperative bargaining solution outperforms the monopolistic solution, if, of course, its efficiency in actually providing and disseminating the information is the same as for the monopolist.

2. What are the conditions for existence of shared information services? We provide explicit expressions for the *critical mass* of users that is necessary to form an information network, both in the cooperative and the monopolistic situation. We introduce the notion of a *critical Nash network size*, which counts the smallest subset of agents that can create a shared information network subject to each agent being at least as well off as without the information.

The outline of this paper is as follows. In the next section, we give an exact definition of the value of information in terms of the *compensating variation* of the agent's utility that makes her indifferent between acquiring the informative signal or not. A definition of the value of information as compensating variation was first introduced by Kihlstrom (1974) in a product-consumption setting and by Treich (1997) for a standard portfolio investment problem. Although not widely spread, this definition

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[2]The Insurance Services Office (http://www.iso.com) acts as a supplier of statistical, actuarial, underwriting, and claims information to the property and casualty insurance industry.

[3]As a current example emphasizing the need for information sharing between federal agencies, consider Informationweek's call that "[s]haring data is key to antiterrorism efforts" (Stahl 2001) in the wake of the September 11 attack on the World Trade Center.

is superior to what is commonly used in the Economics literature, since it takes full account of all wealth effects. We also compare the general effect of the number of participants on the cost of a shared information service, depending on its cost characteristics. In section 3, we determine what each participant should pay as a result of a cooperative bargaining process and how the cost for setting up a shared information service varies with the size of its membership body and associated network externalities. Subsequently we compare this to what a monopolist can charge in terms of an *ultimatum offer*, and we discuss the sources of inefficiency. Section 5 concludes the paper with directions for future research.

2   THE VALUE OF INFORMATION

In the traditional Economics literature, the value of information to a particular agent is assessed, depending on the agent's risk posture, using two different methods (Athey 2000; Marschak and Miyasawa 1968; Raiffa and Schlaifer 1961): (1) for risk-neutral agents, the information value is determined as the difference between the optimal payoff with information and the optimal "default" (or no-information) payoff; (2) for risk-averse agents, the cost of an informative signal is typically modeled as a "utility cost," which is subtracted from the agent's full-information utility, *as if* the amount for the information was paid out of a separate budget. These two common approaches to computing the value of information typically neglect the effect the payment itself has on the agent's budget set and thereby on constraining her feasible actions. Such wealth effects are, as should be expected, of considerable relevance at least in a portion of the wealth domain, irrespective of the assumed risk-aversion characteristics. Also, wealth effects *cannot* generally be eliminated by an "appropriate" choice of a class of utility functions possessing multiplicative or additive separability properties (such as those of constant absolute risk aversion [CARA] as is often claimed). We demonstrate below, using CARA utilities, that wealth effects are significant when the agent's risk aversion, her endowed wealth, or her project risk is relatively small or her confidence in the information is extremely high.

As a consequence, the value of information needs to be carefully defined, taking into account the exact contractual terms governing the transfer of information from the seller to the agent. We define the monetary value or willingness to pay (WTP) for information in terms of Hicks' (1939) measure of welfare change, as the *compensating variation* that moves the agent's expected utility with information to the same level as without information. To simplify the analytical treatment and segregate the "pure" value of information from any specific decision situation as far as possible, we will assume here that a noncontingent fixed payment p for an informative signal $\tilde{s}$ (correlated with the exogenous payoff-relevant random event $\tilde{x}$ is to be made *before* any uncertainty is resolved, and that neither the agent's actions nor her ex-post wealth are contractible.

More specifically, we consider an agent that has the option to invest a portion $a \in [0,1]$ of her endowed wealth $w > 0$ into a project of risky return $\tilde{x}$. The agent's preferences are supposed to be representable by a smooth utility function $u : R \rightarrow R$ that is strictly increasing and concave. The agent's optimal "default" or "no-information" strategy,

$$\hat{a} \in \arg\max_{a \in [0,1]} E_x \left[u\left(w(1 + a\tilde{x}) \right) \right], \tag{1}$$

exists by virtue of u being continuous over the compact action set $[0,1]$ (applying Weierstrass' Theorem) and is unique, since u is concave and strictly monotonic (i.e., one-to-one). The agent can now improve her *default utility*,

$$d(w) = E_x \left[u\left(w(1 + \hat{a}\,\tilde{x}) \right) \right], \tag{2}$$

from this project at least weakly by observing a random signal $\tilde{s}$, correlated with $\tilde{x}$, before choosing her optimal state-contingent action,

$$\hat{a}_s \in \arg\max_{a \in [0,1]} E_x \left[\left(u\left((w - p)(1 + a\tilde{x}) \right) \right) | s \right], \tag{3}$$

where p represents the price of the signal. The agent's expected utility *with* information, v, as a function of her investable wealth $w - p$ is therefore $v(w - p) = E_s \left[E_x \left[\left(u\left((w - p)(1 + \hat{a}_s\tilde{x}) \right) \right) | \tilde{s} \right] \right]$. Hence we *define the value of information* for this agent as the compensating variation $\overline{p}$ that makes her indifferent between observing the signal or not,

$$v(w - \overline{p}) = d(w) \tag{4}$$

In other words, $\overline{p}$ is the highest amount that this agent would be willing to pay for obtaining the right to observe a realization of the signal $\tilde{s}$ prior to choosing her optimal action (see Figure 1).
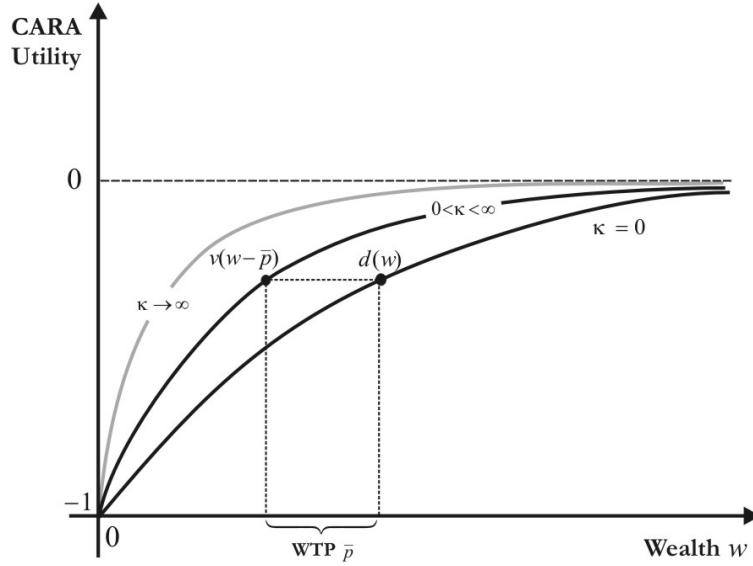

Figure 1. WTP Is Defined as the Compensating Variation $\overline{p}$ in (4)

3 COOPERATIVE INFORMATION SERVICES

We consider now the situation where $N+1$ agents bargain about their respective contributions, $t_0,...,t_N$, to a fixed net present investment $F(N)$ that is needed to set up a shared member-operated information service.[4] This shared service would allow each agent to observe a signal correlated with her respective uncertain future payoff. We examine at first the value of the information service to a single user with CARA utility. Then we analyze the cost structure of a shared information service, which may give rise to both a "critical mass" and a maximal "carrying capacity," imposing a lower and (possibly) upper bound on the size of the information network. At last we provide a characterization of the Nash bargaining solution and examine the resulting *efficient* cost sharing and membership policies.

3.1 Value of the Information Service to a Single User

Let us focus on the case, where a given user has a CARA utility for wealth w of the form $u(w) = -\exp(-\rho w)$ and faces the decision to allocate a fraction $a \in [0,1]$ of her wealth into a risky project of return $\tilde{x}$. This return is assumed to be normally distributed with mean μ and variance σ^2, and her expected utility, given that she invests aw and retains $(1-a)w$, is

$$E[u(w(1+a\tilde{x}))] = -\int_{-\infty}^{\infty} \exp\left[-\rho w(1+a\xi) - \frac{(\xi-m)^2}{2s^2}\right]\frac{d\xi}{\sqrt{2\pi}\sigma}$$

$$= -\exp\left[-\left(\rho w(1+a\mu) - \frac{(a\sigma\rho w)^2}{2}\right)\right],$$

(5)

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[4]To simplify the analysis, we assume that this investment has to be fully committed to *ex ante*, even though in reality it could be spread over time, realizing a real option value of flexibility through "chunkification" of the investment.

so that we obtain the unique no-information maximizer[5]

$$\hat{a} = \left[\frac{\mu}{\sigma^2 \rho w} \right]_{[0,1]},$$ (6)

and consequently for the expected utility of the agent's default option

$$\hat{a} = \begin{cases} -\exp\left[-\left(\rho w + \mu^2 /\left(2\sigma^2\right)\right)\right], & \text{if } w \geq \mu /\left(\sigma^2 \rho\right), \\ -\exp\left[-\left(\rho w(1+\mu) - (\sigma\rho w)^2 / 2\right)\right], & \text{otherwise.} \end{cases}$$ (7)

Now the agent observes the signal $\tilde{s} = \tilde{x} + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is uncorrelated with $\tilde{x}$ and normally distributed with mean zero and variance $1/\kappa$. The positive constant κ, corresponding to the inverse of the variance of $\tilde{\varepsilon}$, represents in Bayesian terms the confidence (or precision) with which the signal informs the decision-maker about the realization of the exogenous random variable $\tilde{x}$. Using Bayes' rule, one can compute the probability density of $\tilde{x} |s,$[6] which is again normally distributed with mean

$$E[\tilde{x} \mid s] = \left(\frac{\kappa\sigma^2}{1 + \kappa\sigma^2} \right)s + \left(\frac{1}{1 + \kappa\sigma^2} \right)\mu,$$ (8)

and variance

$$\text{Var}(\tilde{x} \mid s) = \left(\kappa + \frac{1}{\sigma^2} \right)^{-1}.$$ (9)

The posterior mean $E[\tilde{x} \mid s]$ is a weighted average of signal realization s and prior mean μ. The relative weight of s is increasing in the confidence of the signal, κ. The posterior expected utility is then

$$E[(u(w(1+\tilde{x}a))) \mid s] = -\sqrt{\frac{1+\kappa\sigma^2}{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\rho w(1+a\xi) - \frac{(1+\kappa\sigma^2)(\xi - E[\tilde{x} \mid s])^2}{2\sigma^2} \right)d\xi,$$

which can be maximized with respect to a as before in (5) to yield the optimal state-contingent policy

$$\hat{a}_s = \left[\frac{\kappa\sigma^2 s + \mu}{\sigma^2 \rho w} \right]_{[0,1]}.$$ (10)

Note that $\hat{a}_s$ adjusts the default strategy $\hat{a}$ in (6) with the additive state-contingent term $\kappa s /(\rho w)$, so that the investment intensity is increased, iff the observation s is positive.[7] Hence the investor only reduces her investment in the risky asset if she receives unambiguous "bad" news, i.e., iff $s < 0$, which has a relatively low probability of occurrence. The correction of the default action â is proportional to the investor's confidence in the signal, κ, and inversely proportional to her Arrow-Pratt risk-aversion parameter ρ. As her risk aversion increases, the investor gets more reluctant to adjust her default action, and consequently the

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[5]The $[\cdot]_{[0,1]}$ operator is defined for any $\alpha \in \mathbb{R}$ as $[\alpha]_{[0,1]} = \max\{0, \min\{1, a\}\} = (\min\{1, a\})_+$, truncating the value of α to the admissible interval [0,1].

[6]We denote by s a particular realization of the signal $\tilde{s}$.

[7]This adjustment is additive but *not* linear, since it occurs *inside* the $[\cdot]_{[0,1]}$ operator.

information contained in the observation loses value.[8] The agent's indirect utility contingent on the observation s is therefore (using (10))

$$v_s(w) = \begin{cases} -\exp\left[-\left(\rho w\left(1 + \dfrac{\kappa\sigma^2 s + \mu}{1 + \kappa\sigma^2}\right) - \dfrac{(\sigma\rho w)^2}{2(1 + \kappa\sigma^2)}\right)\right], & \text{if } s \geq \bar{s}, \\[4mm] -\exp\left[-\left(\rho w + \dfrac{(\kappa\sigma^2 s + \mu)^2}{2\sigma^2(1 + \kappa\sigma^2)}\right)\right], & \text{if } s \in [\underline{s}, \bar{s}], \\[4mm] -\exp[-\rho w], & \text{if } s \leq \underline{s}, \end{cases}$$

where we have set $\underline{s} = -\mu/(\kappa\sigma^2)$ and $\bar{s}(w) = (\sigma^2\rho w - \mu)/(\kappa\sigma^2)$ as the two critical observations, above and below which the agent invests respectively all or none of her wealth in the risky asset. In expectation, the agent obtains

$$E_s v_s(w) = -e^{-\rho w}\left(F(\underline{s}) + \int_{\underline{s}}^{\bar{s}} f(\varsigma)\exp\left[-\frac{(\kappa\sigma^2\varsigma + \mu)^2}{2\sigma^2(1 + \kappa\sigma^2)}\right]d\varsigma + \exp\left[-\frac{\mu + \sigma^2\rho w/2}{1 + \kappa\sigma^2}\right]\int_{\bar{s}}^{\infty} f(\varsigma)\exp\left[-\frac{\kappa\sigma^2\rho w\varsigma}{1 + \kappa\sigma^2}\right]d\varsigma\right),$$

where

$$f(s) = \frac{\exp\left[\dfrac{\kappa(s - \mu)^2}{2(1 + \kappa\sigma^2)}\right]}{\sqrt{2\pi(\sigma^2 + 1/\kappa)}}$$

is the probability density of the signal $\tilde{s}$ based on (8)–(9), and F is the associated cumulative distribution function. This yields

$$v(w) = -e^{-\rho w}\left\{F(\underline{s}) + v\exp\left(-\frac{\mu^2}{2\sigma^2}\right)\left[F\left(\frac{\bar{s}}{\lambda} + \mu\right) - F\left(\frac{\underline{s}}{\lambda} + \mu\right)\right] + \exp\left(-\frac{\mu + \sigma^2\rho w/2}{1 + \kappa\sigma^2} - \frac{\sigma^2\rho w(\mu + \sigma^2\rho w/2)}{\sigma^2 + 1/\kappa}\right)\left[1 - F(\bar{s} + \sigma^2\rho w)\right]\right\}, \quad (11)$$

where we have set $\lambda = \sqrt{1 + \kappa\sigma^2}$ and $v(w) = E_s[v_s(w)]$. Provided that both μ and w are large enough, there is a very *simple approximate solution*, since then the interval [,] comprises most of the probability mass. When concentrating on observations s plus or minus three standard deviations, $\pm 3\sqrt{\sigma^2 + 1/\kappa}$, around the prior mean μ, more than 99 percent of the probability mass is concentrated in the critical interval $[\underline{s}, \bar{s}]$ and we can approximate,

$$v(w) \approx \frac{d(w)}{\sqrt{1 + \kappa\sigma^2}}, \quad (12)$$

and from there compute the agent's WTP, $\bar{p}$, for the signal by comparing $v(w - \bar{p})$ to her utility of the no-information strategy, $d(w)$, which yields

$$\bar{p} \approx \frac{\log(1 + \kappa\sigma^2)}{2\rho}. \quad (13)$$

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[8]In fact, the signal is most valuable to a risk-neutral investor who aggressively adjusts the investment policy to $\hat{a}_s \in \{0,1\}$.

A *sufficient condition* for the validity of this approximation is

$$\mu \in \left[\frac{3\sigma^2}{\sqrt{1+\kappa\sigma^2}}, \frac{\rho w - 3\kappa\sqrt{\sigma^2 + 1/\kappa}}{1+\kappa} \right] \tag{14}$$

guaranteeing an error in probability weight of less than one percent. This approximate solution is *not* valid if (1) endowed wealth w is small, (2) signal confidence κ is very high, or (3) the agent is almost risk-neutral (ρ close to zero), since then $\underline{s}$ and $\overline{s}$ will be close, i.e., a small variation in the observation within the interval [$\underline{s}$, $\overline{s}$] induces a large change in the investment behavior. For such cases, the exact computation of the expected utility with information needs to take into account all terms as determined in (11)[9] (see Figure 2).
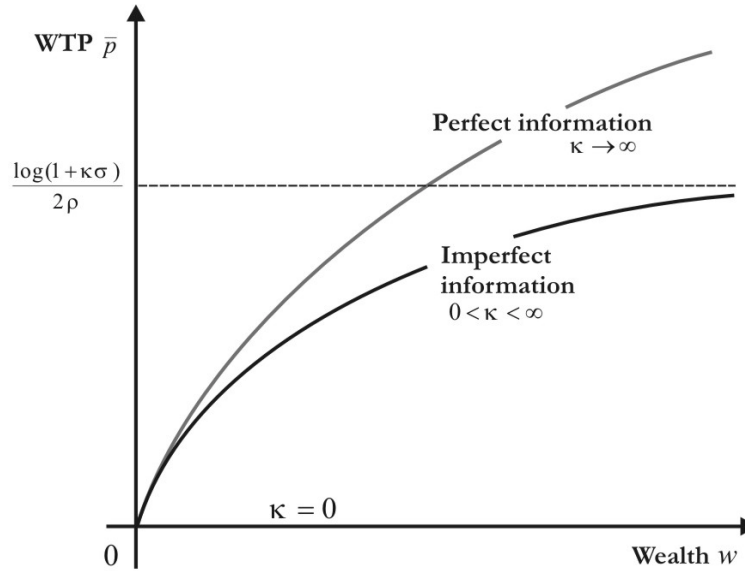

Figure 2. The WTP $\overline{p}$ Can Be Approximated for *w* Large Enough by (13)
(Not Possible for Perfect Information)

Special Case: Perfect Information ($\kappa \to \infty$). Then $\underline{s} = \overline{s} = 0$ and the approximation (12) is *not* valid. From (11) we obtain the value of perfect information,

$$\left(v(w)\right)_{\kappa\to\infty} = -e^{-\rho w}\left[F(0) + \left(1 - F(\sigma^2\rho w)\right)\exp\left(-\rho w\mu - (\sigma\rho w)^2/2\right)\right], \tag{15}$$

which defines an *upper bound for the value of any signal*. It depends strongly on the wealth of the investor and is naturally zero for $w=0$. No information service can rationally charge more for its information than is implied by (15) combined with (4).

3.2 Cost of Shared Information Provision

We suppose that the cost of the information network $F(N)$ contains a *fixed* portion F_0, independent of the number of users participating in the shared information service, and a *variable* component $c(N)$ that is strictly increasing in N.[10] If we further

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[9]Note that in this approximation we have effectively eliminated the dependence on the agent's wealth w in the expression for her WTP (13), but under condition (14) this approximation is justified.

[10]The cost function $c : \mathrm{R} \to \mathrm{R}$ is assumed to be smooth with $c' > 0$ and $c(0) = 0$.

assume for simplicity that the users of the information service are *identical* and each have a WTP of $\bar{p} < F_0$, as determined in (4), then for a *rational* creation of the service we need

$$F(N) = F_0 + c(N) \leq (N+1)\bar{p} \tag{16}$$

If the variable component is linear, $c = \gamma N$ for some nonnegative constant $\gamma < \bar{p}$, then the *critical mass* of users N_c can be computed explicitly,[11]

$$N_c = \frac{F_0 - \bar{p}}{\bar{p} - \gamma}. \tag{17}$$

If the variable cost component c is strictly concave in N (i.e., $c'' < 0$, due, for instance, to economies of scale), then there always exists a finite critical mass. The critical mass decreases if $\bar{p}$ is increasing in N due to positive network externalities (cf. footnote 11).[12] On the other hand, if c contains convexities (i.e., $c'' > 0$ somewhere), these may be balanced by the growth of $\bar{p}(N)$ in N so that even with high fixed cost and increasing complexity in maintaining a shared information network, there may still exist a finite critical mass above which the investment in this shared venture is viable. Figure 3 provides an overview. A maximum carrying capacity can arise in the case of convex costs ($c'' > 0$) if $c > \bar{p}$ from some N on, since then each new member of the information service is willing to contribute less than the community has to pay for adding her.
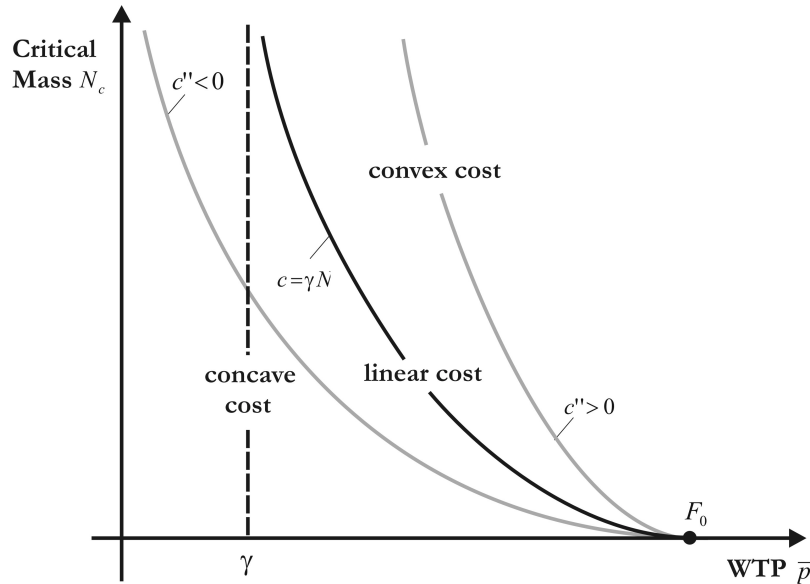

**Figure 3. Critical Mass of the Shared Information Network as a Function of Agents'
Willingness to Pay and Different Marginal Cost Functions**

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[11]If $\gamma > \bar{p}$, then $N_c = \infty$, i.e., the shared information service stands no chance of creation, unless in some way (through additional assumptions) $\bar{p}$ increases in N. This could be the case as additional members create *positive* network externalities on other members by contributing information valuable to everyone.

[12]There may also exist *negative* network externalities, where the average confidence in the signal decreases as more and more agents join the shared information network, because of aggregation effects that decrease the customization value of the signal or because of *strategic* interaction between agents.

3.3 The Bargaining Problem

In this section, we derive an efficient apportioning of the cost of the shared information service among its members. Assume that there are $N+1$ agents that are willing to bargain about their contributions $t_0, t_1, ..., t_N$, where

$$\sum_{i=0}^{N} t_i = F(N). \tag{18}$$

The bargaining solution is chosen here as the benchmark of what can be achieved using a coordinated approach to the shared allocation of costs to the members of a shared information network. Harsanyi (1966) notes that such "[a] cooperative game can always be replaced by a noncooperative game if we incorporate promises and threats in the strategies available to the players" (p. 616). The threats in the cooperative negotiation game correspond to the players' ability to interrupt the bargaining process by forcing the default outcome, an extreme action that is not generally individually rational (cf. Roth 1977) given that entry into the bargaining round was free.[13] More specifically, we assume that each agent $i \in I = \{0,1,..., N\}$ has an endowed wealth of $w_i > 0$, is of constant absolute risk aversion with coefficient $\rho_i > 0$, and pursues a risky project of normal return $\tilde{x}_i$ with mean μ_i and variance σ_i^2. Furthermore, she has confidence $\kappa_i > 0$ in the personalized signal $\tilde{s}_i = \tilde{x}_i + \tilde{\varepsilon}_i$, where the error term $\tilde{\varepsilon}_i$ has mean zero and is statistically independent from the project return $\tilde{x}_i$.[14] If the agent disagrees with a proposed outcome, she can force the default outcome $d_i(w_i) \in \mathrm{R}^{N+1}$ as previously determined in (2) and substituting agent i's parameters. The compact, convex and nonempty set $S \subset \mathrm{R}^{N+1}$ contains all of the feasible utility payoff vectors for the agents in I.[15] Nash (1953) defined a solution to the bargaining game as a function $b: S \to \mathrm{R}^{N+1}$, such that $b(S,\boldsymbol{d}) \in S$ for all possible bargaining situations $(S,\boldsymbol{d})$, where $\boldsymbol{d}=(d_0,...,d_N)$. Given the four axioms (cf. Roth 1979, pp. 6–8) (1) *Indepedence of Equivalent Utility Representations*, (2) *Symmetry*, (3) *Independence of Irrelevant Alternatives*, and (4) *Pareto Optimality*,[16] there exists a unique solution b, such that[17]

$$\mathbf{v} = \arg\max_{\hat{\mathbf{v}} \geq \mathbf{d}} \prod_{i=0}^{N} (\hat{v}_i - d_i), \tag{19}$$

where $\mathbf{v} = (v_0, v_1, ..., v_N)$ and $v_i = v_i(w_i - t_i) = E_s[v_{is}(w_i - t_i)]$ is defined in the same manner as the expected utility for the individual agent in (11). Note that the bargaining set S is defined by (18).

Proposition 1 (COOPERATIVE ALLOCATION) *Consider the bargaining problem* (S,**d**).

(i)   *The solution to (18)–(19) is uniquely characterized by*

$$\frac{v'_j(w_j - t_j)}{v_j(w_j - t_j) - d_j(w_j)} = \frac{v'_0(w_0 - t_0)}{v_0(w_0 - t_0) - d_0(w_0)}, \quad j = 1,..., N, \tag{20}$$

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[13]It may appear unrealistic that any of the players could carry so much weight as to stop the formation of the shared information service. However, the preceding remarks should clarify that this does not generally happen, given the assumption that the players have agreed to bargain about their contribution in the first place. Individual rationality will generally prevent firms from carrying out their threat, which is a mere modeling device to support the efficiency achieved by the Nash bargaining solution (cf. also footnote 16). If a particular member indeed decided to exit the N-firm bargaining, negotiations would subsequently resume with N-1 firms.

[14]Each agent $i \in I$ is only observing her own customized signal $\tilde{s}_i$.

[15]We further assume that all of the projects' payoffs are uncorrelated and that the agents do not form coalitions.

[16]The Pareto-optimality axiom can be relaxed to mere "individual rationality" as shown by Roth (1977). In that case, the solution comprises the Nash solution *and* the default option *d*.

[17]Nash (1953) shows this only for two players. The generalization to N players (without coalitions) is trivial as Roth (1979) points out.

together with

$$t_0 = F(N) - \sum_{i=1}^{N} t_i. \tag{21}$$

(ii) If all agents have the same risk aversion ρ and the approximation (12)–(13) holds for all $i \in I$, , then the optimal cost sharing for the common information service is given by

$$t_j(N) = \frac{1}{N}\left(F(N) - \frac{\log Q}{\rho}\right) + \frac{1}{\rho}\log\left(\frac{q_j}{q_0}\right), \quad j = 1,...,N, \tag{22}$$

and $t_0(N) = (\rho F(N) - \log Q)/(\rho N)$, where $Q = q_1 q_2 \cdots q_N / q_0^N$ and $q_i = \sqrt{1 + \kappa_i \sigma_i^2}$ for all $i \in I$.

(iii) If the risk aversion ρ_j of agent $j \in I$ increases, then—all else being equal—this agent's share t_j increases, iff

$$\frac{\partial \log(v_j(w_j - t_j) - d_j(w_j))}{\partial \rho_j} < \frac{\partial \log v_j'(w_j - t_j)}{\partial \rho_j}. \tag{23}$$

Proposition 1 characterizes the efficient Nash bargaining solution both generally (in terms of a first-order condition), and explicitly for an important special case where all the agents' risk-aversion parameters ρ_i are identical and the approximation condition (14) holds. For differing risk-aversion parameters, a general result by Kihlstrom et al. (1981) states that bargaining performance decreases with increasing risk aversion. However, we find that the agent's absolute monetary contribution to the shared information service does *not* need to increase as a consequence. Indeed, the situation of information procurement produces a countervailing effect, since the value of information generally *decreases* with increasing risk aversion.[18] The *net* change of an agent's contribution is positive if and only if—as a response to an increase of her risk aversion—her demand for information decreases faster than her bargaining power relative to the other agents.[19]

3.4 Effect of Network Externalities

Externalities in information networks do occur as a result of (possibly mediated) interactions of different network members. These externalities can have both positive and negative effects on the members' perceived utility. Riggins et al. (1994) examine the growth of interorganizational systems which can be both hampered and facilitated by differences in perceived network externalities across the different users. *Positive* externalities may be a result of the benefit that comes from sharing information in a nonstrategic way insofar as it increases the signal quality of individual users. On the other hand, *negative* externalities can be a consequence of high user traffic and network capacity limitations that make it harder to retrieve usable signals and thus indirectly reduce the confidence in these signals. Negative externalities may also result from a *strategic* use of information by different members of the same industry against each other.[20] To incorporate such effects in our model, we assume that the firms' respective confidence κ_i in an informative signal retrieved from the shared information service depends on the number of users

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[18]For instance, a risk-averse agent will respond less aggressively to a (imperfect) signal than a risk-neutral investor, and thus is willing to pay less for the information.

[19]It turns out that an agent's monetary contribution to the shared venture is U-shaped in her wealth *w*. For low *w*'s her payment is decreasing whereas for large *w*'s it is increasing. This is due to the fact that for large *w*'s the agent's WTP for the signal is almost constant (since then the approximation condition (14) holds) and the agent's decrease in bargaining power outweighs her decrease in WTP. For low *w*'s the converse is true. "Bargaining power" is meant here in the *implicit* sense of Kihlstrom et al. (1981), who show—as pointed out in the main text—that agents with lower risk aversions obtain a larger surplus in cooperative bargaining situations and therefore have a higher implicit bargaining power. For an *explicit* modeling of bargaining power, see footnote 26.

[20]Such "strategic externalities" are not necessarily negative. They tend to be positive—increasing the incentive of industry participants to share information—when firms' best responses to their actions are strategic complements (cf. Raith 1996; Vives 1984).

N of the information service. For simplicity we thereby assume that $\kappa(N) \geq 0$ is quasiconcave in N or in other words that $\kappa(N)$ is either monotonic or single-peaked in N. For any given network size N, the results of the previous sections remain unaffected; however, incorporating (possibly strategic) network effects may have a dramatic effect on the critical mass and maximum carrying capacity of the shared information network. Under the assumption that (14) holds, we obtain, by combining (13) with (16), for the critical mass under network externalities:[21]

$$N_c^e = \min\left\{ N \geq 1 : F_0 + c(N) \leq \frac{(N+1)\log\left(1 + \kappa(N)\sigma^2\right)}{2\rho} \right\}. \tag{24}$$

For the simple case of a linear network cost expansion path with $c(N) = \gamma N$, one can directly see that any strict increase $\hat{\kappa} > \kappa$ yields a (weak) decrease in the critical mass, as the right-hand side of (17) is strictly decreasing in $\overline{p}$ which in turn is strictly increasing in κ. This naturally generalizes to any network cost function considered in the previous section. For convex c, the quasiconcavity of $\kappa(\cdot)$ ensures that the critical mass in (24) is unique.

3.5 Efficient Cost Sharing and Membership Policies

The Nash bargaining solution is efficient provided that all of the bargaining members need to *either* agree on a common resource allocation *or* force the disagreement outcome *d* (also sometimes appropriately referred to as the "threat point," cf. footnote 13). A sequential enlargement of the membership body does not fulfill this requirement; and naturally the existing members wish to enlarge their user base such that each of their contributions are nonincreasing, which may result in the exclusion of certain agents, if they did not belong to the original network.

4 MONOPOLIST INFORMATION SERVICES

4.1 Pricing and Network Size

Consider now a monopolist that decides about entering a market to sell customized information to a potential client base of M agents. We assume that each agent $m \in \{1,...,M\}$ disposes about enough wealth and is sufficiently risk averse, so that (14) holds and each agent's reservation price is given by (13). Agents are heterogeneous in their risk aversions, $\rho_1, \rho_2,..., \rho_M$, where $\rho_m = m\overline{\rho}/M$ and $\overline{\rho}$ is a positive constant.[22] Corresponding to these different risk aversions are the reservation prices $p_1 < p_2 < \cdots < p_M$, where by (13) and by the definition of ρ_m we have

$$p_m = \frac{\log\left(1 + \kappa_m \sigma_m^2\right)}{2\rho_m} = \frac{M\log(1 + k)}{2m\overline{\rho}}.$$

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[21]To keep the formalism simple, we have here implicitly assumed that the confidences and perceived network externalities are uniform across all firms. Of course, there is no reason for this to be true in any particular situation. In addition to a more complicated form of the expressions for the critical mass due to aggregation of the heterogeneous firms with different quasiconcave confidences $\kappa_i(N), i = 0,..., N$, the fundamental difference is that the feasibility of the shared investment and the surplus of the different players will depend on exactly *who* is bargaining and not just on the number of participants. Firm heterogeneity in perceived network effects generally results in nonmonotonic surplus in the number of participants. To deal with full firm heterogeneity in a proper fashion, it is necessary to endogenize firm entry into the cooperative bargaining situation, an extension that is for space constraints beyond the scope of this paper. It naturally involves determining the fulfilled-expectations equilibria of which there may be several so that coordination and preannouncements may become important dimensions of the problem (cf. Farrell and Saloner 1986).

[22]In other words, we assume here that there is an upper bound for the risk aversion, and that the N agents are uniformly distributed on the interval $[\overline{\rho}/M, \overline{\rho}]$.

Thereby we have assumed that the customization of the information is such that the "risk-adjusted confidence" $\kappa_m \sigma_m^2$ is constant and equal to k for all m.[23] If (for simplicity) the marginal cost of providing one additional agent with the customized information is *constant* (i.e., $c(N) = \gamma N$), we can provide closed-form solutions for the monopolist's optimal price, optimal profits, and market penetration.

Proposition 2 (MONOPOLISTIC ALLOCATION) *Assume that the monopolist rationally enters the market. Then the optimal price is* $p* = 2\gamma$ *and optimal profits are*

$$\Pi* = \frac{M \log(1+k)}{4\bar{\rho}} - F_0, \tag{25}$$

with a market penetration (and resulting network size) of

$$m* = M \min\left\{1, \frac{\log(1+k)}{4\gamma\bar{\rho}}\right\}. \tag{26}$$

The monopolist rationally enters the market iff the critical mass M_c of *potential* users is reached ($M \geq M_c$), so that optimal profits $\Pi*$ in (25) are nonnegative, where $M_c = 4\bar{\rho}F_0 / \log(1+k)$. If the monopolist enters the market (the likelihood of which is inversely proportional to the dispersion $\bar{\rho}$ in the agents' risk aversions), then he in general tends to *restrict* the size of the information network to maximize his profits. Those agents with high risk aversions also have a low WTP and are therefore excluded from the use of the network. Especially when fixed costs F_0 are high, this can lead to market failure, i.e., the monopolist does not enter the market, even though a bargaining solution (which cross-subsidizes the low-value agents) exists, which is the case whenever M is larger than the minimal network size, which we term the critical Nash network size.[24]

Proposition 3 (WELFARE DIFFERENTIALS AND CRITICAL MASS) *Assume that the approximation condition (14) is satisfied for each agent.*

(i) *The total welfare achieved in a non-discriminating monopolistic environment is*[25]

$$W = \begin{cases} \dfrac{M \log(1+k)}{2\bar{\rho}}\left[\dfrac{1}{2} + \displaystyle\sum_{m=1}^{m*} \dfrac{1}{m}\right] - 2\gamma m * - F_0, & \text{if } M \geq M_c, \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

whereas the welfare achieved in the Pareto-optimal Nash bargaining solution is given by

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[23]In that sense the information is horizontally versioned in such a way that the product of confidence in the signal and variance of the risky project is the same across all agents (i.e., if a project is more risky, then the agent can be less sure about the relevance of the signal realization, which corresponds to the notion that the monopolist has "constant" marginal cost to satisfy each additional user of the information network).

[24]The *critical Nash network size* is simply the smallest subset of a given universe of agents that can individually rationally set up an information network (of a given cost structure).

[25]The finite sum can also be expressed in "closed form," using the relation $\sum_{k=1}^{K} 1/k = \Psi(K+1) + \gamma_e$, where Ψ is the Digamma function, related to the standard Gamma function Γ by $\Psi(x) = d \log \Gamma(x)/dx$, and $\gamma_e \approx .5772$ is Euler's constant.

$$\overline{W} = \begin{cases} \dfrac{N\log(1+k)}{2\overline{\rho}}\left(\displaystyle\sum_{n=1}^{N}\dfrac{1}{n}\right) - \gamma N - F_0, & \text{if } N \geq N_c, \\ 0, & \text{otherwise,} \end{cases} \tag{28}$$

(ii) The critical number of potential users of the monopoly shared information service, M_c, and the critical Nash network size N_c are related as follows:

$$N_c = \frac{2\overline{\rho}F_0}{\log(1+k)\left(\displaystyle\sum_{n=1}^{N_c}1/n\right) - 2\overline{\rho}\gamma} = \frac{M_c}{2\left(\displaystyle\sum_{n=1}^{N_c}1/n\right) - \gamma\overline{\rho}/\log(1+k)}. \tag{29}$$

(iii) The critical Nash network size N_c is the smallest possible network size of any shared information network that guarantees individually rational membership, in particular $N_c \leq m^* \leq M_c$.

For small marginal costs γ, the critical Nash network size is less than half as large as M_c, since using (29) we have then:

$$N_c \leq \frac{M_c}{2 - \gamma\overline{\rho}/\log(1+k)} \approx \frac{M_c}{2}. \tag{30}$$

However, M_c just represents the consumer base necessary for the monopolist to serve this heterogeneous market. The actual size of the monopolist's network, m^*, is smaller than that. But still joining the network has to be for each user individually rational, not allowing for a cross-subsidization of low-value agents by high-value agents, as is generally the case with a Pareto-optimal bargaining outcome.

4.2 Sources of Inefficiency

The monopolistic solution is market based, unlike the bargaining equilibrium, which presupposes that each player is telling the truth about her confidence in the signal and that there is common knowledge about the utility functions. In practical situations, an efficiency loss due to asymmetries in the information can be expected, but nevertheless *if* all parties can agree on their respective parameter vectors $(\rho, \kappa, \mu, \sigma)$ and costs for the service are known to be of the form (16), then any disagreements about splitting the investment burden *F* are resolved by the cooperative allocation described in Proposition 1.[26] Thus, even though there is a *nominal* efficiency loss by implementing the market-based solution, it may actually be preferable whenever the expected differences in welfare are small.

5 DISCUSSION

We have examined shared information services from both a cooperative bargaining viewpoint and the perspective of a monopolist supplier. To approximate the *efficient* outcome generated by Nash bargaining, the pricing of information should be contingent on the agent's confidence in the signal, her project risk, and her risk aversion. Even though each agent's demand for information decreases as her risk aversion increases, in a cooperative bargaining situation her contribution share may actually increase, an effect that a market-based mechanism implemented by a monopolistic information supplier cannot exploit. We have also shown

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[26]An additional hidden assumption is that the agents all have equal (explicit) bargaining power (cf. also footnote 19). Situations with asymmetric bargaining power can be dealt with in a manner already suggested by Nash (1953) by introducing positive exponents β_i, $i \in I$ into the Nash function (19) proportional to the agent's bargaining power, such that $\beta_1 + \beta_2 + \cdots + \beta_N = N$. This modification makes the model's prescribed sharing rule potentially contentious whenever not all β_i's are equal to one.

that wealth effects are significant for low-wealth agents, due to the strong impact of a prepayment on their investable wealth. This suggests that pricing should be significantly different for agents with a low endowed wealth, but should be essentially independent of wealth after a certain threshold (given by condition (14)) is reached. We also discussed the *critical mass* of users necessary for the creation of shared information services in the presence of network externalities. Clearly any prospective information provider or cooperating group of agents should verify that they meet these minimum requirements before setting up the information service in order to avoid failure. We identified the ingredients of an efficient pricing scheme and a (future) manager of a shared information service network, given that it is somehow possible to isolate the salient agent characteristics (e.g., their confidence in the informative signal), knows *how* an efficient pricing scheme can be structured to maximize welfare or—in the case of monopolistic information provision—profits.

Future theoretical contributions could evolve along the following three axes: (1) to endogenize entry in the presence of uncertainty about the costs of the shared information network and examine the associated fulfilled-expectations equilibria; (2) to examine monopolistic information selling mechanisms that involve various screening techniques based on versioning; (3) to introduce informational asymmetries in the bargaining solution to capture parameter uncertainty and differences in bargaining power among the agents.

6   ACKNOWLEDGEMENTS

7   REFERENCES

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Appendix: Proofs

Proof of Proposition 1. (i) Define $\bar{u}_i(t_i) = v_i(w_i - t_i)$ for all $i \in I$. Then using (18) the first-order optimality conditions for the problem (19) can be written as[27]

$$-\bar{u}_0'\left(F - \sum_{i=1}^{N} t_i\right)\prod_{i=1}^{N}\left(\bar{u}_i(t_i) - d_i\right) + \bar{u}_j'(t_j)\prod_{\substack{i=1 \\ (j \neq i)}}^{N}\left(\bar{u}_i(t_i) - d_i\right) = 0, \qquad j = 1,...,N,$$

which is equivalent to (20)–(21). (ii) From (12)–(13) and (20) we obtain

$$\frac{-\rho d_j(w_j - t_j)}{d_j(w_j - t_j) - \sqrt{1 + \kappa_j \sigma_j^2} d_j(w_j)} = \frac{-\rho d_0(w_0 - t_0)}{d_0(w_0 - t_0) - \sqrt{1 + \kappa_0 \sigma_0^2} d_0(w_0)},$$

which, after using the multiplicative separability of the CARA utilities implying that $\bar{d}_j(w_j - t_j) = e^{\rho t_j} d_j(w_j)$ for all $i \in I$, is equivalent to

$$q_j e^{-\rho t_j} = q_0 e^{-\rho t_0}, \qquad j = 1,...,N,$$

where we have set the q_i's as in the proposition. From the last relation we can conclude using (21) that

$$t_j = t_0 + \frac{1}{r}\log\frac{q_j}{q_0}, \qquad j = 1,...,N. \tag{31}$$

Summing up the t_j's from 1 through to N one obtains t_0 as given in the proposition. Substituting this expression for t_0 into (31) we obtain (22). (iii) If we substitute (21) into the first-order condition (20) and differentiate both sides (totally) with respect to ρ_j ($j > 0$ without loss of generality) we obtain (using the abbreviation $\bar{u}_i(t_i) = v_i(w_i - t_i)$ as above)[28]

$$\frac{\left(\bar{u}_j''t_j' + \partial_{\rho_j}\bar{u}_j'\right)\left(\bar{u}_j - d_j\right) - \left(\bar{u}_j't_j' + \partial_{\rho_j}\bar{u}_j - \partial_{\rho_j}d_j\right)\bar{u}_j'}{(\bar{u}_j - d_j)^2} = \frac{-\bar{u}_0''t_j'(\bar{u}_0 - d_0) + (\bar{u}_0')^2 t_j'}{(\bar{u}_0 - d_j)^2},$$

which is equivalent to

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[27]The second-order conditions are satisfied as a consequence of the concavity of $\bar{u}_i$ for all $i = 0, 1,..., N$.

[28]Here we omit the arguments for simplicity. Note that the arguments in for utilities with the "zero" index are $(F\text{-}t_1\text{-}t_2\text{-}\cdots\text{-}t_N)$.

$$\left(\frac{(\bar{u}_j - d_j)\bar{u}_j'' - (\bar{u}_j')^2}{(\bar{u}_j - d_j)^2} + \frac{(\bar{u}_0 - d_0)\bar{u}_0'' - (\bar{u}_0')^2}{(\bar{u}_0 - d_0)^2} \right) t_j' = \frac{-(\bar{u}_j - d_j)\partial_{\rho_j}\bar{u}_j' + \bar{u}_j'\partial_{\rho_j}(\bar{u}_j - d_j)}{(\bar{u}_j - d_j)^2}. \tag{32}$$

It can be readily verified that the bracketed expression on the LHS of (32) is always *negative*, so that the sign of t_j' depends solely on the sign of the numerator on the RHS, characterized by (23). QED

Proof of Proposition 2. The monopolist's profit function is

$$\Pi(p) = \left(\sum_{m=1}^{M} \mathbf{1}_{\{p_m \geq p\}} \right)(p - \gamma) - F_0,$$

where $\mathbf{1}$ denotes the indicator function. Since there are only a finite number of agents, maximizing profits is equivalent to finding $m^* \in \{1, 2, ..., M\}$, such that $\Pi(p_{m^*}) = \max_p \{\Pi(p)\}$, i.e.,

$$m^* \in \arg \max_m \left\{ m \left(1 - \frac{\gamma}{p_m} \right) \right\}, \tag{33}$$

which yields (26) as the unique solution. Hence the optimal price is $p^* = 2\gamma$ and profits are given by (25). QED

Proof of Proposition 3. (i) Let us first compute the consumer surplus in the monopolistic situation,

$$CS = \sum_{m=1}^{m^*} (p_m - p_{m^*}) = \frac{M \log(1 + k)}{2\bar{\rho}} \left(\sum_{m=1}^{m^*} \frac{1}{m} \right) - 2\gamma m^* \geq 2\gamma m^* \left(\sum_{m=2}^{m^*} \frac{1}{m} \right), \tag{34}$$

where the last inequality binds, whenever the *unconstrained* solution of (33) lies inside [0,*M*], i.e., whenever the monopolist actively *excludes* potential users from the network. Adding CS in (34) and (25) we obtain the expression (27). On the other hand, the welfare (in monetary terms) for the Pareto-optimal case can be—given that the approximation condition (14) holds—simply determined by adding up the WTPs (13) over all agents $i=1,...,N$, and subtracting the cost of the network $F_0 + \gamma N$,

$$\overline{W} = \log(1 + k) \sum_{i=1}^{N} \frac{1}{2\rho_m} - \gamma N - F_0 = \frac{N \log(1 + k)}{2\bar{\rho}} \sum_{n=1}^{N} \frac{1}{n} - \gamma N - F_0,$$

as long as $N \geq N_c$ and zero otherwise, which yields (28). (ii) The critical number M_c of potential users of the monopolistic network have been computed in the main text exactly offsetting the monopolist's total cost. The Nash network size is analogously determined by setting $\overline{W}$ to zero. From this, equation (29) follows directly. (iii) Clearly, since the Nash bargaining solution is welfare maximizing, and at N_c each agent is as low in her utility that removing a single agent would prompt at least one user to force the default outcome *d*. QED