The Question of Ownership in a Sharing Economy

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Abstract
The sharing of durable goods in a dynamic ownership economy is attractive, since it has the potential to realize gains from trade via short-term transfers of usage rights. We develop a model in which a set of agents, who are heterogeneous in their likely need of a durable good, make purchase decisions and then have the option to participate in a sharing market contingent on a realized need. The agents’ purchase decisions are compared to a situation where ex-post sharing is impossible. The impact of sharing on product sales is ambiguous: for low-price products sales may drop, while for high-price products the number of consumers who decide to become owners may actually increase. Our analysis extends to a sharing market in which prices are negotiated bilaterally in a Nash-bargaining framework. The resulting negotiated-sharing equilibrium allows for a realistic supply-demand imbalance in the sharing market.

1. Introduction
The recent emergence of intermediaries that enable the collaborative consumption of durable goods has fuelled the rapid growth of a sharing economy in which assets can be exchanged among peers on a short-term basis. The scope of what can be shared is continually widening, beyond the first-wave applications of collaborative housing (e.g., via AirBnB, Wimdu, 9Flats) or peer-to-peer car/ride sharing (e.g., via Lyft, ShareACar, WhipCar), to include a priori any durable good (e.g., via Ecomodo, e-Loue, Zilok). This realization led The Economist to conclude that “[o]n the Internet, everything is for hire.”1 Nielsen (2014) finds that more than two-thirds of about 30,000 worldwide internet respondents are willing to participate in the sharing economy; for any given category of goods (such as electronics, power tools, bicycles, clothing, and household items), the willingness to participate exceeds 20%.

In an economy where everything is shared the question arises, who will own the various items, and who will rely on others to obtain access when needed? In this paper, we probe the issue by developing a two-period model where agents of varying current and future needs first decide about buying an item and then choose whether to participate in a sharing economy. The question of sharing versus buying is different from that of renting versus buying because sharing happens contingent on the realization of a need (or absence thereof), whereas renting amounts primarily to a transformation of a cash-flow stream in time, e.g., to smooth consumption. By allowing for a short-term exchange after the purchase decision, the sharing economy provides mutual insurance by enabling the owners who do not need an item to rent it out to a non-owner who can put it to productive use. Conversely, non-owners can rely to some degree on exchange in a sharing market to get access to the item, should the need arise. Thus, a functioning market for sharing increases the likelihood of matching individuals’ needs with access to the assets that cater to those needs.

While it may be intuitively clear that a market for sharing improves the agents’ well-being by realizing gains from trade in an exchange economy via temporary adjustments of usage rights, its impact on ownership incentives is somewhat ambiguous. In the purchasing period, one agent reasons that the possibility of sharing will afford access to the item in question if needed later on and thus forgo a purchase. In that case, the presence of a sharing market would lead to a decrease in sales. Given a functioning peer-to-peer exchange, another agent thinks that the item may now well be worth buying because renting it out when not needed remains an option. This points to an increase in sales. The model addresses the above tradeoff in a precise way. Specifically we show that a sharing economy does not need to lead to a decrease in sales. While it is possible that less people will own a given item when it is obtainable via a sharing agreement, this is not necessarily true. The effect of sharing on product sales depends on the usage characteristics of the good and the price of ownership.

In our discussion, we abstract largely from possible frictions in the sharing economy, e.g., from matching problems, moral hazard and the related lack of trust. Both in practice and in theory these problems have been addressed (Weber 2014a,b). At the margin, the effects of these imperfections amount in our model to an in-
crease in price and/or a decrease in the probability that a given sharing transaction will take place.

1.1. Literature

Felson and Spaeth (1978) introduce the idea of “collaborative consumption” as an act of consuming while engaging in joint activities with others. Their idea that collaborative consumption requires coordination proves important. In our model, sharing occurs only among certain individuals (low-need owners and high-need non-owners); a market serves to reveal those individuals to each other and thus facilitates matching. Botsman and Rogers (2010) update the notion of collaborative consumption to refer essentially to sharing, lowering the significance of the social context in which the sharing might take place.

Belk (2007) points out that “rather than distinguishing what is mine and yours, sharing defines something as ours” (p. 127). In our model, this amounts to solving an allocational problem so as to increase efficiency by augmenting the likelihood that an item ends up in the hands of an individual who needs it most. Economic incentives and the role of intermediaries in a sharing economy are discussed by Weber (2014a,b). An intermediary can solve moral-hazard issues when the sharing parties are risk-neutral.

In this paper, we abstract from incentives arising from informational issues, and focus our attention on the agents’ buy-versus-borrow (or lend-versus-use) decisions. In that sense, the analysis here is reminiscent of the early studies of buying versus renting, for example concerning a home (Shelton 1968) or land (Reiss 1972). Naturally, sharing imposes no binding constraint on the owner when the good to be shared is nonrival, such as an information good which can be reproduced without significantly affecting its use value. Varian (2000) employs a static model to examine the effect of sharing on the production and consumption of information goods. Sharing in that context happens because some consumers are forming ex ante a “club,” e.g., in the form of a library, to share goods. In a sharing economy, the sharing decisions happen ex post, contingent on the realization of a need or its absence. In that sense, our base model with market clearing does have the flavor of an exchange economy for contingent claims in the spirit of Arrow (1953). In our setting, the goods and usage rights act as securities which help the agents to efficiently allocate risk. Yet sharing markets do not necessarily clear, because supply and demand are determined by random need and extant endowment (resulting from past purchases) and, on the other hand, prices are negotiated rather than determined by an invisible auctioneer presumed for market clearing. As an extension of the base model, we present an alternative solution based on Nash bargaining, in which the long side of the sharing market (which could be either the potential borrowers or the potential lenders) is randomly rationed, resulting in a transaction probability of less than one for that side.

1.2. Outline

The remainder of this paper is organized as follows: Section 2 introduces the model primitives. Section 3 examines the baseline case when there is no sharing economy and agents decide whether to buy (or rent) an item or not. In Section 4, we discuss sharing transactions conditional on an installed base of purchased products and contingent on the consumers’ respective need realizations. Section 5 examines the purchasing decision in the presence of a sharing market. Section 6 presents a model extension allowing for asymmetries between the numbers of potential borrowers and potential lenders. Section 7 concludes.

2. Model

Consider a continuum of autonomous agents (consumers), who take economic decisions at the discrete time periods \( t \in \{0, 1\} \). Without loss of generality, the total number of agents can be normalized to 1. A particular agent is characterized by his “type” \( \theta \in [0, 1] \) which describes his subjective probability that he will need to use a certain durable good (e.g., a power drill or a car) in the period \( t \in \{1\} \). The types \( \theta \) are distributed on \([0, 1]\) according to the (continuous, increasing) cumulative distribution function \( F(\cdot) \), with \( F(0) = 0 \) and \( F(1) = 1 \). The need is described by a state \( s \in \{L, H\} \) which can be either high \( (H) \) or low \( (L) \), so that

\[
\text{Prob}(s = H) = \theta.
\]

The agent’s realized subjective state \( s \) determines his demand for the item in the current period. At any time \( t \), an agent’s indirect utility \( v_i(y, s) \) depends on his income level \( y > 0 \), the subjective realization of his state of the world \( s \), and whether he disposes of the item or not (corresponding to \( i = 1 \) or \( i = 0 \), respectively).

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2Their logic is motivated by the notion of symbiosis as a mutual dependence of organisms which fulfill different functions and commensalism as describing relationships between organisms concerning similar functions (Hawley 1950).

3Belk (2010) provides a qualitative overview of the various contexts in which sharing can occur.

4In the context of information goods, Gopal et al. (2006) formulate a model where consumers can decide to purchase given an exogenous supply of pirated copies, which can also serve as a signal for an ex-ante uncertain consumption value of the good.

5The indirect utility \( v_i(y, s) \) is the agent’s utility level after having maximized his utility with respect to all other consumption goods, subject to the budget constraint (which specifies that the value of the consumed goods cannot exceed the income \( y \)). We assume that the agent’s utility is continuously differentiable and locally nonsaturated, so his indirect utility is increasing (and differentiable) in income.
We assume that the good is beneficial (at least weakly) under all circumstances, so
\[ v_1(y, s) \geq v_0(y, s) \]
for any income level \( y \) and any state realization \( s \). For \( s = H \) the item adds more value than for \( s = L \). That is,
\[ v_1(y, H) - v_0(y, H) > v_1(y, L) - v_0(y, L), \]
irrespective of the income level \( y \). If the item is needed, then possessing it has a larger impact on an agent’s well-being than when it is not needed.

At time \( t = 0 \) (“purchasing period”), any agent chooses whether to acquire the item at a positive price \( p \) based on his type \( \theta \) and given rational expectations about what will happen in the future. The purchasing decision is restricted to the initial time period. Based on the purchasing decision, consumption of all goods also takes place in that period, leading to an indirect utility of \( v_1(y - p, s) \) for owners and \( v_0(y, s) \) for non-owners in state \( s \). At time \( t = 1 \) (“sharing period”), any agent takes a consumption decision, which may include borrowing or lending the item at a rate \( r \) from or to another agent, depending on whether the agent is an owner or a non-owner. The discount factor between periods is \( \beta \in (0, 1) \).

To solve the model, we first consider the benchmark case without sharing (Section 3) and proceed by backward induction, analyzing first the sharing period (Section 4) and then the purchasing period (Section 5).

3. **Purchasing without Sharing**

At the price \( p > 0 \), let \( \theta_s \) be the type indifferent between purchasing the item or not, contingent on the current state \( s \). In the absence of future sharing, the expected payoff (net present value) without the item,
\[ v_0(y, s) + \beta (\theta_s v_0(y, H) + (1 - \theta_s) v_0(y, L)), \]
should equal the expected payoff with the item,
\[ v_1(y - p, s) + \beta (\theta_s v_1(y, H) + (1 - \theta_s) v_1(y, L)), \]
for \( s \in \{L, H\} \), whence
\[ \theta_s = \frac{\frac{v_0(y, s) - v_1(y - p, s)}{v_1(y, L) - v_0(y, L)} - 1}{\frac{v_1(y, H) - v_0(y, H)}{v_1(y, L) - v_0(y, L)} - 1}. \]
The indifferent type \( \theta_s \) does not necessarily lie in the type space \( \Theta = [0, 1] \). If \( \theta_s < 0 \), then no agent in state \( s \) would like to buy the item. If \( \theta_s > 1 \), then no agent in state \( s \) would like to buy the item. The findings in the main text are illustrated by a series of examples, all in the same setting.

**Example 1.** Fix \( \alpha > 1 \) and \( \gamma > 0 \), and let
\[ v_i(y, s) = \begin{cases} y + (\alpha - 1)iy + i\gamma, & \text{if } s = H, \\ y + i\gamma, & \text{if } s = L, \end{cases} \]
for \( i \in \{0, 1\} \). The parameter \( \alpha \) specifies the “synergy” for the agent, in the sense that it makes the rest of his consumption more enjoyable.

For instance, having a power drill produces a better productivity for the agent’s other tools and his paintings (when hung from the hooks in the drilled holes). The assumption that \( \alpha > 1 \) implies a complementarity in state \( H \) between the rest of remaining income and the item under consideration. The parameter \( \gamma \) denotes the “standalone value” of the item, i.e., the added value that its possession affords the agent, irrespective of the underlying need state. For example, contributing to the standalone value of a power drill is that it can always be used as a paper weight in addition to its standard complementary usefulness when enabling nails to hold objects up. To avoid degenerate results, it is reasonable to assume that the standalone value does not exceed an agent’s income, i.e., \( \gamma \leq y \). Lastly, we note that in order to simplify the algebra, at a slight loss of generality, the agent’s utility without the item is assumed independent of the need state. With this, one obtains the indifferent types,
\[ \theta_L = \frac{1}{\beta} \left( 1 + \frac{\beta \gamma}{\alpha - 1} \right) = \theta_H + \frac{1}{\beta} \left( 1 - \frac{p}{y} \right). \]
This implies that a higher income increases demand of both types for the item.

The demand for the item depends on an agent’s need state in the purchasing period \( t = 0 \). The number of agents of types greater than a given \( \theta \), who find themselves in state \( H \), is given by
\[ G(\theta) = \int_0^1 dF(\theta). \]
Provided that \( 0 \leq \theta_H \leq \theta_L \leq 1 \), the demand for the product is
\[ D = \int_{\theta_L}^1 dF(\theta) + \int_{\theta_H}^{\theta_L} \theta dF(\theta) = 1 - F(\theta_L) + G(\theta_H) - G(\theta_L). \]
This means that all agents between \( \theta_L \) and 1 buy the item. Additionally, of the agents with types in \( [\theta_H, \theta_L] \) the ones who are in state \( H \) at time \( t = 0 \) also purchase the good.

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5 There is no banking between periods, i.e., income cannot be saved and there are no bank loans between periods. This simplification avoids confounding the findings about sharing markets with possible incentives to smooth consumption. The renting-versus-buying problem discussed at the end of Section 3 illustrates the effect of restructuring the cash-flow stream when converting a one-time upfront payment into two equal loan repayments.
Example 2. To the assumptions in Ex. 1 we add that the types are uniformly distributed, i.e., \( F(\theta) = \theta \) on \([0, 1] \). Then \( G(\theta) = (1 - \theta^2)/2 \), and
\[
D = \frac{1 - \theta_H^2 + (1 - \theta_L)^2}{2} \in [0, 1]
\]
is the demand for the purchase of the durable good at time \( t = 0 \), in the absence of sharing. \( \square \)

The per-period willingness to pay \( w_s = w_s(y) \) in state \( s \) (“static valuation”) is such that
\[
v_1(y - w_s, s) = v_0(y, s).
\]
The willingness to pay for complete ownership in state \( s \) (“dynamic valuation”) is \( \bar{w}_s = \bar{w}_s(y, \theta) \), and
\[
v_1(y - \bar{w}_s, s) = v_0(y, s) - \beta \left( \bar{v}_1(y, \theta) - \bar{v}_0(y, \theta) \right),
\]
where
\[
\bar{v}_1(y, \theta) = E[v_1(y, \bar{s})|\theta] = \theta v_1(y, H) + (1 - \theta) v_1(y, L).
\]

In a more compact form, the linear relationship between static and dynamic values becomes
\[
\begin{bmatrix} \bar{w}_L \\ \bar{w}_H \end{bmatrix} = \begin{bmatrix} 1 + \beta \left( \frac{1 - \theta}{(1/\alpha) \theta} \right) \end{bmatrix} \begin{bmatrix} w_L \\ w_H \end{bmatrix},
\]
where \( I \) denotes the identity matrix. Both \( \bar{w}_H \) and \( \bar{w}_L \) are increasing in \( \theta \), since \( w_H > w_L/\alpha \). Moreover, the difference \( \bar{w}_H - \bar{w}_L \) decreases in \( \theta \), since
\[
\frac{\partial (\bar{w}_H - \bar{w}_L)}{\partial \theta} = \beta(\alpha - 1) \left( \frac{w_L}{\alpha} - w_H \right) < 0.
\]

The dynamic value in state \( H \) exceeds the dynamic value in state \( L \), i.e., \( \bar{w}_H \geq \bar{w}_L \), if and only if
\[
\theta \leq \frac{y - (1 + \beta)\gamma}{\beta(\alpha - 1)y} = \theta_m.
\]
Note that \( \theta_m \) in the last condition is strictly less than 1, as long as
\[
\alpha > 1 + \frac{1}{\beta} \left( 1 - (1 + \beta)\gamma \right).
\]

The last inequality is true for large \( \alpha \) or small income values \( y \). Specifically, when \( \alpha > 1 + (1/\beta) \), it holds irrespective of the other parameter values. At small incomes the opportunity cost of the capital outlay at time \( t = 0 \) is larger in state \( H \) than in state \( L \). Note that an increase in the standalone value \( \gamma \) has the same effect as a decrease in income, corresponding to a deflation of the currency in real terms. For \( y > (1 + \beta)\gamma \), it is \( \theta_m > 0 \). Fig. 1 illustrates the situation. \( \square \)

The demand elasticity is
\[
\varepsilon = p \cdot \frac{(1 - \theta_L) f(\theta_L) + \theta_H f(\theta_H) \theta_H}{1 - F(\theta_L) + G(\theta_H) - G(\theta_L)}.
\]

Given a constant marginal cost \( c \), the optimal monopoly price \( p^* \) (see, e.g., Tirolo 1988) is therefore such that
\[
p^* = c + \frac{1 - F(\theta_L) + G(\theta_H) - G(\theta_L)}{(1 - \theta_L) f(\theta_L) \theta_L + \theta_H f(\theta_H) \theta_H} |_{p = p^*},
\]
where we have used the fact that \( G'(\theta) = -\theta f(\theta) \).

Example 4. Differentiating the indifferent types \( \theta_L, \theta_H \) with respect to \( p \) yields \( \theta_L' = \frac{(\alpha - 1)\beta y}{(\alpha - 1)\beta y + (1 - \theta_L)/\theta_L} \) and \( \theta_H' = \alpha \theta_H \), respectively, so the optimal monopoly price \( p^* \) is characterized by
\[
p^* = c + \frac{(\alpha - 1)\beta y}{2} \cdot \frac{1 - \theta_H + (1 - \theta_L)^2}{\alpha \theta_H + 1 - \theta_L} |_{p = p^*},
\]
where \( \theta_L \) and \( \theta_H \) are as in Ex. 3. \( \square \)
Renting vs. Buying. When renting an item across two periods at the rate $\rho$, a type-$\theta$ agent obtains the expected utility
\[ v_1(y - \rho, s) + \beta \bar{v}_1(y - \rho, \theta), \]
conditional on being in state $s$ at time $t = 0$. This needs to be compared to the utility of the same agent in the role of an owner in state $s$,
\[ v_1(y - p, s) + \beta \bar{v}_1(y, \theta). \]
If the rental price $\rho \leq w_L$, then a non-owner would always rent. If $\rho \in (w_L, w_H]$, then for a non-owner renting is optimal, conditional on being in the high-need state. Lastly, for $\rho > w_H$, a non-owner would never rent.

The reservation rent $\bar{\rho}_s = \bar{\rho}_s(\theta)$ is the highest rental rate a type-$\theta$ agent in state $s$ is willing to commit to in the purchasing period, so
\[ v_1(y - \bar{\rho}_s, s) + \beta \bar{v}_1(y - \bar{\rho}_s, \theta) = v_0(y, s) + \beta \bar{v}_0(y, \theta), \]
for all $s \in \{L, H\}$. This definition of $\bar{\rho}_s$ is analogous to the willingness to pay for complete ownership $\bar{w}_s$ introduced earlier in this section (see also footnote 7 below).

Example 5. For $s = L$, an agent of type $\theta$ has the reservation rent
\[ \bar{\rho}_L = \frac{(1 + \beta)\gamma + \theta \beta (\alpha - 1) y}{1 + \beta (1 + \theta (\alpha - 1))} \leq \frac{\bar{w}_L}{1 + \beta}. \]
For $s = H$, the reservation rent becomes
\[ \bar{\rho}_H = \frac{(\alpha - 1)(1 + \theta \beta) y + (1 + \beta)\gamma}{(1 + \theta \beta)\alpha + (1 - \theta)\gamma} \geq \frac{\bar{w}_H}{1 + \beta}. \]
Hence, in the low state ($L$) the discounted value of the reservation rent, $(1 + \beta)\bar{\rho}_L$, is below the dynamic valuation $\bar{w}_L$. Conversely, in the high state ($H$) the discounted value of the reservation rent, $(1 + \beta)\bar{\rho}_H$, exceeds the dynamic valuation $\bar{w}_H$. In sum,
\[ \frac{\bar{w}_H}{\bar{\rho}_H} \leq 1 + \beta \leq \frac{\bar{w}_L}{\bar{\rho}_L}, \]
so that a monopolist would like to price discriminate by offering a sales contract to agents in the low state and a rental contract to agents in the high state. From the agents’ perspective, the intuition for this is that giving up income is easier in the low than in the high state.

4. A Market for Sharing

In the sharing period (at $t = 1$), all agents are either owners or non-owners. The split depends on the agents’ buying decisions in the purchasing period (at $t = 0$). A non-owner in state $s$ is willing to borrow the item at the rate $r$ if and only if this rate does not exceed his willingness to pay in that state, i.e., if and only if $r \leq w_s$. Conversely, an owner in state $s$ is willing to lend out the item if the price for sharing matches at least his willingness to accept, i.e., if and only if $r \geq a_s$.

An agent’s willingness to accept in state $s$ is the equivalent variation $a_s$ such that
\[ v_0(y + a_s, s) = v_1(y, s). \]

From Weber (2010, Prop. 8) it follows that there is no trade between different agents in the same state $s$, provided that $v_1(\cdot, s) - v_0(\cdot, s)$ is nondecreasing, and $v_1(\cdot, s)$ is concave for $i \in \{0, 1\}$. The only possible transaction with gain is therefore between owners in state $L$ and non-owners in state $H$. 

Example 6. Using the above definition, one finds in the setting of our example that an agent’s willingness to accept, given a state $s \in \{L, H\}$, is
\[ a_H = (\alpha - 1)y + \gamma \quad \text{and} \quad a_L = \gamma, \]
respectively. Comparing this with the results in Ex. 3 shows that $a_s \geq w_s$ for $s \in \{L, H\}$, consistent with the normative endowment effect (see footnote 7).

At the rate $r \in [a_L, w_H]$, let $D_1$ denote the demand for shared items and $S_1$ the supply of these items in the sharing period. $D_1$ corresponds to all non-owners in state $H$, while $S_1$ includes all owners in state $L$. It is important to note that ex post, i.e., after the purchasing period, demand and supply of shared items do not depend on the specific value of the rental rate $r$, as long as it is between $a_L$ and $w_H$.

We assume here that the sharing market clears, so that
\[ S_1 = D_1. \]

This assumption is relaxed in Section 6. If we denote by $\bar{\theta}_L$ and $\bar{\theta}_H$ (with $\bar{\theta}_L \geq \bar{\theta}_H$) the indifferent types in the purchasing period ($t = 0$), given a sharing market at $t = 1$, then the supply of sharers (lenders) is the fraction of owners in state $L$, at time $t = 1$,
\[ S_1 = \int_{\bar{\theta}_L}^{\bar{\theta}_H} (1 - \theta) dF(\theta) + \int_{\bar{\theta}_H}^{\gamma} (1 - \theta) dF(\theta). \]

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Footnote 7: The Hicksian welfare measures of compensating variation (willingness to pay) and equivalent variation (willingness to accept) are fundamental in determining the value of nonmarket goods. Provided that one of the welfare measures is increasing in income, the other is also increasing in income, and the willingness to accept exceeds the willingness to pay. The latter is also referred to as the “normative endowment effect” (Weber 2010).

Footnote 8: Since Edgeworth (1897) and Pareto (1909) complementarity is associated with the property of increasing differences.
On the other hand, the demand of sharers (borrowers) at time \( t = 1 \) is the fraction of non-owners in state \( H \),
\[
D_1 = \int_{\hat{\theta}_H}^{\theta_H} \theta \, dF(\theta) + \int_{\hat{\theta}_L}^{\hat{\theta}_L} (1 - \theta) \, dF(\theta).
\]
Market clearing requires therefore
\[
\int_{\hat{\theta}_L}^{1} (1 - \theta) \, dF(\theta) = \int_{\hat{\theta}_H}^{\theta_H} \theta \, dF(\theta).
\]

**Example 7.** Given a uniform type distribution, market clearing requires that
\[
S_1 = 1 - \frac{1}{2} \hat{\theta}_L \left( 1 - \frac{\hat{\theta}_L}{2} \right) = \frac{\hat{\theta}_H^2}{2} = D_1,
\]
which is equivalent to the simple condition that
\[
1 - \hat{\theta}_L = \hat{\theta}_H.
\]

In other words, in order for the sharing market to be balanced, the number of agents who acquire the good in the low-need state \( L \) must be equal to the number of agents who do not buy the good in the high-need state \( H \), in the purchasing period.

### 5. Purchasing with Sharing

Under our earlier assumption of rational expectations, agents will correctly anticipate the rental price \( r \) in the sharing market when making their initial purchasing decisions. At the fulfilled-expectations rental price \( r \), a non-purchaser (and thus non-owner) in state \( s \) obtains the expected payoff of
\[
v_0(y, s) + \beta (\theta v_1(y - r, H) + (1 - \theta)v_0(y, L)).
\]
Similarly, a purchaser at price \( p \) obtains
\[
v_1(y - p, s) + \beta (\theta v_1(y, H) + (1 - \theta)v_0(y + r, L)).
\]
As a result, the indifferent type in state \( s \) becomes
\[
\hat{\theta}_s = \frac{1}{\beta} \left( \frac{v_0(y, s) - v_1(y - p, s)}{v_1(y + r, L) - v_0(y, L)} \right) - 1.
\]

The demand for the good in the purchasing period (at time \( t = 0 \)) is
\[
\hat{D} = 1 - F(\hat{\theta}_L) + G(\hat{\theta}_H) - G(\hat{\theta}_L).
\]

The following example provides the basis for a generic comparison between the demand for the durable good with sharing (\( \hat{D} \)) and without sharing (\( D \)).

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**Example 8.** In the parametrization of our ongoing example, one finds the following indifferent types:
\[
\hat{\theta}_L = \frac{1}{\beta} \left( \frac{p - \gamma - \beta r}{(\alpha - 1)r} \right) = \frac{\beta}{(\alpha - 1)r} = \frac{p - a_L}{(\alpha - 1)r},
\]
in the low state and
\[
\hat{\theta}_H = \hat{\theta}_L - \frac{y - p}{\beta r} = \frac{\alpha(p - w_H)}{(\alpha - 1)r} = \frac{\beta - r}{(\alpha - 1)r},
\]
in the high state. Fig. 2 depicts the segments in the product market (at \( t = 0 \)) and at the sharing stage.\(^9\) Using the results in Ex. 2 and the market-clearing condition in

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\(^9\) For \( \hat{\theta}_H \geq 0 \) it is necessary that \( p \geq w_H \). In addition, \( \hat{\theta}_L \geq \hat{\theta}_H \) if and only if \( p - a_L \geq \alpha(p - w_H) \), which is equivalent to \( p \leq y \). The assumption that the purchase price cannot exceed the per-period income is necessary given that no banking is available; see footnote 6.
Ex. 7, the demand for purchasing the good at \( t = 0 \) in the presence of sharing is constant, 
\[
\hat{D} = \frac{1 - \hat{\theta}_H^2 + (1 - \hat{\theta}_L)^2}{2} = \frac{1}{2}.
\]
That is, 50% of all agents purchase the good, provided that the sharing market clears. Together with the expressions for the indifferent types the market-clearing condition also implies that 
\[
1 + \frac{y - p}{\beta r} = 2\hat{\theta}_L = \frac{2p - \gamma - \beta r}{\beta(\alpha - 1)r}.
\]
Solving the last expression for \( r \) yields the equilibrium sharing rate \( r^* \), 
\[
r^* = \frac{1}{\beta} \left( p - \frac{\alpha - 1}{\alpha + 1} y - \frac{2\gamma}{\alpha + 1} \right).
\]
Since \( r^* \) needs to lie in \( [a_L, w_H] \) for a transaction to take place, market clearing in the sharing market obtains if the price \( p \) of the durable good is such that 
\[
\frac{\alpha w_H + a_L}{\alpha + 1} + \beta a_L \leq p \leq \frac{\alpha w_H + a_L}{\alpha + 1} + \beta w_H.
\]
At a lower price the sharing rate \( r^* \) drops below what a potential lender would be willing to accept. Conversely, if the price of the durable good is too high, then \( r^* \) exceeds what a potential borrower is willing to pay in a state of high need.

The last example shows that the sharing market may not necessarily clear. Because of the relatively recent advent of sharing markets there is also no reason to believe that manufacturers of durable goods have set prices with the potentiality of ex-post sharing in mind. Section 6 explores a relaxation of market clearing based on bilateral bargaining in the sharing period.

**Impact of Sharing on Product Sales.** The generic impact of the sharing economy on product sales (at a constant price point) amounts to comparing the demand \( D \) in Ex. 2 (without sharing) and the demand \( \hat{D} \) in Ex. 8 (with sharing). Thus, \( D > \hat{D} \) is equivalent to 
\[
1 - \theta_L > \theta_H,
\]
or in other words, 
\[
\frac{\alpha w_H + a_L}{\alpha + 1} > \frac{p}{1 + \beta}.
\]
To interpret the last inequality it is convenient to check the admissible interval for the price of the durable good \( p \) in Ex. 8. Consistent with this interval, as implied by the last inequality, the demand without sharing exceeds the demand with sharing if and only if 
\[
p < \frac{\alpha w_H + a_L}{\alpha + 1} + \beta \left( \frac{a_L}{\alpha + 1} + \frac{\alpha w_H}{\alpha + 1} \right).
\]
Hence, only when the purchase price of a good is relatively low will introducing a market for sharing negatively impact demand \( (D > \hat{D}) \). When the price of ownership is high, i.e., for relatively expensive items, a sharing market tends to increase the desire for ownership and thus product sales will increase with sharing \( (D < \hat{D}) \).

It is important to note that the previous result depends on the fact that the sharing market is assumed to clear, which may not be the case in general (see Section 6). The finding is consistent with the casual observation that clubs for the sharing of luxury sports cars have coexisted with a sustained increase in sales of such vehicles. The sharing of relatively cheap cars is a more recent phenomenon, attributable to an increase of sharing due to the overcoming of market imperfections such as moral hazard (Weber 2014a,b).

### 6. Extension: Negotiated Sharing

Market clearing may be difficult to achieve in a model where the demand within need segments is inelastic, as long as the transaction rate \( r \) lies between the willingness to accept \( a_L \) and the willingness to pay \( w_H \). Supply-demand imbalances can arise naturally in new markets where the price discovery is still quite decentralized and based on bilateral negotiation rather than market clearing.\(^{10}\)

#### 6.1. Bargaining Transactions

Assume that the transaction price \( r \) between matching parties is subject to negotiation. Suppose further that the relative magnitude of demand vs. supply determines the negotiation power \( \lambda \) of borrowers vs. lenders,\(^{11}\)
\[
\lambda = \frac{D_l}{D_l + S_l} \in [0, 1].
\]
Let the transaction price \( r^* \) be determined using Nash bargaining (Nash 1950; Roth 1979), whence 
\[
r^* \in \arg \max_{r \in [a_L, w_H]} N(r),
\]
where 
\[
N = (v_1(y-r, H) - v_0(y, H))^{1-\lambda}(v_0(y+r, L) - v_1(y, L))^{\lambda}
\]
\(^{10}\)In reality, the available supply of goods in the sharing market may be quite heterogeneous due to variations in prior use, different vintages, and different product types. All of these factors compound the need for bilateral negotiation.

\(^{11}\)The introduction of a parameter for the relative bargaining power between parties is standard in axiomatic bargaining theory (Roth 1979).
is the Nash product. The solution is naturally constrained to be located between the willingness to accept in the low state and the willingness to pay in the high state.

**Remark 1.** The Nash bargaining solution relies on four axioms, namely the independence of equivalent utility representations (i.e., solution is invariant with respect to positive linear transformations of the utility), symmetry (i.e., if all players’ utilities are the same, including the payoffs when bargaining fails, then the solution is symmetric), independence of irrelevant alternatives (i.e., enlarging the set of possible negotiated outcomes by adding non-chosen alternatives does not change the solution), and Pareto-optimality (i.e., the solution is such that no other feasible outcome can improve a player’s payoff without making somebody else worse off). The maximizer of the Nash product (with respect to all feasible bargaining outcomes) describes the unique outcome that is consistent with the four axioms. This Nash bargaining solution is widely used to predict the outcome of standard bargaining situations with symmetric information.

To solve the Nash bargaining problem, it is convenient to take the logarithm of the objective function. The corresponding optimality condition for \( r \) is

\[
\frac{v_0(y + r, L) - v_1(y, L)}{v_1(y - r, H) - v_0(y, H)} = \frac{\lambda}{1 - \lambda} \frac{v_0(y + r, L)}{v'_1(y - r, H)},
\]

which has a solution in \([a_L, w_H]\).

**Example 9.** The first-order optimality condition becomes

\[
y + r - (y + \gamma) \frac{1}{\alpha(y - r) + \gamma - y} = \lambda \frac{1}{1 - \lambda},
\]

whence

\[
r = (1 - \lambda)\gamma + \lambda \cdot \frac{(\alpha - 1)y + \gamma}{\alpha}.
\]

Thus, \( r = (1 - \lambda)a_L + \lambda w_H \). As \( \lambda \) goes from 0 to 1, the rental rate increases from the lender’s willingness to accept in the low state (\( a_L \)) to the borrower’s willingness to pay in the high state (\( w_H \)). □

When supply and demand is in disequilibrium, i.e., when \( S_1 \neq D_1 \), the probability \( q \) that a sharing transaction takes place becomes less than one for agents in the majority faction. For owners in state \( L \) (i.e., potential lenders) the transaction probability is

\[ q = \min\{1, S_1/D_1\} = \min\{1, \frac{1 - \lambda}{\lambda}\}. \]

For non-owners in state \( H \) (i.e., potential borrowers) the transaction probability is

\[ \bar{q} = \min\{1, D_1/S_1\} = \min\{1, \frac{\lambda}{1 - \lambda}\}. \]

Note that \( \max\{q, \bar{q}\} = 1 \) and \( \min\{q, \bar{q}\} = q\bar{q} \).

**Remark 2.** In general, the sharing-transaction probabilities \( q, \bar{q} \) also depend on the quality of matching in the sharing market. In addition, they could depend on the effectiveness with which inherent problems of sharing transactions are handled in the market.

### 6.2. Negotiated-Sharing Equilibrium

Given the anticipated sharing-transaction probabilities \( q, \bar{q} \) and the fulfilled-expectations rental price \( r \), a non-purchaser (and thus non-owner) in state \( s \) obtains the expected payoff of \( v_0(y, s) \) plus

\[ \beta(\theta(qv_1(y - r, H) + (1 - q)v_0(y, H)) + (1 - \theta)v_0(y, L)). \]

On the other hand, a purchaser obtains \( v_1(y - p, s) \) plus

\[ \beta(\theta v_1(y, H) + (1 - \theta)(qv_0(y + r, L) + (1 - q)v_1(y, L))). \]

Fig. 3 compares the payoffs of owners and non-owners using standard event trees. The indifferent type is determined in the usual way,

\[
\hat{\theta}_s = \frac{\frac{1}{2} v_0(y, s) - v_0(y - p, s)}{\frac{\Delta(y, H)}{\Delta(y, L)} - 1}.
\]
where \( \pi = (q, 1 - q) \), \( \bar{\pi} = (\bar{q}, 1 - \bar{q}) \), and

\[
\Delta(y, r, L) = \begin{pmatrix} v_0(y + r, L) - v_0(y, L) \\ v_1(y, L) - v_0(y, L) \end{pmatrix},
\]

\[
\Delta(y, r, H) = \begin{pmatrix} v_1(y, H) - v_1(y-r, H) \\ v_1(y, H) - v_0(y, H) \end{pmatrix}.
\]

In principle, one could consider the transaction probabilities \( q \) and \( \bar{q} \) as independent parameters (see also Remark 1), and for \( q = \bar{q} = 0 \) there is no sharing market, so that \( \bar{\theta}_s \) reduces to the indifferent type \( \theta_s \) determined in Section 2, for all \( s \in \{L, H\} \).

**Example 10.** It is \( \Delta(y, r, L) = (r, \gamma)^\top = (r, a_L)^\top \) and \( \Delta(y, r, H) = (\alpha r, (\alpha - 1)y + \gamma)^\top = \alpha(r, w_H)^\top \),

\[
\bar{\theta}_L = \frac{p - a_L}{\alpha(q r + (1-q)a_L)} - (qr + (1-q)a_L).
\]

The expression for \( \bar{\theta}_H \) is similar,

\[
\bar{\theta}_H = \frac{\alpha(p - w_H)}{\alpha(q r + (1-q)a_L)} - (qr + (1-q)a_L).
\]

As already noted in footnote 9, these indifferent types are such that \( \bar{\theta}_L \geq \bar{\theta}_H \) if and only if \( p \leq y \), which naturally holds in the absence of banking.

The transaction probabilities \( q \) and \( \bar{q} \) can be written as a function of the price \( r = (1-\lambda)a_L + \lambda w_H \) in the sharing market,

\[
q = \min \left\{ 1, \frac{w_H - r}{r - a_L} \right\} \quad \text{and} \quad \bar{q} = \min \left\{ 1, \frac{r - a_L}{w_H - r} \right\}.
\]

Their values depend on whether sharing happens in a lenders’ market \((S_1 \geq D_1, q = 1)\) or in a borrowers’ market \((D_1 \geq S_1, \bar{q} = 1)\). The former regime obtains, i.e., sharing happens in a lenders’ market, if \( r \geq \frac{a_L + w_H}{2} \).

The sharing-equilibrium condition becomes

\[
\frac{w_H - r}{r - a_L} = \frac{S_1}{D_1} \equiv \varphi(r),
\]

where the supply-demand ratio \( \varphi(r) \) is a continuous function with positive values for all \( r \in (a_L, w_H) \). This implies the fixed-point problem

\[
r = \frac{\varphi(r) a_L}{1 + \varphi(r)} + \frac{w_H}{1 + \varphi(r)} \in [a_L, w_H],
\]

which has a solution \( r^* \) by Brower’s fixed-point theorem (Aubin and Ekeland 1984, p. 46). The solution is unique because the left-hand side is increasing while the right-hand side is decreasing in \( r \) (with \( \varphi(a_L) = 0 \) and \( \varphi(w_H) = \infty \)).

---

**Example 11.** Using the results in Ex. 7 and Ex. 10, an equilibrium in the sharing market with bargaining obtains for

\[
\frac{w_H - r}{r - a_L} = \left( \frac{1 - \bar{\theta}_L}{\bar{\theta}_H} \right)^2 \equiv \varphi(r),
\]

which implies a unique equilibrium sharing rate \( r^* \), as noted earlier.

The equilibrium price \( r^* \) in the sharing market (at \( t = 1 \)), which depends on the given price \( p \) of ownership, determines the agents’ choices in the purchasing period (at \( t = 0 \)). It implies the indifferent types

\[
\bar{\theta}_s^* = \bar{\theta}_s |_{r=r^*},
\]

for \( s \in \{L, H\} \). The equilibrium demand for ownership in the presence of a sharing market with bargaining is therefore

\[
\bar{D}^* = 1 - F(\bar{\theta}_L^*) + G(\bar{\theta}_H^*) - G(\bar{\theta}_L^*),
\]

analogous to the corresponding expression in Section 5.

**Example 12.** Consider the case where \((\alpha, \beta, \gamma) = (3, 0.8, 1)\) and the agents’ income is \( y = 5 \). At a price of \( p = 4 \), the demand without sharing market is \( D = 61/80 \approx 0.7625 \). With (negotiated) sharing, the demand for ownership decreases by about 2.1% to \( \bar{D}^* \approx 0.7464 \). In this case, the equilibrium price in the sharing market is \( r^* \approx 1.9286 \), and there are significantly more lenders than borrowers:

\[
S_1 \approx 0.2522 > 0.0058 \approx D_1,
\]

so the supply-demand ratio \( S_1/D_1 \) exceeds 40. After an increase in the price of ownership to \( p = 4.5 \), the equilibrium price in the sharing market drops to \( r^* \approx 1.4927 \). The demand without sharing \( D \approx 0.6969 \) decreases by about 1.7% to \( \bar{D}^* \approx 0.6848 \). The supply-demand ratio in the sharing market is now below 10 \((S_1/D_1 \approx 0.2055/0.0207 \approx 9.9)\).

The last example illustrates that despite a fairly symmetric setup ex ante (e.g., uniform type distribution) supply and demand in the sharing market may be severely imbalanced.

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\(^{12}\) \( ^\top \) denotes transposition of a vector or matrix.

\(^{13}\) Despite a relatively high price \( p \) (close to \( y \)), the demand for ownership drops with the introduction of sharing (which would also happen if market clearing were imposed; see Section 5).
7. Conclusion

When deciding whether to buy a durable good in the presence of sharing, an agent faces the basic tradeoff that ownership is generally not aligned with future need. A sharing market provides mutual insurance to the various agents, who are heterogeneous with respect to their beliefs about the likelihood of needing the item. As shown in Section 5, the introduction of a sharing market can tip the proverbial scales for consumers either way. Whether overall demand increases or decreases when a sharing market is introduced depends on the product characteristics and on the product price. With market clearing, the introduction of a sharing market can increase demand for ownership when prices are relatively high. This runs counter to the intuition that production is bound to always decrease when durables are shared.

Our model of ownership decisions in the presence of sharing does not depend on the number of lenders and borrowers to match in equilibrium. A certain imbalance in the sharing market should be naturally expected in reality, since sharing markets have not existed for long in many domains. For example, the installed base of a product such as cars might be very large, so a potential borrower faces possibly many more lenders than competition for those offers from other borrowers. In a sharing market with negotiated prices the agents’ relative bargaining power depends on the balance between borrowers and lenders. The analysis in Section 6 shows that under fairly minimal conditions there is a unique equilibrium price, which enables a positive number of sharing transactions until at least one side of the market (i.e., borrowers or lenders) is exhausted.

References


