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A continuum of commitment

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HIGHLIGHTS

- Conditions for intermediate commitment in a general vector-valued setting and the relation to regulatory commitment.
- Conservation, as side-payment to the agent, of any adjustment cost may be suboptimal both privately and socially.
- Examples which show the main effects.

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ABSTRACT

We examine a generic three-stage game for two players with alternating moves, where the first player can choose the level of adjustment cost to be paid in the last period to modify the action she announced in the first period. In the resulting continuum of commitment options, convexifying the choice between first-mover and second-mover advantage in pure strategies, we characterize when an intermediate adjustment-cost level is chosen in equilibrium. We show that the wastefulness of the adjustment cost may be in the players' best interest, improving both of their individual net payoffs over making any fraction of the adjustment cost a side-payment from the first to the second player.

1. Introduction

Frequently an agent's preferred choice depends on the ability of others to commit to their actions. For example, a regulator might want to encourage the agent to make an irreversible investment in an environmentally clean technology by announcing a tax on pollution at a sufficiently high level. For the agent's investment decision, the regulator's propensity to make ex-post changes to the announced policy is of key importance. The corner solutions of either full or no commitment have been well examined in Stackelberg-duopoly or entry games (see, e.g. Bagwell and Wolinsky, 2002). To determine which of these "extreme" commitment options is better for the first player amounts to determining whether there is a first-mover or a second-mover advantage, which in turn depends on whether both players' actions are strategic complements or substitutes. In reality, the extreme commitment options are neither available nor would they always be optimal. In a policy setting, this means that a regulator cannot completely prevent the current legislation from being modified in the future and that any change will in reality involve some adjustment cost. In this paper, we find that extreme forms of commitment may not be in the regulator's best interest, and that the seemingly wasteful expense of a positive adjustment cost in equilibrium can be preferable to internalizing any fraction of it, even from a social point of view.

Formally, the ability to commit is equivalent to the existence of adjustment costs which render an ex-post modification of an earlier choice expensive. Without commitment ability adjustment costs vanish. We analyze a two-player sequential-moves game, where the first player (hereafter: the "principal") selects an

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observable action, to which the second player (hereafter: the "agent") reacts. Finally, the principal can adjust the initially chosen action, which may therefore also be referred to as a mere announcement. At the beginning of the game, the principal has the option to commit to a nonnegative adjustment-cost function, enabling her to effectively select from a continuum of commitment levels, thus seamlessly linking the extremes of full commitment (for infinite adjustment cost) and no commitment (for zero adjustment cost). We provide conditions under which an interior level of commitment is optimal for a quadratic adjustment cost of variable magnitude.

In the absence of commitment, the principal's initial announcement is inconsequential and can be adjusted freely after the agent moves. The agent then chooses his action by offsetting the direct effect (i.e., the marginal utility of his own action) against the strategic effect (i.e., the marginal utility of changing the principal's reaction). By increasing her own adjustment cost, the principal can diminish the magnitude of the strategic effect in the agent's payoff-maximization problem. At the same time, the importance of an initially nonexisting strategic effect in the principal's payoff-maximization problem increases. Finally, in the limiting case of infinite adjustment cost, the agent's strategic effect vanishes while the magnitude of the principal's strategic effect of anticipating the agent's action is maximal. At each extreme commitment option one player's strategic effect vanishes. The continuum of commitment options in between the extremes balances the incentives, and the principal's payoff can attain its maximum at an interior level.

In addition to the commitment continuum generated by varying the principal's adjustment cost the set of attainable payoff vectors can be further augmented by allowing for a fraction of the adjustment cost to be paid to the agent. This added flexibility includes budget balance, where all of the principal's adjustment cost becomes a side-payment to the agent. A somewhat surprising result is that the attempt to conserve even a fraction of the principal's adjustment cost in the form of a side-payment to the agent may be inefficient for *both* players. The reason is that a side-payment to the agent inadvertently augments the relative importance of the strategic effect in his payoff-maximization problem. Thus, to optimally balance strategic and direct effects in both players' decision problems, the wasteful nature of adjustment costs may be essential.

1.1. Literature

The importance of the move order in a two-period duopoly was pointed out by von Stackelberg (1934). Gal-Or (1985) showed that the advantage of moving first over moving second in a duopoly with identical players is determined by the slope of the reaction functions: when the players' actions are strategic substitutes moving first is better.[1] In this setting, moving first means to be able to fully commit to a strategy, in the sense of an infinite adjustment cost. Yet, Schelling (1960, p. 34) in a broader context already noted that "it may be difficult to conceive of a really firm commitment."

The tradeoff between commitment and flexibility has been discussed for problems of self-control where an individual or a government, by at least partially committing to a future course of action, attempts to improve the time-consistency of behavior (Gharad et al., 2010), e.g., in the context of intertemporal consumption problems (Amador et al., 2006). A plethora of "commitment bonds" is available to enable individuals, firms, and governments to achieve a degree of commitment (Abramowicz and Ayres, 2012), for instance using tax earmarking in environmental policy (Marsiliani and Renström, 2000).

In a strategic setting, the strength of commitment can vary with the size of an irreversible investment (Dixit, 1980), or with the perishability of a durable good for a monopolist (Cho, 2007). Henkel (2002) shows that intermediate commitment might sometimes be optimal when players' actions are strategic complements. Our (multidimensional) analysis does not depend on assumptions of strategic complementarity or substitutability. The price for this generality is that we focus on quadratic adjustment costs, for which explicit solutions can be obtained.[2] Caruana and Einav (2008) examine the question of endogenous commitment, that is: *who* decides to commit? In their model, commitment can be achieved through waiting and thereby raising a switching cost which is bound to increase over time. In the present paper, we focus on the question of *how* to best commit, providing optimality conditions and examining the possibility of conserving the self-imposed adjustment cost by making it a side-payment to the agent.

1.2. Outline

In Section 2 we introduce the commitment problem and its solution for a given commitment-cost parameter, via backward induction and simultaneous equations (Proposition 1). In Section 3 the optimal commitment level is characterized (Proposition 2). We also provide conditions under which at least partial conservation of the adjustment cost is optimal (Proposition 3). Applications are discussed in Section 4. Section 5 concludes.

2. Model

A principal and an agent play an alternating-moves game over three time periods $t \in \{0, 1, 2\}$. At time $t = 0$, the principal (she) selects an action $\alpha \in \mathbb{R}^n$ and an adjustment-cost parameter $\beta \geq 0$. At time $t = 1$, the agent (he) observes the principal's time-0 choice $\theta = (\alpha, \beta)$ and picks an action $x \in \mathbb{R}^m$. At time $t = 2$, the principal observes the agent's time-1 action and then has the option to revise her action from α to $y \in \mathbb{R}^n$ at the adjustment cost[3]

$$K(y, \theta) \triangleq \beta \|y - \alpha\|^2/2.$$

Subsequently, the agent obtains the payoff $U(x, y)$, and the principal obtains the payoff

$$W(x, y, \theta) \triangleq V(x, y) - K(y, \theta),$$

where the functions $U, V : \mathbb{R}^{m+n} \to \mathbb{R}$ are assumed to be thrice continuously differentiable (at least piecewise). For any (x, y), we assume that $U(\cdot, y)$ be strictly quasiconcave and $V(x, \cdot)$ strongly concave.[4]

Remark 1. For simplicity, the model abstracts from constraints on the players' actions. This simplification is without loss of generality, for any relevant constraints can be integrated into the players' payoffs as convex costs (barrier functions).

[1] There is a related discussion in the literature for games where one party moves first and then several players move simultaneously. Fudenberg and Tirole (1984) provide the corresponding taxonomy. The notion of strategic substitutes (and strategic complements) was introduced by Bulow et al. (1985).

[2] Henkel's analysis focuses on 0/1-type adjustment costs, which leads to a negative result for strategic substitutes, stating that intermediate commitment cannot lead to an improvement in equilibrium (Henkel, 2002, Prop. 2). For quadratic adjustment costs, we provide an example for the optimality of intermediate commitment when the players' actions are strategic substitutes; see Example 2 in Section 4.

[3] $\|\cdot\|$ denotes the Euclidean 2-norm.

[4] These assumptions are inessential and can be relaxed. It is enough that, for any $(x, y) \in \mathbb{R}^{m+n}$, each of the functions $U(\cdot, y)$ and $V(x, \cdot)$ attains its global maximum at a unique point.

2.1. Backward induction

To ensure that all threats in the above three-period commitment game are credible, we restrict attention to subgame-perfect Nash equilibria, which can be found via backward induction. At time $t = 2$, given a tuple (x, θ) the principal solves

$$\hat{g}(x, \theta) \in \arg\max_y W(x, y, \theta).$$

By assumption, the solution $\hat{g}(x, \theta)$ exists and can be obtained as the unique solution of the first-order necessary optimality condition,[5]

$$W_y(x, \hat{g}(x, \theta), \theta) = 0. \tag{$**$}$$

Differentiating the last equation with respect to x implies that

$$\hat{g}_x(x, \theta) = -W_{yy}^{-1}(x, \hat{g}(x, \theta), \theta)\, W_{xy}(x, \hat{g}(x, \theta), \theta),$$

where $W_{yy} < 0$ by assumption. At time $t = 1$, given θ the agent solves

$$f(\theta) \in \arg\max_x U(x, \hat{g}(x, \theta)).$$

Thus, using the abbreviation $g(\theta) \triangleq \hat{g}(f(\theta), \theta)$, the corresponding first-order necessary optimality condition is

$$\begin{aligned} &U_x(f(\theta), g(\theta)) - U_y(f(\theta), g(\theta))\, W_{yy}^{-1}(f(\theta), g(\theta), \theta) \\ &\quad \times W_{xy}(f(\theta), g(\theta), \theta) = 0. \end{aligned} \tag{$*$}$$

The first term on the right-hand side of Eq. $(*)$ is referred to as the *direct effect* the agent's action has on his payoff, the second term (including the minus sign) denotes the *indirect effect* of his action, through the principal's anticipated reaction (Fudenberg and Tirole, 1984). At time $t = 0$, the principal solves

$$\theta^* \in \arg\max_{\theta \in \Theta} W(f(\theta), g(\theta), \theta),$$

where $\Theta \subseteq \mathbb{R}^m \times \mathbb{R}_+$ is nonempty; as long as Θ is compact, a solution to the principal's time-0 problem exists by the Weierstrass theorem (Bertsekas, 2003, p. 85). Determining a solution for this problem via the first-order optimality condition is complicated by the need for the derivative of $(f(\theta), g(\theta))$, which may be obtained from Eq. $(*)$. Instead, it is more convenient to use a first-order approach and solve an optimization problem subject to conditions $(*)$ and $(**)$ as incentive constraints.

2.2. First-order approach

An alternative method of dealing with the principal's problem is to use a first-order approach, disaggregating the principal's objective function $W = V - K$, thus considering the problem

$$\max_{x,y,\theta} \{V(x, y) - K(y, \theta)\},$$

subject to

$$\begin{aligned} M(x, y, \beta) &\triangleq U_x(x, y) + U_y(x, y)\left(\beta I_n - V_{yy}(x, y)\right)^{-1} V_{xy}(x, y) \\ &= 0, \end{aligned} \tag{1}$$

where I_n is an $n \times n$ identity matrix, and

$$N(x, y, \theta) \triangleq V_y(x, y) - K_y(y, \theta) = 0. \tag{2}$$

The corresponding Lagrangian is of the form

$$\mathcal{L} \triangleq V - K - \mu \cdot M - \nu \cdot N,$$

where $\mu \in \mathbb{R}^m$ and $\nu \in \mathbb{R}^n$ are the Lagrange multipliers associated with the incentive constraints (1) and (2); the latter correspond to the constraints $(*)$ and $(**)$ above. To determine the principal's optimal level of commitment β^* one can first solve her problem given any fixed positive cost parameter β.

[5] Partial derivatives are denoted by subscripts.

Proposition 1. *For any fixed $\beta > 0$, an equilibrium strategy profile $(x^*(\beta), y^*(\beta), \theta^*(\beta))$, with $\theta^*(\beta) = (\alpha^*(\beta), \beta)$, is such that*[6]

$$(x^*(\beta), y^*(\beta)) \in \mathcal{C}_\beta,$$

$$\mathcal{C}_\beta = \left\{(x, y) \in \mathbb{R}^{m+n} : C(x, y, \beta) \triangleq \begin{bmatrix} M(x, y, \beta) \\ V_y(x, y) - \beta\hat{\nu}(x, y, \beta) \end{bmatrix} = 0\right\},$$

and

$$\alpha^*(\beta) = y^*(\beta) - \nu^*(\beta),$$

given that $\nu^*(\beta) = \hat{\nu}(x^*(\beta), y^*(\beta), \beta)$, *with*

$$\hat{\nu} \triangleq V_x\left(M_x + M_y^\top(\beta I_n - V_{yy})^{-1}V_{xy}\right)^{-1} M_y(\beta I_n - V_{yy})^{-1}.$$

This last result provides a representation of all attainable payoff-relevant strategy profiles in equilibrium, as a function of β. It also shows that the problem of determining the set of equilibria $\mathcal{C}_\beta$ in (x, y)-space can be separated from the question of finding the principal's time-0 action $\alpha^*(\beta)$. Furthermore, the costly equilibrium deviation $y^*(\beta) - \alpha^*(\beta)$ is equal to the Lagrange-multiplier value $\nu^*(\beta)$, which can be computed using the function $\hat{\nu}(x, y, \beta)$, independent of α. Hence, in this representation the principal obtains all attainable outcomes $(x^*(\beta), y^*(\beta))$ (see Fig. 1 for Example 2 in Section 4), together with the corresponding "implementation cost"

$$\kappa(\beta) \triangleq K(y^*(\beta), \theta^*(\beta)) = \beta\nu^*(\beta). \tag{3}$$

Proof. Let $\beta > 0$ be fixed. The optimality conditions

$$\mathcal{L}_x = V_x - \mu M_x^\top - \nu N_x^\top = 0, \tag{4}$$

$$\mathcal{L}_y = -\mu M_y^\top - \nu N_y^\top \mu = 0, \tag{5}$$

can, after transposition, be written in the form

$$\begin{bmatrix} M_x & N_x \\ M_y & N_y \end{bmatrix} \begin{bmatrix} \mu^\top \\ \nu^\top \end{bmatrix} = \begin{bmatrix} V_x^\top \\ 0 \end{bmatrix}.$$

Using the block-matrix inversion formula by Banachiewicz (1937), the last equation yields

$$\mu^\top = \left(M_x - N_x N_y^{-1} M_y\right)^{-1} V_x^\top,$$

$$\nu^\top = -N_y^{-1} M_y \left(M_x - N_x N_y^{-1} M_y\right)^{-1} V_x^\top.$$

Since the square matrices M_x and N_y are by the Schwarz theorem (Zorich, 2004, p. 459) symmetric, one obtains that

$$\mu = V_x\left(M_x + M_y^\top(\beta I_n - V_{yy})^{-1}V_{xy}\right)^{-1},$$

$$\nu = V_x\left(M_x + M_y^\top(\beta I_n - V_{yy})^{-1}V_{xy}\right)^{-1} M_y(\beta I_n - V_{yy})^{-1},$$

where $V_{yx}^\top = V_{xy}$. Consider now the optimality condition

$$\mathcal{L}_\alpha = -K_\alpha + \nu K_{y\alpha} = \beta(y - \alpha) - \beta\nu = 0. \tag{6}$$

Since $\beta \neq 0$, Eq. (6) implies that

$$\nu = y - \alpha.$$

From the preceding arguments it becomes clear that for any given (x, y), the optimality conditions (4)–(6) yield the corresponding tuple (α, μ, ν). Moreover, in equilibrium it is $\beta\nu = \beta\hat{\nu}(x, y, \beta) = \beta(y - \alpha)$, so that $C(x, y, \beta) = 0$ is equivalent

[6] The gradients M and N are of dimension $1 \times m$ and $1 \times n$, respectively. The multipliers μ, ν are row vectors with corresponding dimensions. The transpose of a matrix X is denoted by $X^\top$.

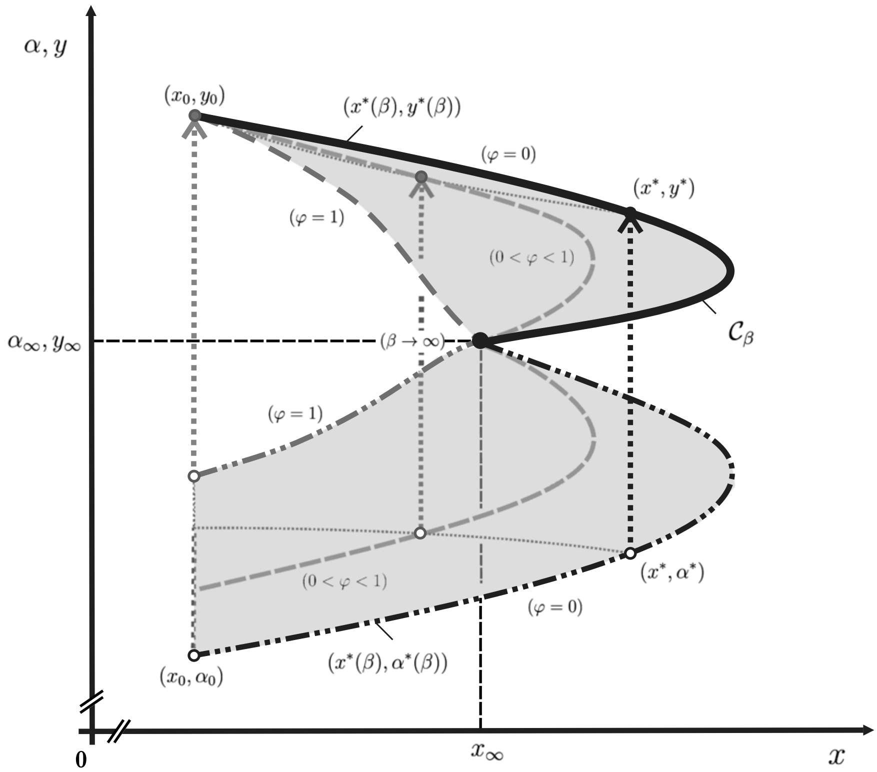

Fig. 1. Implementable strategy profiles in Example 2, for $\beta \in \bar{\mathbb{R}}_+$ and $\varphi \in [0, 1]$.

to the incentive constraints (1) and (2), which together determine $(x^*(\beta), y^*(\beta)) \in \mathcal{C}_\beta$. Hence, taking into account the earlier expression for ν, the principal's optimal time-0 action becomes

$$\alpha^*(\beta) = y^*(\beta) - \hat{\nu}(x^*(\beta), y^*(\beta), \beta) = y^*(\beta) - \nu^*(\beta),$$

which completes the proof. □

Remark 2. (i) In the absence of any commitment, i.e., for $\beta = 0$, it is

$$\mathcal{C}_0 = \left\{(x, y) \in \mathbb{R}^{m+n} : M(x, y, 0) = 0 \text{ and } V_y(x, y) = 0\right\}.$$

Since the adjustment cost vanishes, the principal's time-0 action α_0 is inconsequential and therefore undetermined. Without loss of generality, it can be set to the limit of $\alpha^*(\beta)$ in Proposition 1 for $\beta \to 0^+$, whence

$$\alpha_0 = y_0 - \hat{\nu}(x_0, y_0, 0).$$

(ii) With full commitment, i.e., for $\beta \to \infty$, one obtains

$$\mathcal{C}_\infty = \Big\{(x, y) \in \mathbb{R}^{m+n} : U_x(x, y) = 0 \text{ and } V_y(x, y) - V_x(x, y)U_{xx}^{-1}(x, y)U_{yx}(x, y) = 0\Big\}.$$

In that case, the principal's time-0 action is $\alpha_\infty = \lim_{\beta\to\infty} \alpha^*(\beta) = y_\infty$ because the principal's equilibrium adjustment, $\nu^*(\beta) = y^*(\beta) - \alpha^*(\beta)$, tends to zero as $\beta \to \infty$.

Remark 3. Consider the one-dimensional case where $m = n = 1$. Let $\lambda = \frac{V_{yy}}{V_{yy}-\beta} \in [0, 1]$ for all $\beta \in \mathbb{R}_+ \triangleq [0, \infty]$.[7] Then the incentive constraints (1) and (2) become

$$\frac{U_x}{U_y} = \lambda \left(\frac{V_{xy}}{V_{yy}}\right) \quad \text{and} \quad \frac{V_y}{V_x} = (1 - \lambda) \left(\frac{\rho_x}{\rho_y} - \frac{U_x}{U_y}\right)^{-1},$$

where

$$\rho \triangleq (-V_{yy}) \left(\frac{V_{xy}}{V_{yy}} - \frac{U_x}{U_y}\right) \left(\frac{U_x}{U_y}\right)^{-1}$$

is the (normalized) relative difference between the strategic and the direct effect for the agent's problem. These constraints determine the players' direct rates of substitution U_x/U_y and V_y/V_x, as well as the (second-order) strategic rates of substitution V_{xy}/V_{yy} and ρ_x/ρ_y, in equilibrium. By eliminating λ one obtains that the implementable strategy profiles (x, y) satisfy

$$F(x, y) \triangleq \left[\left(\frac{U_x}{U_y} - \frac{V_{xy}}{V_{yy}}\right) - \left(\frac{V_{xy}}{V_{yy}}\right)\left(\frac{U_x}{U_y} - \frac{\rho_x}{\rho_y}\right)\left(\frac{V_y}{V_x}\right)\right]\Bigg|_{(x,y)} = 0.$$

The *commitment continuum* $\mathcal{C} \triangleq \bigcup_{\beta\in\mathbb{R}_+} \mathcal{C}_\beta$ is a subset of

$$\left\{(x, y) : F(x, y) = 0\right\}.$$

3. Optimal commitment

For any feasible commitment-cost parameter β, the principal's equilibrium payoff is by Proposition 1 and Eq. (3) equal to

$$\bar{W}(\beta) = V(x^*(\beta), y^*(\beta)) - \kappa(\beta),$$

where $(x^*(\beta), y^*(\beta)) \in \mathcal{C}_\beta$.[8] At the two extreme commitment levels (for $\beta = 0$ or $\beta \to \infty$) the principal obtains

$$\bar{W}_0 \triangleq V(x_0, y_0) \quad \text{and} \quad \bar{W}_\infty \triangleq V(x_\infty, y_\infty),$$

[7] For details on the (affine) extension of the real numbers, $\bar{\mathbb{R}} = [-\infty, +\infty]$, see, e.g., Aubin (1977, Sec. 1.3).

[8] By the maximum theorem (Berge, 1963, p. 116) the value function $\bar{W}(\cdot)$ is continuous. If $\mathcal{C}_\beta$ is a singleton for all β, it is also differentiable as a composition of differentiable functions, by virtue of the implicit function theorem (Rudin, 1976, p. 224–225). Milgrom and Segal (2002) provide a more general version of the envelope theorem for arbitrary choice sets.

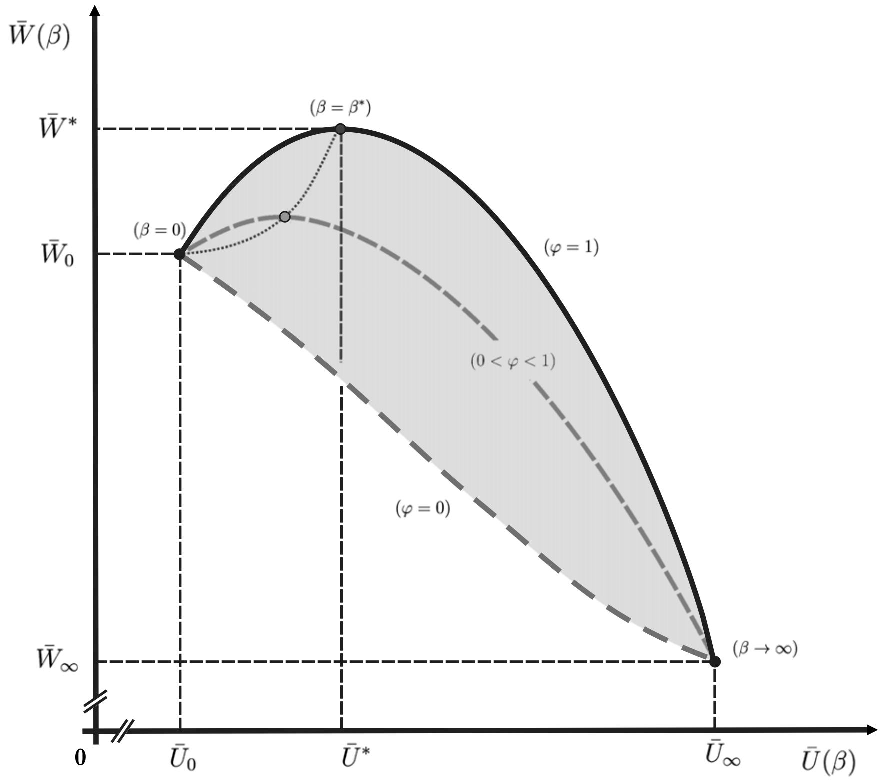

Fig. 2. Attainable payoffs in Example 2, for $\beta \in \bar{\mathbb{R}}_+$ and $\varphi \in [0, 1]$.

respectively, where (x_0, y_0), (x_∞, y_∞) are given in Remark 2, and $\kappa(0) = \lim_{\beta\to\infty} \kappa(\beta) = 0$. At intermediate commitment levels $\beta \in (0, \infty)$, the variation of the principal's value function can be determined explicitly. Analogous to the earlier definition of $\nu^*(\beta)$, let $\mu^*(\beta) = \hat{\mu}(x^*(\beta), y^*(\beta), \beta)$, where $\hat{\mu} = V_x(M_x - M_y^\top (V_{yy} - \beta I_n)^{-1})V_x^\top$.

Proposition 2. (i) *For any $\beta > 0$, the change in the principal's value function is*

$$\bar{W}'(\beta) = \frac{\|\nu^*(\beta)\|^2}{2} - \mu^*(\beta) V_{xy}^\top \left(\beta I_n - V_{yy}\right)^{-2} U_y^\top \Big|_{(x^*(\beta), y^*(\beta))}.$$

(ii) *An optimal commitment level $\beta^* \in \bar{\mathbb{R}}_+$ exists and satisfies*

$$\beta^* \in \arg\max_{\beta \in \mathcal{B}} \bar{W}(\beta),$$

where $\mathcal{B} = \{\beta > 0 : \bar{W}'(\beta) = 0\} \cup \{0, \infty\}$ and $\bar{W}(\infty) \triangleq \bar{W}_\infty$.

The last result allows finding the principal's optimal commitment level β^*, which could possibly arise at an extreme. If the optimal commitment level β^* is interior, it can be computed as a root of the closed-form expression for $\bar{W}'(\cdot)$ in part (i) of Proposition 2.

Proof. (i) By the envelope theorem, the change of the principal's value function is

$$\bar{W}'(\beta) = \mathcal{L}_\beta(x^*(\beta), y^*(\beta), \theta^*(\beta)) \triangleq \mathcal{L}_\beta^*(\beta).$$

By virtue of Proposition 1, the variation of the Lagrangian along the optimal trajectory (x^*, y^*) with respect to β is

$$\mathcal{L}_\beta^* = -\frac{\|\nu^*\|^2}{2} - \mu^* U_y(x^*, y^*) \left[\beta I_n - V_{yy}(x^*, y^*)\right]^{-2} V_{xy}(x^*, y^*) + \|\nu^*\|^2,$$

which implies the given formula for $\bar{W}'(\beta)$. (ii) By Fermat's lemma at any local extremum $\hat{\beta} \in (0, \infty)$ of $\bar{W}(\cdot)$ its slope must vanish (Weber, 2011, p. 247), so $\bar{W}'(\hat{\beta}) = 0$.[9] Comparing the principal's payoff $\bar{W}(\hat{\beta})$ for all $\hat{\beta}$ for which $\bar{W}'(\beta) = 0$ and the values $\bar{W}_0$ (for $\beta = 0$) and $\bar{W}_\infty$ (for $\beta \to \infty$) yields an optimal commitment level β^* in $\bar{\mathbb{R}}_+$ at which the principal's equilibrium payoff is maximized. □

Remark 4. Consider the one-dimensional case, where $m = n = 1$. The commitment level $\beta \in (0, \infty)$ is extremal,[10] i.e., is an element of $\mathcal{B}$, if and only if

$$\beta = \left[V_{yy} - \left(\frac{V_x M_y}{2U_y} + \frac{V_{xy} M_x}{M_y} \right)^{-1} V_{xy}^2 \right] \Bigg|_{(x^*(\beta), y^*(\beta))} > 0.$$

By varying the level of commitment the principal can implement a continuum of outcomes (x, y) and associated payoff vectors (U, V). Her own net payoff $W = V - K$ is computed including the adjustment cost. Fig. 1 shows the attainable net payoff vectors (U, W) for Example 2 in Section 4.

Adjustment-cost conservation. In order to further increase the payoff-attainability set (shaded region in Fig. 2), the principal can pay a fraction $\varphi \in [0, 1]$ of the adjustment cost to the agent at time $t = 2$, as a direct compensation for any change in the initially

[9] $\bar{W}'(\beta)$ also vanishes for $\beta \to 0^+$ and for $\beta \to \infty$, so $\mathcal{B} = \{\beta \in \bar{\mathbb{R}}_+ : \bar{W}'(\beta) = 0\}$. Indeed, for $\beta \to \infty$ it is $\nu^*(\beta) \to 0$ and $\mu^*(\beta) \to -U_{xx}^{-1}(x_\infty, y_\infty)V_x(x_\infty, y_\infty)$, and therefore $\mathcal{L}_\beta^* \to 0$ as $\beta \to \infty$.

[10] A commitment level β is *extremal* if it is a critical point of the principal's net payoff $\bar{W}$ (i.e., $\beta \in \mathcal{B}$); it is *extreme* for either zero or full commitment (i.e., $\beta = 0$ or $\beta = \infty$).

announced action. For example, a regulator (the principal) could hope to strengthen the power of her commitment as a motivational tool that prompts the agent to invest in a clean technology. Under this cost-conservation scheme, the agent's payoff becomes

$$\hat{U} \triangleq U + \varphi K,$$

and the question is, under what condition the equilibrium fraction $\hat{\varphi}^*$ is positive.

Proposition 3. *In equilibrium, $\hat{\varphi}^* > 0$ if $V_{xy}^\top(\beta^* I_n - V_{yy})^{-2} M_y|_{(x^*,y^*,\beta^*)} > 0$, where $x^* = x^*(\beta^*)$, $y^* = y^*(\beta^*)$, and β^* describe an equilibrium for $\varphi = 0$.*

Intuitively, the motivational force of the adjustment cost increases when in part paid to the agent, if the slopes of the incentive constraints with respect to the other player's action, i.e., M_y and $N_x = V_{xy}^\top$, are aligned. This means that there is a "sequential strategic symmetry", in the sense that the players' actions are either "sequential strategic complements" or "sequential strategic substitutes." As opposed to the standard notion of strategic complementarity and substitutability (Bulow et al., 1985), the sequential version here includes the strategic effect experienced by the agent; in particular, M_y depends on the principal's payoff function V. The alignment of the (matrix-valued) cross-effects M_y and N_x is normalized by the principal's Hessian (with respect to his action vector y).

Proof. For $\varphi > 0$, the principal's objective and her incentive constraint (2) remain unchanged compared to the base case where $\varphi = 0$. The agent's incentive constraint (1) becomes

$$\hat{M} \triangleq M + (\varphi\beta)(y - \alpha)(\beta I_n - V_{yy})^{-1} V_{xy} = 0.$$

The Lagrangian in the presence of possible side-payments is

$$\hat{\mathcal{L}} \triangleq \mathcal{L} + \mu \cdot \left(\hat{M} - M\right).$$

Hence

$$\hat{\mathcal{L}}_\varphi^* = \beta\mu V_{xy}^\top(\beta I_n - V_{yy})^{-1}\nu^\top = \beta\mu\left[V_{xy}^\top(\beta I_n - V_{yy})^{-2}\hat{M}_y\right]\mu^\top,$$

where

$$\mu = V_x\left(\hat{M}_x + \hat{M}_y^\top(\beta I_n - V_{yy})^{-1}V_{xy}\right)^{-1}.$$

Thus, for $\beta > 0$, $\hat{\mathcal{L}}_\varphi^*$ is a quadratic form, which is positive if the positive-definiteness condition in Proposition 3 is satisfied, which completes our proof.[11] □

Remark 5. In the one-dimensional case where $m = n = 1$, a positive side-payment fraction $\hat{\varphi}^* > 0$ is optimal for the principal in equilibrium if $N_x M_y = V_{xy} M_y > 0$.

4. Applications

We now discuss two applications of the results developed in Sections 2 and 3.

Example 1. As shown by Gal-Or (1985), the standard symmetric Cournot duopoly, where $U(x, y) = (1 - x - \gamma y)x$ and $V(x, y) = U(y, x)$, for $\gamma \in [-1, 1]$, exhibits a first-mover advantage for strategic substitutes ($\gamma > 0$) and a second-mover advantage for strategic complements ($\gamma < 0$).[12] Accordingly, Proposition 2 yields that $\beta^* = 0$ for $\gamma \in [-1, 0)$, $\beta^* = \infty$ for $\gamma \in (0, 1]$, and $\beta^* \in \bar{\mathbb{R}}_+$ for $\gamma = 0$. An intermediate commitment level is never strictly optimal.

Example 2. Suppose that $U(x, y) = (15 - 2x - 3y)x$ and $V(x, y) = (x + y - 2)(7 - y) - (x^2/2)$. For any fixed $\beta > 0$, by Proposition 1 the attainable equilibria in $\mathcal{C}_\beta$ are obtained where both M and N vanish, i.e., where

$$(x^*(\beta), y^*(\beta)) \in \mathcal{C}_\beta = \left\{\left(\frac{6\left(5\beta^2 + 11\beta + 2\right)}{17\beta^2 + 29\beta + 8}, \frac{3\left(15\beta^2 + 29\beta + 10\right)}{17\beta^2 + 29\beta + 8}\right)\right\},$$

and

$$\alpha^*(\beta) = \frac{9\left(5\beta^2 + 6\beta + 1\right)}{17\beta^2 + 29\beta + 8}.$$

For the extreme commitment options of $\beta = 0$ and $\beta \to \infty$, one obtains by Remark 2 or, alternatively, by taking the corresponding limits in the formulas for $(x^*(\beta), y^*(\beta), \alpha^*(\beta))$:

$$(x_0, y_0, \alpha_0) = \left(\frac{3}{2}, \frac{15}{4}, \frac{9}{8}\right) \quad \text{and}$$

$$(x_\infty, y_\infty, \alpha_\infty) = \left(\frac{30}{17}, \frac{45}{17}, \frac{45}{17}\right).$$

Fig. 1 depicts the continuum of "implementable" strategy profiles for $\beta \in \bar{\mathbb{R}}_+$, including (in the shaded region) the possibility of a transfer of adjustment costs from the principal to the agent for $\varphi \in (0, 1]$. For $\varphi = 0$, the corresponding profit for the principal is

$$\bar{W}(\beta) = \frac{1}{2}\,\frac{304\beta^2 + 571\beta + 151}{17\beta^2 + 29\beta + 8}.$$

Furthermore, $\bar{W}_0 = 151/16 \approx 9.44$ and $\bar{W}_\infty = 152/17 \approx 8.94$. By Proposition 2, the optimal commitment level is $\beta^* = 1/3$, maximizing the principal's payoff, $\bar{W}^* = \bar{W}(\beta^*) = 211/22 \approx 9.59$. This presents a 1.6% improvement over no commitment and a 7.3% improvement over full commitment. Fig. 2 shows the set of attainable net payoff vectors (U, W) including a possible adjustment-cost transfer to the agent of proportion $\varphi \in [0, 1]$. Any positive fraction $\varphi > 0$ is Pareto-dominated by zero adjustment-cost conservation, $\varphi = 0$, which highlights that the wastefulness of the adjustment expense can be desirable from a social viewpoint.[13] Fig. 3 depicts the improvement of intermediate commitment relative to the iso-V contours for extreme commitment.

5. Discussion

The ability to commit derives from a possibly self-imposed inertia with respect to an initial decision. The propensity for inaction can be quantified by the adjustment cost at which the decision may be modified. In the generic three-stage commitment game discussed in the preceding sections, the first-moving principal is able (at time 0) to influence both the agent's (at time 1) and her own decision problem (at time 2). Augmenting the adjustment cost accomplishes two things: it lowers the strategic

[11] For $\varphi = 0$, it is $\hat{M}_x = M_x$ and $\hat{M}_y = M_y$. By Proposition 1, $\alpha^*(\beta^*) = y^*(\beta^*) - \hat{\nu}(x^*(\beta^*), y^*(\beta^*), \beta^*)$.

[12] The results apply symmetrically to the Bertrand duopoly with differentiated goods.

[13] In practice, one may divert adjustment-cost-related funds to socially beneficial but strategically unrelated causes, e.g., a reduction of the budget deficit when the principal is the government.

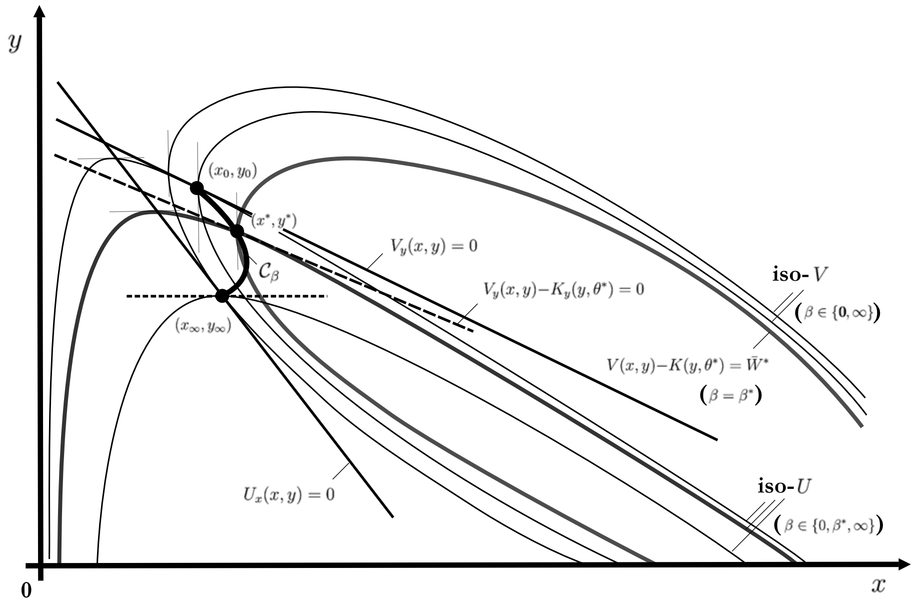

Fig. 3. Implementable outcomes $(x^*(\beta), y^*(\beta))$ for $\beta \in \{0, \beta^*, \infty\}$ in Example 2.

effect of her action in the agent's decision problem, and, at the same time, it increases the strategic effect of his (i.e., the agent's) action in her decision problem. At the extremes of the choice spectrum, the principal obtains either no commitment as a second mover (forgetting about the initial choice in the last period) or full commitment as a first mover (completely locking in the initial choice). Specifying her own adjustment cost affords the principal the flexibility of choosing from a continuum of implementable outcomes (see Fig. 1). In practice, adjustment costs can be specified, for example, in the form of voting thresholds for the change of a regulatory policy, a price guarantee or controlled obsolescence in a durable-goods monopoly, as well as commitment bonds of varying denominations issued by third parties.

A somewhat surprising outcome, highlighted by Example 2, is that the wastefulness of the adjustment cost may be both privately and socially desirable. If the agent is subsidized by a fraction of the principal's adjustment cost, then the agent's response in anticipation of the principal's time-2 deviation is moderated, which may result in a tragedy-of-the-commons decreasing overall surplus due to the tighter coupling between the players' marginal incentives. Specifically, since the principal's marginal adjustment cost is, in equilibrium, equal to the marginal gross benefit of her action (the direct effect), adding it to the agent's incentive constraint makes the principal's direct effect a part of the agent's strategic effect. In Proposition 3 we have shown that the principal finds the cost conservation as side-payment to the agent beneficial if there is "sequential strategic symmetry" in the players' actions. From a practical viewpoint, it is remarkable that intermediate commitment options can generate noticeable payoff improvements, despite the positive adjustment-cost payout in equilibrium. Hence, when a regulatory commitment is sought to secure the agent's cooperation with the principal's objective, an optimum may well be achieved by means of moderate rather than extreme (non-)commitment.

References

Abramowicz, M., Ayres, I., 2012. Commitment bonds. Georgetown Law J. 100 (3), 605–656.
Amador, M., Werning, I., Angeletos, G.-M., 2006. Commitment vs. flexibility. Econometrica 74 (2), 365–396.
Aubin, J.-P., 1977. Applied Abstract Analysis. Wiley, New York, NY.
Bagwell, K., Wolinsky, A., 2002. Game theory and industrial organization. In: Aumann, R., Hart, S. (Eds.), Handbook of Game Theory with Economic Applications, Vol. 3. Elsevier, Amsterdam, Netherlands, pp. 1851–1895.
Banachiewicz, T., 1937. Zur Berechnung der Determinanten, wie auch der Inversen und zur darauf basierten Auflösung der Systeme linearer Gleichungen. Acta Astron. Ser. C 3, 41–67.
Berge, C., 1963. Topological Spaces. Oliver and Boyd, London, UK (Reprinted by Dover Publications, Mineola, NY, in 1997).
Bertsekas, D.P., 2003. Convex Analysis and Optimization. Athena Scientific, Belmont, MA.
Bulow, J.I., Geneakoplos, J.D., Klemperer, P.D., 1985. Multimarket oligopoly: strategic substitutes and complements. J. Political Economy 93 (3), 488–511.
Caruana, G., Einav, L., 2008. A theory of endogenous commitment. Rev. Econom. Stud. 75 (1), 99–116.
Cho, I.-K., 2007. Perishable durable goods, Working Paper, Department of Economics, University of Illinois at Urbana-Champaign.
Dixit, A.K., 1980. The role of investment in entry-deterrence. Econ. J. 90 (357), 95–106.
Fudenberg, D., Tirole, J., 1984. The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. Amer. Econ. Rev. 74 (2), 361–366.
Gal-Or, E., 1985. First mover and second mover advantages. Internat. Econom. Rev. 26 (3), 649–653.
Gharad, B., Karlan, D., Nelson, S., 2010. Commitment devices. Annu. Rev. Econ. 2, 671–698.
Henkel, J., 2002. The 1.5th mover advantage. Rand J. Econ. 33 (1), 156–170.
Marsiliani, L., Renström, T.I., 2000. Time inconsistency in environmental policy: tax earmarking as a commitment solution. Econ. J. 110 (462), 123–138.
Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. Econometrica 70 (2), 583–601.
Rudin, W., 1976. Principles of Mathematical Analysis, third Ed. McGraw-Hill, New York, NY.
Schelling, T.C., 1960. The Strategy of Conflict. Harvard University Press, Cambridge, MA.
von Stackelberg, H., 1934. Marktform und Gleichgewicht. Springer, Vienna, Austria.
Weber, T.A., 2011. Optimal Control Theory with Applications in Economics. MIT Press, Cambridge, MA.
Zorich, V.A., 2004. Mathematical Analysis I. Springer, New York, NY.