Sustainable Production and Consumption 19 (2019) 79–96

Contents lists available at ScienceDirect

Sustainable Production and Consumption

journal homepage: www.elsevier.com/locate/spc

Research article

Strategic durability with sharing markets

Maryam Razeghian [a], Thomas A. Weber [b,*]

[a] École Polytechnique Fédérale de Lausanne, Station 5, CH-1015 Lausanne, Switzerland
[b] Chair of Operations, Economics and Strategy, École Polytechnique Fédérale de Lausanne, Station 5, CH-1015 Lausanne, Switzerland

ARTICLE INFO

Article history:
Received 12 October 2018
Received in revised form 3 February 2019
Accepted 9 February 2019
Available online 27 February 2019

JEL classification:
C72
D11
D23
D61

Keywords:
Collaborative consumption
Durability
Peer-to-peer markets
Planned obsolescence
Sharing economy

ABSTRACT

Electronic sharing markets are contributing to a paradigm shift, from consuming products to accessing products. This paper studies the effects of sharing markets on the prices for new products and on product design in terms of durability. In a dynamic economy with overlapping generations, consumers take strategic purchasing decisions, anticipated by a durable-goods monopolist. Without sharing, the optimal durability increases in the production cost. For low-cost products, a producer prefers to limit durability, effectively disabling a secondary sharing market. The presence of sharing never decreases the incentives to provide durability thus contributing to sustainable product design.

1. Introduction

In the face of a paradigm shift in consumer preferences from ownership to access, manufacturers need to strategically rethink product design. Emerging peer-to-peer sharing markets, run on platforms such as eLoue, LocalTools, or ZipCar, now include a broad variety of products and enjoy a sizeable transaction volume. The liquidity of sharing markets depends on the absolute number of lenders and borrowers as well as the balance between them. A key determinant for a good to be available on a sharing market, ready to be used by a peer, is its *durability*, i.e., the likelihood with which it can be used again in a future period. Durability is not an intrinsic product characteristic, but can rather be considered a design feature, subject to deliberate choice and technological feasibility. This paper investigates durability as a strategic choice in addition to the product price, in the presence of a functioning peer-to-peer exchange which offers the possibility for aftermarket transactions.

Anecdotal evidence suggests that manufacturers are beginning to actively respond to sharing markets. Some embrace existing platforms while others offer their own sharing alternative. For example, Toyota has recently invested over $100 million in Uber, thus planning – as part of a strategic alliance – to offer leases to Uber drivers (MacMillan, 2016). General Motors and Volkswagen have put substantial amounts in Lyft ($300 million) and Gett ($500 million), respectively (ibid.). Meanwhile, BMW is operating DriveNow since 2011 as a joint venture with Sixt, whereas Audi and General Motors have started their own proprietary sharing platforms.[1] Because of the inertia a firm has to overcome when modifying product design and the resulting delay, as well as the inherent lag when trying to measure product failure, observations of companies' sharing-related product-design decisions are – as of now – not (yet) readily available in the public domain. We therefore take an agnostic position and analyze a firm's economic incentives to adapt the durability of their goods as a consequence of sharing markets. The model, which is concerned with the provision of product durability (and not the organization of proprietary sharing markets), shows that depending on the production cost and the consumers' level of patience, a firm's benefits from sharing and concomitant incentives to provide product durability vary significantly.

Although earlier studies have attempted to capture the effect of sharing on product sales, purchase price, and product quality (Jiang and Tian, 2018; Weber, 2016), to the best of our knowledge the effectiveness of durability as a deliberate strategic tool for

* Corresponding author.
E-mail address: thomas.weber@epfl.ch (T.A. Weber).

[1] See, e.g., Audi press release in the USA (at audiusa.com on April 27, 2015), and Hawkins (2016) on the launch of GM's car-sharing platform Maven in Boston, Chicago, and Washington, DC.

https://doi.org/10.1016/j.spc.2019.02.007
2352-5509/

a manufacturer in the presence of a sharing market has not been explored thus far. By means of a dynamic model featuring overlapping generations of heterogeneous consumers, we analyze a firm's rational behavior with and without sharing markets. To illustrate some of the tradeoffs, consider the transaction volume on the sharing market. Without durability, there can be no secondary exchange because products are effectively disposable. This decreases the value of the product for consumers and thus the demand for ownership, all else equal. An increase in durability therefore also increases demand for the firm's product and the transaction volume on the secondary sharing market, at least up to a point. For larger levels of durability, there are fewer defective products that need short-term replacement which in turn decreases demand in the sharing market. This diminishes the *ex post* benefit to those owners who do not need the item and are ready to share, thus decreasing their willingness to pay *ex ante* for the new product. We examine the firm's rational balance between the various effects, also as a function of its commitment power.

1.1. Literature review

Chamberlin (1953) was among the first to recognize durability as a product attribute worthy of optimization[2]: "the problem is to find that length of life for [the firm's] product which will maximize [its] profit" (p. 24). Martin (1962) proposed a solution to this problem by separating the demand for ownership of the good and the demand for the service the good renders. In the benchmark setting with vanishing cost of durability, a monopolist would find it optimal to provide fully durable products to sufficiently patient consumers (with high discount factors), while when serving impatient consumers it is best for the firm to cut down on durability. Our model produces findings consistent with this early result; yet, it turns out that production costs play a decisive role in the firm's optimal product design because they determine its attitude towards sharing markets. Swan (1972) contends that since profit maximization requires cost minimization, the cheapest way for a monopolist to deliver a given flow of service is to provide products at an efficient durability level. Bulow (1986) argues that this type of analysis may be flawed because of dynamic inconsistency; the reoptimization of output (or price, as it may be in our model) leads the firm to generally opt for an inefficient level of durability. In fact, the issue of dynamic inconsistency cannot arise in our model with overlapping generations, as the firm is naturally drawn to a stationary policy, although it could in principle change its product design every period.[3] Fudenberg and Tirole (1998) use a two-period model to examine a monopolist's interaction with a second-hand market using upgrades and the possibility of repurchase.[4] Orbach (2004) notes that durable-goods monopolists face a commitment problem because they are selling bundles of present and future consumption that consumers may be reluctant to purchase, given uncertainty about future needs. We argue that sharing markets can ameliorate this situation by matching consumption need and availability of the good in the economy. In other words, there are situations – as we show, for high-cost products – in which a monopolist prefers the existence of sharing markets and then has an incentive to produce highly durable goods.

The recent interest in sharing markets from an economic viewpoint begins with the excellent conceptual overview by Benkler (2004). Belk (2007, 2010) discusses various modes of sharing of which we restrict attention here to behavior compatible with the *homo economicus* who compares the cost and benefits of sharing and does not derive extra utility from altruism. Recent studies on consumer behavior relative to sharing have been conducted by Ozanne and Ballantine (2010) on a toy library, Bardhi and Eckhardt (2012) on car sharing, and Lamberton and Rose (2012) in the contexts of sharing cars, cell-phone minutes, and bikes. Fraiberger and Sundararajan (2015) calibrate a dynamic-programming model to empirically identify a shift from car ownership to sharing for below-median income consumers. The intermediation aspect of sharing markets by a platform have been investigated by Weber (2014) as well as Benjaafar et al. (2015). Razeghian and Weber (2015) construct a diffusion model for sharing markets, including the effect of a commission charged by an intermediary. Here we abstract from the effect of a self-interested market maker, taking the view that asymmetric-information problems can be solved (e.g., using reputation or bitcoin-type protocols). The preceding study includes transaction costs, the effect of which is also considered by Horton and Zeckhauser (2016) who show that in the presence of a "bringing-to-market cost" ownership incentives increase and the transaction volume on the sharing market decreases.

This paper builds on the overlapping-generations model by Weber (2016) which shows that an active sharing market tends to augment the purchase price by positive sharing premium. However, there the focus is solely on pricing, and the product design remains entirely exogenous. Jiang and Tian (2018) study a manufacturer's pricing and quality decisions in the presence of a sharing market using a two-period model. In that setting, the sharing market has a positive effect on the firm's price and product quality, and the manufacturer enjoys a positive added profit in the presence of sharing. Our results differ in two ways: first, controlled durability (or induced obsolescence) is different from product quality as it includes product failure, thus potentially preventing future use for any consumer generation. Second, we obtain the critical role of durability as a strategic tool to choke off the sharing market, which has not been examined in the literature before. The firm's possible desire to shut down sharing means in fact that its profits without sharing may well be higher than with sharing, in contrast to the earlier findings relative to product quality.

1.2. Outline

The remainder of this paper is organized as follows. Section 2 introduces the model primitives: a monopolist's design choices and overlapping generations of heterogeneous consumers. Section 3 provides an equilibrium analysis of the consumers' dynamic choice behavior, the firm's preferred product design, and the resulting equilibrium in a sharing market—provided it exists. The company's choice depends on its ability to commit, and it may include the deliberate shutdown of sharing markets. Section 4 examines the aggregate impact of the peer-to-peer economy on consumers and on society, for the different commitment regimes. Section 5 concludes with a discussion of the

---

2 Waldman (2003) and Swan (2006) provide surveys of the durable-goods theory and its relation to practice.

3 Intra-period commitment issues do play a role in equilibrium for our model, but those yield an intuition in the spirit of the conjecture by Coase (1972): commitment ability helps the firm to extract revenue; see Remark 1.

4 The key difference between a sharing market in our model and a standard second-hand market in the literature – beyond the obvious lack of a transfer of ownership when sharing – is that sharing markets operate with the implicit purpose of mutual insurance, in the spirit of Arrow (1953), based on the agents' realized (ex-ante random) needs. In the context of the two-period overlapping-generations model developed in this paper, there is also a decisive structural difference: At any period agents have the option to rent goods from peers, rather than having to purchase them. In particular, an agent who decides to rent a good in his early consumption phase will *not* have the good at his disposal in his late consumption phase as it would be the case with a purchased second-hand good—modulo failure in the absence of perfect durability. Thus, first-period renters from a sharing market can reappear in the next period only on the demand side rather than on the supply side, as they might in the role of first-period buyers in a second-hand market. In addition, all goods disappear from the sharing market with their owners, while second-hand goods might well outlive their owners, traveling from generation to generation.

strategic nature of product design in view of the peer-to-peer economy, as well as model limitations together with managerial and societal implications of the results.

2. Model

We begin by considering a monopolist who is maximizing profits over an infinite time horizon. The monopolist has a constant unit production cost $c \in [0, \bar{c}]$, and at each time period $t \in \{0, 1, 2, \ldots\}$, the firm chooses both the product's price r_t and its durability $q_t \in [0, 1]$.[5] Over its maximum lifetime of two periods, which corresponds to the lifetime of any consumer in our model, failure after a first consumption period t occurs at the rate $1 - q_t$. That is, of n_t units available at the beginning of time t, at the end of this period $(1 - q_t) \cdot n_t$ units fail, and $q_t n_t$ units remain intact in period $t + 1$. As long as the units are functional, consumers derive full utility, as they would from newly purchased units.[6]

As in Weber (2016), we use an overlapping-generations model to describe the consumers' life cycles. This is illustrated in Fig. 1. At any time $t \geq 0$, a new generation of consumers (agents) is born and lives for two periods. Correspondingly, time t is referred to as the *early consumption phase* of generation t (denoted by $\mathcal{C}_0^t$), while $t + 1$ is called the *late consumption phase* of generation t (denoted by $\mathcal{C}_1^t$). Consequently, total product sales at time $t \geq 1$ is the sum of sales Ω_0^t to generation t in its early consumption phase $\mathcal{C}_0^t$, and sales Ω_1^{t-1} to generation $t - 1$ in its late consumption phase $\mathcal{C}_1^{t-1}$. For simplicity, we assume that no later generation inherits products from its ancestors.

The agents in the economy have heterogeneous preferences. Any consumer ("he") is characterized by a type (θ, ν) in the unit square $\mathcal{Q} = \Theta \times \mathcal{V}$. The *likelihood* $\theta \in \Theta \triangleq [0, 1]$ specifies the agent's (subjective) probability with which he will find himself in a state of high need in any future time period. More specifically, it describes the distribution of the agent's need state $\tilde{s}_j^t$ in his consumption phase $\mathcal{C}_j^t$, for $j \in \{0, 1\}$, with realizations s_j^t in $\mathcal{S} = \{0, 1\}$; the need state can be either "low" ($s_j^t = 0$) or "high" ($s_j^t = 1$), and *ex ante*:

$$P(\tilde{s}_j^t = 1) = \theta.$$

The *value* $\nu \in \mathcal{V} = [0, 1]$ describes the payoff the agent derives from having access to the item in a high-need state. For simplicity we assume that the type distribution is uniform on $\mathcal{Q}$, that each consumer's type is persistent, and that the need states are uncorrelated. Given a stationary population trend, the total number of agents in a given generation is normalized to 1, without loss of generality.

At any time t, an agent's indirect utility $u_i(y, s|\nu)$ depends on his income level y, the realization of his need state s, and whether he has the item at his disposal or not (corresponding to $i = 1$ or $i = 0$, respectively). As in Razeghian and Weber (2015) we assume that in the low-need state the agent's utility vanishes, no matter if the item is available for use or not, so

$$u_i(y, s|\nu) = \begin{cases} y + i\nu, & \text{if } s = 1, \\ y, & \text{if } s = 0. \end{cases}$$

[5] To avoid biased results, it is assumed that the product's durability can be changed at no cost. In reality, depending on the base version of the product, it may be costly to either increase or decrease the product's durability from its nominal value. The former is a standard product upgrade, while the latter would amount to a "damaged good" in the sense of Deneckere and McAfee (2005) without the benefit of product differentiation implied there.

[6] This (classical "one-hoss shay") assumption is regularly used in the durable-goods literature (see, e.g., Swan, 1972; Stokey, 1981; Bulow, 1986; Fethke and Jagannathan, 2002; Goering, 2007).

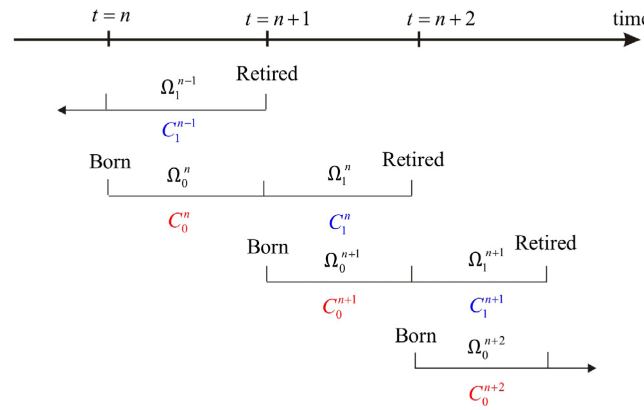

Fig. 1. Overlapping-generations model.

Agents are risk-neutral, and they are rational so as to maximize their objectives given all subjectively available information. In the early consumption phase, a newly-born agent realizes his type and observes his early need state (s_t^0). He then decides about purchasing the item at the price r_t. In the next period, the agent observes his late need state (s_t^1) and realizes whether his item has failed (conditional on being an owner). He can then renew his purchase decision.

In the presence of an active peer-to-peer market, all non-owners have the option of renting the item from the sharing market at a clearing price p_t. At the same time, any owner may decide to *either* keep and use the item *or* offer it on the sharing market to others in view of enjoying the additional income $y_t = p_t$. We thereby assume that the item cannot be shared on the market in the very same period in which it was acquired.[7]

Remark 1. The fact that the different overlapping consumer generations co-exist induces stationary model behavior which allows us to focus on the effect of the sharing economy on product durability, abstracting from complications arising from nonstationarities in combination with the Coase problem (Coase, 1972; Gul et al., 1986). Still, in our setting the firm's commitment ability matters for the equilibrium, and this dependence is examined in detail (see Section 3).

3. Equilibrium analysis

To derive predictions for the firm's and the agents' behavior in the underlying dynamic game of complete information, with and without sharing markets, we use the concept of subgame-perfect Nash equilibrium by Selten (1965). Due to the limited lifetime and coexistence of the two consumer generations in every period, the dynamic equilibrium of the supergame will be stationary. Indeed, the firm faces the same distribution of agents in each period and therefore will find the same choice for its durability and price optimal. While consumption choices are nonstationary for each generation, in aggregation they are stationary. Thus, the time index is omitted for convenience.

[7] Getting an item ready for the sharing market (including cleaning, preparation and installation of sharing-specific features, and creating a listing for the item with a sharing intermediary) takes time and usually comes at a cost; see Razeghian and Weber (2015) for a model which considers such transaction costs explicitly.

3.1. Consumption and product design without sharing

3.1.1. Isolated consumption decisions

In the absence of a sharing market, ownership is the sole mode of consumption. The agents' optimal consumption decisions are obtained by backward induction.

In the late consumption phase $\mathcal{C}_1$, an owner of type (θ, ν) (with a functioning item) obtains the payoff

$$\hat{V}_{s_1} = \nu s_1, \tag{1}$$

which is positive only in the high-need state. A non-owner of type (θ, ν), on the other hand, can decide to acquire the item or else forgo use at a zero payoff. Thus, a purchase transaction takes place if and only if $\nu s_1 - r \geq 0$, resulting in the non-owner's optimal payoff,

$$\hat{U}_{s_1} = \max\{0, \nu s_1 - r\}. \tag{2}$$

In the early consumption phase ($\mathcal{C}_0$), the purchase decisions are based on each agent's rational assessment of his present and future prospects. The latter correspond to the agent's expected payoffs in the late consumption phase ($\hat{V}_{s_1}$ and $\hat{U}_{s_1}$) in the anticipated role of owner or non-owner, respectively. To compute the agent's possible payoffs, we first note that in a low-need state he would never purchase the item because of the standing option to buy the item in the next consumption phase at the same stationary price, should a high-need state realize then. In other words, without a present high need no agent would be willing to invest in uncertain future consumption. Hence, purchasers are necessarily in their high-need state. An agent of type (θ, ν), in the high-need state, would acquire the item in his early consumption phase if the expected discounted payoff of ownership exceeds the expected payoff of non-ownership,

$$\nu - r + \delta\theta\left((1-q)\hat{U}_1 + q\hat{V}_1\right) \geq \delta\theta\hat{U}_1, \tag{3}$$

where $\delta \in (0, 1]$ is a discrete-time discount factor, common for all agents. The preceding relation yields a purchasing threshold in terms of the likelihood of future need:

$$\theta \geq \theta_0(\nu|q, r) \triangleq \frac{\max\{0, r-\nu\}}{\delta q \nu}. \tag{4}$$

Agent types with sufficiently high θ purchase as long as $\nu \geq r/(1+\delta q)$; an agent type with any θ purchases if $\nu \geq r$. As a result, sales to consumers in the early consumption phase are

$$\hat{\Omega}_0(q, r) = \int_{\mathcal{Q}} \mathbf{1}_{\{\theta \geq \theta_0(\nu|q,r)\}}\, d(\theta, \nu).$$

In the late consumption phase ($\mathcal{C}_1$), the demand for ownership consists of two groups of agents in their high-need state (both with values $\nu \geq r$): (i) first-time purchasers who were in the low-need state in their early consumption phase; (ii) second-time purchasers who were owners in $\mathcal{C}_0$ but ended up with a defective item at the end of their early consumption phase. Sales to this generation are

$$\hat{\Omega}_1(q, r) = \int_{\mathcal{Q}} \mathbf{1}_{\{\nu \geq r\}} \left[(1-\theta)\theta + (1-q)\theta^2\right] d(\theta, \nu). \tag{5}$$

In any given period, the aggregate demand for ownership ($\hat{\Omega}$) is the sum of the respective demands by the young generation ($\hat{\Omega}_0$) and by the mature generation ($\hat{\Omega}_1$).

Proposition 1 (*Demand for Ownership without Sharing*)**.** *Let $q \in [0, 1]$ and $r \in [0, 1+\delta q]$. In the absence of sharing, the demand for ownership is $\hat{\Omega}(q, r) \triangleq \hat{\Omega}_0(q, r) + \hat{\Omega}_1(q, r)$,*[8] *where*

$$\hat{\Omega}_0(q, r) = \frac{1}{2} + \frac{\max\{0, r^2-1\}}{2(\delta q)^2} - \frac{r}{\delta q}\left(1 - \frac{1}{\delta q}\ln\left(\frac{1+\delta q}{\max\{1, r\}}\right)\right), \tag{6}$$

and

$$\hat{\Omega}_1(q, r) = \max\{0, 1-r\}\left(\frac{1}{2} - \frac{q}{3}\right). \tag{7}$$

When durability goes up, then *ceteris paribus* the demand by young consumers increases, whereas the demand by mature consumers decreases.

Corollary 1. *For any $(q, r) \in [0, 1] \times \mathbb{R}_+$, the generational demands for product ownership, $\hat{\Omega}_0$ and $\hat{\Omega}_1$, are such that*

$$\frac{\partial \hat{\Omega}_1(q, r)}{\partial q} \leq 0 \leq \frac{\partial \hat{\Omega}_0(q, r)}{\partial q}.$$

The reason for the somewhat counterintuitive negative dependence of $\hat{\Omega}_1$ on q is that a higher durability tends to deplete the pool of second-time purchasers.

Remark 2. We concentrate on the interesting case where the unit production cost is low enough, such that the monopolist is able to offer a unit purchase price $r \leq 1$ for all $q \in [0, 1]$. This case is particularly interesting because the choice of durability affects the sales in both consumption phases (early and late). If the purchase price exceeds unity, for each generation sales occur only in the early consumption phase.

3.1.2. Isolated product-design decisions

The firm maximizes its expected discounted profit with respect to durability and price. In any given period, the (stationary) profit $\hat{\Pi}(q, r)$ is the product of the markup $(r-c)$ and the sales $\hat{\Omega}(q, r)$ to both generations, so

$$\hat{\Pi}(q, r) = (r-c)\,\hat{\Omega}(q, r) = (r-c)\left(1 - \frac{q}{3} - r\rho(q)\right), \tag{8}$$

where $\rho(q) \triangleq 1/2 - q/3 + [1 - \ln(1+\delta q)/(\delta q)]/(\delta q)$ and $c \in [0, \bar{c}]$.[9] While simultaneous optimization with respect to both q and r is possible, more insight is obtained by considering the optimal pricing problem and the optimal durability problem sequentially which ultimately leads to the same result. Yet, arguably it is easier for the firm to adjust the retail price than to change the durability characteristics of its product. Durability, as a product-design decision that affects the hardware of the product, tends to be more sticky than the posted price. A sequential solution approach thus also reveals, in addition to the optimal product design, the optimal price for any given durability level. Hence, via backward induction, we begin by determining the optimal product price $\hat{r}(q)$ for all $q \in [0, 1]$ and subsequently determine the optimal durability $\hat{q}$. The resulting optimal product design $(\hat{q}, \hat{r})$ with $\hat{r} = \hat{r}(\hat{q})$ is the same as if both instruments (price and durability level) had been optimized simultaneously.

Optimal pricing problem. The first-order necessary optimality condition for the maximization of the firm's profit in Eq. (8) with respect to r yields the optimal price as a function of a given durability level q:

$$\hat{r}(q) = \frac{c}{2} + \frac{1}{2\rho(q)}\left(1 - \frac{q}{3}\right). \tag{9}$$

[8] One can verify that $\hat{\Omega}(q, r)$ is twice continuously differentiable on the interior of its domain.

[9] For the purchase price to not exceed the unity given all $\delta \in (0, 1]$, the production cost needs to be in the interval $[0, (12\ln(2) - 10)/(6\ln(2) - 7)] \approx [0, 0.59]$.

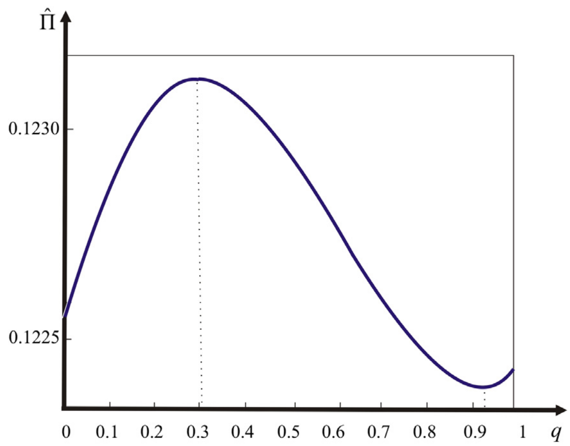

Fig. 2. Isolated profit as a function of durability, for $(\delta, c) = (0.6, 0.3)$.

This price is increasing in both the marginal cost c and the provided level of durability q. Furthermore, it is remarkable that in absolute terms the firm's price is more sensitive to changes in the agents' discount factor δ than to changes in the durability q, even though the consumption decisions depend only on their product (δq), as shown in Section 3.1.2.

Lemma 1. *For any $\delta, q \in (0, 1)$, the optimal price in Eq. (9) is such that*

$$0 \leq \hat{r}'(q) \leq \frac{\partial \hat{r}(q)}{\partial \delta}.$$

An exogenous change in consumers' level of patience has a more profound impact on the price than an increase in the durability, which is moderated by the consumers' patience.

Optimal durability problem. For any given price, the firm's optimal level of durability maximizes the demand $\hat{\Omega}$ for the product.[10] Given that the monopolist uses the optimal price in Eq. (9) for any feasible $q \in [0, 1]$, the firm's profit in Eq. (8) becomes

$$\hat{\Pi}(q, \hat{r}(q)) = \frac{(1 - c\rho(q) - q/3)^2}{2\rho(q)}. \tag{10}$$

Since the latter is generally nonconcave in q (see Fig. 2), and the objective function involves a transcendental function, the optimal durability problem,

$$\hat{q} \in \arg\max_{q \in [0,1]} \hat{\Pi}(q, \hat{r}(q)), \tag{11}$$

cannot be solved in closed form. A more detailed analysis reveals that for small production costs, the firm has an incentive to provide low levels of durability. When the unit production cost is high enough, then the firm finds it optimal to produce a fully durable product (i.e., $\hat{q} = 1$).

Proposition 2 (*Optimal Durability without Sharing*)**.** *Let $(\delta, c) \in (0, 1] \times [0, \bar{c}]$ and set $\check{c} \triangleq (1-\delta)/(1+\delta)$. Then there exists $\hat{c} \in (\check{c}, \bar{c})$ such that (i) $\hat{q} = 0$ for $c \leq \check{c}$; (ii) $\hat{q} \in (0, 1)$ for $c \in [\check{c}, \hat{c}]$; (iii) $\hat{q} = 1$ (perfect durability) for $c > \hat{c}$.*

In other words, it is optimal for the monopolist to make low-cost products *disposable*, and in stark contrast, to make high-cost products *perfectly durable*.[11] The thresholds in Proposition 2 can also be formulated in terms of the discount factor. When consumers heavily discount future payoffs, the firm has an incentive to produce disposable products ($\hat{q} = 0$). At the other end of the spectrum, when consumers are very patient because δ is close to 1, the firm finds it optimal to produce a fully durable product ($\hat{q} = 1$).

Corollary 2. *Let $c \in [0, \bar{c}]$ and set $\check{\delta} \triangleq (1-c)/(1+c)$. Then there exists $\hat{\delta} \in (\check{\delta}, 1]$ such that (i) $\hat{q} = 0$ for $\delta \leq \check{\delta}$; (ii) $\hat{q} \in (0, 1)$ for $\delta \in [\check{\delta}, \hat{\delta}]$; (iii) $\hat{q} = 1$ for $\delta > \hat{\delta}$.*

The following result can be used to compute the optimal durability level $\hat{q}$. The corresponding optimal price $\hat{r} = \hat{r}(\hat{q})$ follows from Eq. (9).

Corollary 3. *Let $(\delta, c) \in (0, 1] \times [0, \bar{c}]$. For $c \in (\check{c}, \hat{c})$, or equivalently $\delta \in (\check{\delta}, \hat{\delta})$, the optimal durability $\hat{q}$ satisfies*

$$-\left(1 + c\rho(\hat{q}) - \frac{\hat{q}}{3}\right)\frac{\rho'(\hat{q})}{\rho(\hat{q})} = \frac{2}{3}, \tag{12}$$

and it is intermediate (i.e., $0 < \hat{q} < 1$).

In line with the intuition provided by Proposition 2, the optimal durability depends monotonically on the consumers' patience and the firm's production cost.

Proposition 3 (*Durability Drivers*)**.** *The optimal durability $\hat{q}$ is increasing the discount factor δ and the production cost c, for all $(\delta, c) \in (0, 1] \times [0, \bar{c}]$.*

More patient consumers expect better-quality products and are willing to pay more at the margin, thus leading the firm to increase durability. Similarly, to sustain a substantial markup over an increasing production cost, a firm would find it best to also increase the level of durability. Consider now the firm's optimal profit $\hat{\Pi}^* \triangleq \hat{\Pi}(\hat{q}, \hat{r})$.

Lemma 2. *The optimal profit $\hat{\Pi}^*$ is increasing in the consumers' discount factor δ and decreasing in the production cost c, for all $(\delta, c) \in (0, 1] \times [0, \bar{c}]$.*

The fact that the firm's optimal profit increases in the consumers' level of patience has some interesting business implications if there are indirect ways for the monopolist to decrease the implied interest rate for the customer. By offering financial services (e.g., loans) to facilitate the purchase, the consumers' discount rate may effectively decrease, implying a higher discount factor. Depending on the firm's cost of capital, this may result in a net profit increase.[12] The optimal durability and profit are shown in Fig. 3, as a function of (δ, c).

Remark 3. Maximizing profits with respect to durability for a *given* price r is equivalent to maximizing the demand for ownership $\hat{\Omega}(q, r)$ with respect to q. An interior solution is such that

$$\hat{r}(q) = -\frac{1}{3\rho'(q)}. \tag{13}$$

Hence, by combining Eqs. (9) and (13), one obtains a fixed-point problem satisfied by an interior optimal durability $\hat{q} \in (0, 1)$:

$$\hat{q} = 3\left(1 + c\rho(\hat{q})\right) + 2\left(\rho(\hat{q})/\rho'(\hat{q})\right). \tag{14}$$

This condition is equivalent to Eq. (12).

10 By Corollary 1 this means that it is (locally) optimal to increase the durability as long as the resulting marginal increase of $\hat{\Omega}_0$ outweighs the marginal decrease of $\hat{\Omega}_1$.

11 The cost thresholds $\check{c}$ and $\hat{c}$ in Proposition 2 are such that $\check{c} \leq \left.\frac{\sigma - 2/3}{(1-\sigma)\sigma}\right|_{\sigma = \sqrt{\rho(1)}} \leq \hat{c}$. The algebraic details are in Appendix A.

12 While the firm's per-period profit is stationary, the net present value of the monopolist's operations depends on its own discount factor δ'. The latter is generally different from the consumers' δ; usually $\delta' < \delta$. Yet, because of the stationarity in the equilibrium in the overlapping-generations model, δ' is of no particular concern here.

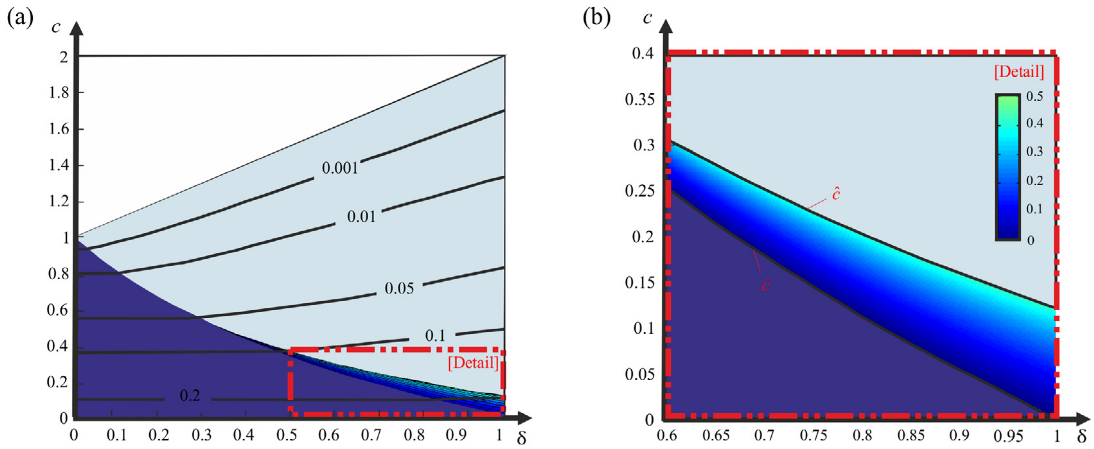

Fig. 3. Optimal durability and profit without sharing as a function of (δ, c).

3.2. Consumption and product design with sharing

In the presence of an active sharing market, the monopolist faces consumers that can choose between ownership and mere access to the products. In addition, owners have the option to rent out their items when they are not needed. As pointed out by Weber (2016), the value created by this flexibility creates an incentive for the firm to increase the retail price, at least for perfectly durable goods. The question answered here is how durability interacts with the firm's preferred retail price and how much durability is optimal to provide for the monopolist when consumers can share their goods.

3.2.1. Consumption decisions: Access versus ownership

For transactions on the sharing market to exist, the price p of renting a product for one period cannot exceed the (stationary) purchase price r set by the firm.[13] In other words, for sharing to take place it is necessary that

$$p < r. \tag{15}$$

Remark 4. In case of a tie between the price of ownership r and the rental price p, we assume that consumers prefer ownership, for a variety of possible reasons. The latter may include avoiding informational problems inherent to rental markets and the resulting potential for ex-post disputes, as well as the enjoyment of residual claims (e.g., the unmodeled possibility of resale).

Hence, for a *non-owner* in his late consumption phase, the sharing market is more attractive than purchasing, independent of his type (θ, ν); his resulting payoff is

$$U_{s_1} = \max\{0, \nu s_1 - p\}, \tag{16}$$

given the realized need state $s_1 \in \{0, 1\}$. By contrast, an *owner* in the late consumption phase can choose whether to use his item or become a supplier on the sharing market. Because of the vanishing use value in the low-need state, he would participate in the sharing market whenever $s_1 = 0$. On the other hand, in the high-need state he would be willing to share, as long as the clearing price p exceeds his own utility when using the item privately. This implies an owner's payoff of the form

$$V_{s_1} = \max\{\nu s_1, p\}, \tag{17}$$

contingent on the realization of his need state $s_1 \in \{0, 1\}$. Ownership decisions are based on an evaluation of the lifecycle payoffs, analyzed next.

In the early consumption phase, each agent observes his need state s_0, evaluates his expected future payoff, and decides whether it is best to purchase, to rent, or to do nothing. For an agent of type (θ, ν), the state-dependent total discounted payoff is therefore

$$T_{s_0} = \max\left\{\delta\bar{U}, \nu s_0 - p + \delta\bar{U}, \nu s_0 - r + \delta(q\bar{V} + (1-q)\bar{U})\right\},$$

where $\bar{U} \triangleq (1-\theta)U_0 + \theta U_1$ and $\bar{V} \triangleq (1-\theta)V_0 + \theta V_1$. Purchasing is best when the expected payoff of ownership outweighs the expected return from non-ownership in the high-need state:

$$\nu - p + \delta q\bar{U} \le \nu - r + \delta q\bar{V}. \tag{18}$$

Substituting Eqs. (16)–(17) in Eq. (18) and then rearranging the terms yields the ownership criterion

$$r \le \min\{\nu, p\} + \delta pq. \tag{19}$$

In the presence of an active collaborative economy, an agent's purchase decision depends only on his value ν, not on the likelihood of future need θ. This underlines the fact that sharing markets allow for hedging of need-state-specific payoff variations, thus providing the consumer base with the possibility of mutual insurance.

The ownership criterion in Eq. (19) implies that no agent is willing to make a purchase if the retail price strictly exceeds the effective net present value of the item when accessed via the sharing market in consecutive periods, i.e., if $r > (1 + \delta q)p$, so that necessarily

$$\frac{r}{1+\delta q} \le p. \tag{20}$$

Remark 5. Only agents in the high-need state become owners. In the early consumption phase, no agent in a low-need state is willing to purchase. The latter can be shown by contradiction. Suppose an agent who observed his need state $s_0 = 0$ makes a purchase. This can be optimal only if $-p + \delta\bar{U} \le -r + \delta(q\bar{V} + (1-q)\bar{U})$, or equivalently if $r \le \delta pq$,[14] which (for $\delta q < 1$) violates the liquidity requirement (15). This condition also implies that no purchases are made in the late consumption phase as long as there is a functioning sharing market.

Based on the preceding remark and the ownership criterion (19), early-generation agents in the high-need state with

[13] If $p \ge r$, then there is no sharing, and the results in Section 3.1 do apply.

[14] This inequality is equivalent to the ownership criterion (19) when $\nu = 0$ (as the benefit vanishes in the low-need state).

values $v \geq r - \delta pq$ are willing to buy the product, resulting in the steady-state demand for ownership,

$$\Omega(q, r; p) = (1 - (r - \delta pq))\bar{\theta} = \frac{1 + \delta pq - r}{2}, \tag{21}$$

where $\bar{\theta} \triangleq \int_0^1 \theta\, d\theta = 1/2$. As will become clear below, the motivation for ownership differs across agents, depending on v. Consumers with high values $v \in [p, 1]$ buy the item primarily for their personal use, and they act as suppliers only if they end up in a low-need state during their late consumption phase. Supra-marginal consumers with intermediate values $v \in [r - \delta pq, p]$ purchase the item to benefit from the additional income they can gain by offering it on the sharing market in their late consumption phase, regardless of the need state.

3.2.2. Equilibrium in the sharing market

By combining the requirements (15) and (20) the clearing price p of a functioning sharing market for a product with characteristics (r, q) must satisfy the *liquidity condition*

$$\frac{r}{1 + \delta q} \leq p < r. \tag{L}$$

Of the buyers in their early consumption phase, only the fraction q are expected to still own a functioning item at the beginning of their late consumption phase. Of these residual owners, all could in principle act as a supplier, except those who are in a high-need state for the second time in a row and whose valuation v exceeds the rental price p in the sharing market. Aggregate supply is therefore

$$\begin{aligned} S(q, r; p) &= q\left((1-p)\int_0^1 \theta(1-\theta)\, d\theta + (p - (r - \delta pq))\int_0^1 \theta\, d\theta\right) \\ &= \frac{q}{2}\left(\frac{1}{3} + \left(\frac{2}{3} + \delta q\right)p - r\right), \end{aligned} \tag{22}$$

where the number of owners $\Omega(q, r; p)$ is specified in Eq. (21). Conversely, the demand for the shared item consists of all non-owners with value $v \geq p$ in their late consumption phase:

$$\begin{aligned} D(q; p) &= (1-p)\left(\int_0^1 (1-\theta)\theta\, d\theta + (1-q)\int_0^1 \theta^2\, d\theta\right) \\ &= \frac{1-p}{2}\left(1 - \frac{2q}{3}\right), \end{aligned} \tag{23}$$

independent of the purchase price r. The first term in the middle of Eq. (23) captures the non-owners in $\mathcal{C}_0$ who find themselves in a high-need state in $\mathcal{C}_1$. The second term reflects the additional demand created due to product failure in $\mathcal{C}_1$, as experienced by the fraction $1 - q$ of initial owners. The sharing market clears if supply equals demand, i.e., if

$$S(q, r; p) = D(q; p), \tag{24}$$

which in turn determines the price p in the sharing market as a function of the firm's product design (q, r). In this context, it is important to note that the liquidity condition (L) guarantees a functioning sharing market by bracketing the clearing price p. It can also be stated in terms of bounds on the retail price r, as follows:

$$\underline{r}(q) < r \leq \bar{r}(q), \tag{L'}$$

where $\underline{r}(q) \triangleq (1-q)/(1-q+\delta q^2)$ and $\bar{r}(q) \triangleq 1 + \delta q$; see Fig. 4. For maximum durability ($q = 1$), the liquidity condition (L') is automatically satisfied, as $\underline{r}(1) = 0$ and $\bar{r}(1) = 1 + \delta$.

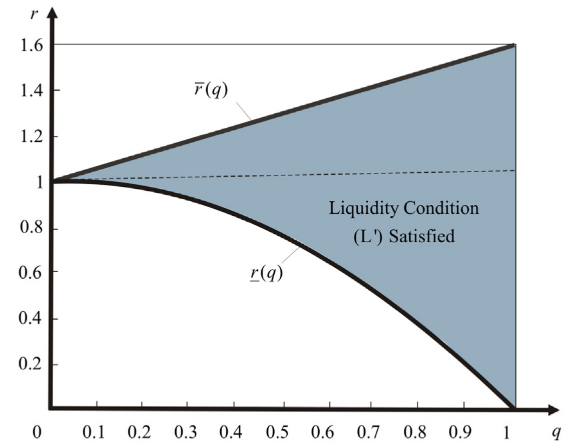

Fig. 4. Retail-price liquidity thresholds in Eq. (L') as a function of durability ($\delta = 0.6$).

Proposition 4 (*Equilibrium in the Sharing Market*)**.** *Given a product design (q, r) which satisfies the liquidity condition (L'), the equilibrium price in the sharing market is*

$$p(q, r) = \frac{1 - (1-r)q}{1 + \delta q^2}, \tag{25}$$

resulting in the equilibrium transaction volume

$$Q(q, r) = \frac{q}{2}\left(1 - \frac{2q}{3}\right)\left(\frac{\bar{r}(q) - r}{1 + \delta q^2}\right), \tag{26}$$

for any given discount factor $\delta \in (0, 1]$.

By construction, the transaction volume can only be nonnegative if the liquidity condition (L') holds. The latter can be broken by the firm's product design, leading to a deliberate "sharing shutdown", further examined in Section 3.3. The sensitivity of the transaction volume in the sharing market to the product design is illustrated by the next result.

Lemma 3. *Consider an active sharing market for product designs (q, r) that satisfy the liquidity condition (L'). (i) The clearing price $p(q, r)$ is decreasing in the durability q and increasing in the retail price r. (ii) The transaction volume $Q(q, r)$ is unimodal in q and decreasing in r.*

For lower levels of durability, the scarcity arising from the concomitant planned obsolescence produces a higher price in the sharing market, which in turn dissuades some agents from participating in the peer-to-peer economy. A higher retail price also tends to increase the clearing price in the sharing market, thus reducing the transaction volume. Perhaps the most interesting point of Lemma 3 is that the volume of the sharing transactions is maximal for items of intermediate durability. When products are disposable, there is nothing to share. Perfectly durable goods, on the other hand, ultimately decrease demand because of the inherent lack of product failure and replacement, thus resulting in a somewhat lower sharing volume than the maximally attainable transaction volume on the peer-to-peer market.

3.2.3. Market responsiveness and commitment regimes

The monopolist's best actions depend on the responsiveness of the sharing market. An isomorphic and more classical way to view this dependency, is that the firm's optimal product design is influenced by its ability to commit to its choice. To understand the importance of commitment, let $\Omega(q, r; p)$ denote the demand for ownership, given a price p in the sharing market and a product-design choice (q, r).

1. *Stackelberg play (SP)*: With full ability to commit to a product design, the firm can capitalize on the anticipated market response $p(q,r)$ as specified in Proposition 4, and maximize its "Stackelberg profits"

$$\Pi_{\mathrm{SP}}(q,r) = (r-c)\,\Omega_{\mathrm{SP}}(q,r), \tag{27}$$

where $\Omega_{\mathrm{SP}}(q,r) \triangleq \Omega(q,r;p(q,r))$ is the demand for ownership in this sequential-move Stackelberg play. This commitment regime is relevant unless the sharing market is very responsive to changes in the retail price or product durability or if the firm has no adjustment cost for design changes.[15]

2. *Simultaneous-move play (SMP)*: Without commitment ability and fast adjustment, the sharing market can be viewed as a player all by itself who *reacts* to the firm's product design so as to maximize the welfare of its participants. Indeed, by the first fundamental welfare theorem an exchange economy produces an efficient outcome for its participants, and thus in aggregate can be viewed as a single rational "player" interacting repeatedly with the firm, at each time $t \geq 0$ in simultaneous moves. In this (lack-of-)commitment regime, the firm maximizes its "simultaneous-move profits"

$$\Pi_{\mathrm{SMP}}(q,r;p) = (r-c)\,\Omega_{\mathrm{SMP}}(q,r;p), \tag{28}$$

so as to determine its best response $(q(p), r(p))$ to any viable market price p that satisfies the liquidity condition (L).

3. *Simultaneous-move play with Durability Commitment (DC)*: While the SP and SMP commitment regimes can be viewed as extremes on a continuum of possible commitment levels, the DC regime presents a perhaps more realistic intermediate situation. Durability is a built-in product feature, arguably much more difficult to adjust in most cases than the product price. The simultaneous-move play with durability commitment regime allows the firm to partially commit to its design feature q and maximize its "durability-commitment profits"

$$\Pi_{\mathrm{DC}}(q,r;p) = (r-c)\,\Omega_{\mathrm{DC}}(q,r;p). \tag{29}$$

The induced durability level q may or may not be equal to the best-response product design of the SMP game.

We compare and contrast the analysis for the three regimes. Overall, similar to the prediction of the Coase conjecture (Coase, 1972; Gul et al., 1986), the firm does best with full commitment, i.e., with Stackelberg play. The authors feel that this regime reflects reality fairly well because the price adjustment on sharing markets is arguably limited (Razeghian and Weber, 2015). The following auxiliary result specifies the demand for ownership with and without a sharing market.

Lemma 4 (*Demand for Ownership*)**.** *For any admissible product design* (q,r)*, the demand for ownership is* $\Omega(q,r;p) \triangleq \Omega_0(q,r) + \Omega_1(q,r)$*, where*

$$\Omega_0(q,r;p) = \begin{cases} (1-r+\delta p q)/2, & \text{if } \underline{r}(q) < r \leq \bar{r}(q), \\ \hat{\Omega}_0(q,r), & \text{otherwise}, \end{cases} \tag{30}$$

and

$$\Omega_1(q,r;p) = \begin{cases} 0, & \text{if } \underline{r}(q) < r \leq \bar{r}(q), \\ \hat{\Omega}_1(q,r), & \text{otherwise}. \end{cases} \tag{31}$$

All agents can acquire ownership in either consumption phase ($\mathcal{C}_0$ or $\mathcal{C}_1$), and the total per-period demand for ownership obtains as the sum for both coexisting generations. With active sharing markets no products are acquired in the late consumption phase. For this reason, the monopolist may resort to product designs that intentionally shut down the sharing markets, as discussed in Section 3.3.

3.2.4. Product design in the Stackelberg regime

At the beginning of each period, the monopolist selects a product design (q,r) so as to maximize the Stackelberg profits in Eq. (27). As in Section 3.1, we consider (without any loss of generality) the monopolist's decisions about durability and price sequentially.

Optimal pricing problem. The firm anticipates the sharing price in Proposition 4 and for a given product design q chooses a retail price so as to maximize the expected per-period profit $\Pi_{\mathrm{SP}}(q,r)$ by solving

$$r(q) \in \arg\max_{r} \left\{ \frac{1}{2}\left(1 - r + \delta q\left(\frac{qr + (1-q)}{1+\delta q^2}\right)\right)(r-c) \right\}. \tag{32}$$

The corresponding first-order necessary optimality condition yields the unique solution

$$r(q) = \frac{c}{2} + \frac{1+\delta q}{2}, \tag{33}$$

for any given level of durability $q \in [0,1]$, so that (q,r) satisfies the liquidity condition (L'). Benefiting from its commitment ability, the firm is able to syphon off the value that the sharing market adds for consumers, in the form of a "sharing premium."

Lemma 5. *In the Stackelberg regime, the price with sharing* $r(q)$ *exceeds the price without sharing* $\hat{r}(q)$ *by a nonnegative sharing premium* $\pi(q)$*. More specifically,* $r(0) = \hat{r}(0) = (1+c)/2$*, and*

$$\pi(q) \triangleq r(q) - \hat{r}(q) = \frac{1}{2}\left(\frac{1-q/3}{\rho(q)} - (1+\delta q)\right) \quad (\geq 0)$$

is increasing, for all $q \in [0,1]$ *such that the product design* $(q, r(q))$ *satisfies the liquidity condition (L').*

It is remarkable that the sharing premium is *independent* of the production cost. This is in contrast to the absolute price levels, both with and without a sharing market, which clearly depend on c. Thus, for low-cost products the relative weight of the sharing premium is likely to be substantially larger in relative terms than for high-cost products.

Optimal durability problem. Given the optimal price $r(q)$ in Eq. (33), the firm's Stackelberg profit becomes a function of durability only:

$$\Pi_{\mathrm{SP}}(q, r(q)) = \frac{1}{8}\,\frac{(1+\delta q - c)^2}{1+\delta q^2}.$$

As this function is strictly increasing in q, sharing markets provide strong incentives for companies to make their products durable.[16]

Proposition 5 (*Optimal Product Design (SP)*)**.** *In the Stackelberg regime, the optimal product design is* (q^*_{SP}, r^*_{SP})*, with* $q^*_{SP} = 1$ *(perfect durability) and* $r^*_{SP} = (1+\delta+c)/2$*.*

15 Usually an additional costly production run and design changes are required to change the product durability. Price adjustments can also be costly because of the expense to update catalogues, relabeling of physical products, communication in the retail network and delays.

16 Since durability is in general costly to provide implies that the marginal profit with respect to durability needs to equal the marginal cost of durability provision; see also Section 5 for further discussion.

By virtue of its perfect durability, the product design $(q^*_{\mathrm{SP}}, r^*_{\mathrm{SP}})$ always satisfies the liquidity condition (L'). The firm's resulting optimal profit is

$$\Pi^*_{\mathrm{SP}} \triangleq \Pi_{\mathrm{SP}}(q^*_{\mathrm{SP}}, r^*_{\mathrm{SP}}) = \frac{1}{8}\frac{(1+\delta-c)^2}{1+\delta}, \tag{34}$$

for all $(\delta, c) \in (0, 1] \times [0, \bar{c}]$.

Remark 6. The firm's payoff is increasing in the consumers' level of patience (as measured by the discount factor δ), and it is decreasing in the firm's production cost c. More precisely:

$$\frac{\partial \Pi^*_{\mathrm{SP}}}{\partial c} = -\frac{1}{4}\left(1-\frac{c}{1+\delta}\right) < 0 < \frac{1}{8}\left(1-\frac{c^2}{(1+\delta)^2}\right) = \frac{\partial \Pi^*_{\mathrm{SP}}}{\partial \delta},$$

for all $(\delta, c) \in (0, 1) \times (0, 1+\delta)$. Thinking creatively, the firm has therefore an interest to augment an agent's patience, e.g., by offering advantageous financing. Because of the added benefit through profit increase, the company can in principle afford offering a loan below the market rate, slightly cross-subsidizing the anticipated extra rent.

3.2.5. Product design with simultaneous-move play

Consider now a setting with a very efficient and fast sharing market and where the company cannot commit to its product design at the beginning of the period because it expects a quasi-instant response by the sharing market. For each sharing price, the company determines a best response in terms of its product design. Together with the price response of the sharing market (see Proposition 4), this determines a Nash-equilibrium outcome. The viability of the sharing market and thus the existence of the Nash equilibrium is, however, subject to the liquidity constraint.

Best-response product design. Given a clearing price p in the sharing market, consider the firm's product-design problem. A solution is obtained by maximizing the simultaneous-move profit $\Pi_{\mathrm{SMP}}(q, r; p)$ as specified in Eq. (29) with respect to the product design (q, r).

Lemma 6. *Given any (nonnegative) sharing price p, the firm's best-response product design is $(q_{SMP}(p), r_{SMP}(p))$, where*

$$r_{SMP}(p) = \begin{cases} \max\{c, (1+\delta)p\}, & \text{if } p \in [0, (1+c)/(2+\delta)], \\ (1+c+\delta p)/2, & \text{if } p \in ((1+c)/(2+\delta), \\ & \quad (1+c)/(2-\delta)], \\ \hat{r}(\hat{q}), & \text{otherwise}, \end{cases} \tag{35}$$

and

$$q_{SMP}(p) = \begin{cases} 1, & \text{if } p \in [0, (1+c)/(2+\delta)], \\ 1, & \text{if } p \in ((1+c)/(2+\delta), (1+c)/(2-\delta)], \\ \hat{q}, & \text{otherwise}; \end{cases} \tag{36}$$

$\hat{r}(\cdot)$ is the optimal retail-price schedule without sharing specified in Eq. (9), and $\hat{q}$ is the optimal product durability without sharing as described in Proposition 2.

For small levels of p, the firm makes no sale. Thus, instead of charging marginal cost, any retail price is also optimal. In this case, the firm does not really care about the particular level of durability (which can therefore also be different from 1). Note also that the requirement

$$\frac{1+c}{2+\delta} < p \le \frac{1+c}{2-\delta} \tag{L''}$$

is equivalent to the liquidity conditions (L) and (L').

Nash equilibrium without commitment. By intersecting the best-response product design in Lemma 6 with the market-price response in Proposition 4 one obtains a unique Nash equilibrium as prediction for the repeated stage-game interaction between the firm and the sharing market. Even though the firm's equilibrium design is stationary, it is fundamentally influenced by the fact that it could – if necessary – easily change this design. The players' best responses are shown in Fig. 5, intersecting at a unique Nash equilibrium.

Proposition 6 (*Nash Equilibrium (SMP)*)**.** *The unique Nash-equilibrium sharing-price and product-design profile $(p^*_{SMP}, (q^*_{SMP}, r^*_{SMP}))$ are such that $p^*_{SMP} = (1+c)/(2+\delta)$, $q^*_{SMP} = 1$ (perfect durability), and $r^*_{SMP} = (1+c)(1+\delta)/(2+\delta)$, for any $c \in [0, \bar{c}]$.*

Given the product design $(q^*_{\mathrm{SMP}}, r^*_{\mathrm{SMP}})$ in the simultaneous-move game with the sharing market, the firm obtains the equilibrium profit

$$\Pi^*_{\mathrm{SMP}} \triangleq \Pi_{\mathrm{SMP}}(q^*_{\mathrm{SMP}}, r^*_{\mathrm{SMP}}) = \frac{1}{2}\frac{(1+\delta-c)^2}{(2+\delta)^2},$$

for any $c \in [0, \bar{c}]$. The firm's payoffs are monotonic in the salient model parameters.

Remark 7. The firm's profit is increasing in the consumers' discount factor δ and decreasing in the production cost c.

$$\frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial c} = -\frac{1+\delta-c}{(2+\delta)^2} < 0 < \frac{(1+c)(1+\delta-c)}{(2+\delta)^3} = \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial \delta},$$

for all $\delta \in (0, 1]$ and $c \in [0, \bar{c}]$, similar to the Stackelberg regime. Again, increasing the consumers' level of patience turns out to be beneficial for the firm (see Remark 6).

3.2.6. Product design with durability commitment

We now provide a more general result, allowing the firm to partially commit to a durability level q which may or may not be equal to the perfect durability induced by the best-response product design of the simultaneous-move game.

Lemma 7. *Given any (nonnegative) sharing price p, and precommitment on durability $q \in [0, 1]$, the firm's best-response retail price is*

$$r_{\mathrm{DC}}(p; q) = \begin{cases} \max\{c, (1+\delta q)p\}, & \text{if } p \in [0, (1+c)/(2+\delta q)], \\ (1+c+\delta p q)/2, & \text{if } p \in ((1+c)/(2+\delta q), \\ & \quad (1+c)/(2-\delta q)], \\ \hat{r}(q), & \text{otherwise}, \end{cases} \tag{37}$$

where $\hat{r}(\cdot)$ is the optimal retail-price schedule without sharing specified in Eq. (9).

Intersecting the best-response price schedule $r_{\mathrm{DC}}(\cdot; q)$ in Lemma 7 with the sharing-market response $p(\cdot, q)$ in Proposition 4 yields the Nash equilibrium outcome with durability commitment.

Proposition 7 (*Nash Equilibrium with Durability Commitment*)**.** *Given a durability commitment $q \in [0, 1]$ and a production cost $c \ge \underline{c}(q)$, the Nash-equilibrium market-price/retail-price profile $(p_{DC}(q), r_{DC}(q))$ is such that*

$$p_{DC}(q) = \frac{2-(1-c)q}{2+\delta q^2} \quad \text{and}$$
$$r_{DC}(q) = \frac{1}{2}\left(1+c+\delta q\left(\frac{2-(1-c)q}{2+\delta q^2}\right)\right), \tag{38}$$

whereby the cost threshold is $\underline{c}(q) \triangleq \max\{0, (1-q-\delta q)/(1-q+\delta q^2)\}$. In the case where $c < \underline{c}(q)$, the sharing market collapses.

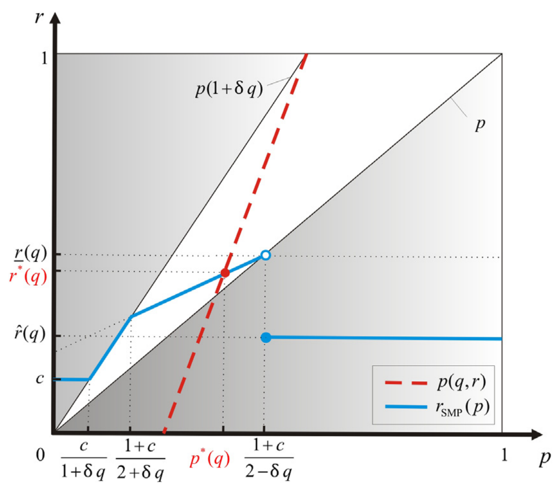

Fig. 5. Best responses in a simultaneous-move game between firm and sharing market.

The firm's Nash-equilibrium profit with durability commitment q becomes

$$\Pi_{\mathrm{DC}}(q) \triangleq \Pi_{\mathrm{SMP}}(q, r_{\mathrm{DC}}(q)) = \frac{1}{2}\left(\frac{1+\delta q-c}{2+\delta q^2}\right)^2 .$$

It is clear that durability precommitment can only be beneficial to the monopolist, since by committing to perfect durability ($q = 1$) the firm obtains the Nash-equilibrium outcome without precommitment. In this setting, it is always in the firm's best interest to commit to a level of durability which may not necessarily be equal to 1.

Proposition 8 (*Product Design with Durability Commitment*)**.** *In the simultaneous-move regime with durability commitment $q \in [0, 1]$, the optimal durability is*

$$q^*_{\mathrm{DC}} = \min\left\{1, \frac{\sqrt{(1-c)^2+2\delta}-(1-c)}{\delta}\right\} \in \arg\max_{q\in[0,1]} \Pi_{\mathrm{DC}}(q);$$

perfect durability ($q^*_{\mathrm{DC}} = 1$) is optimal for $c \in [\delta/2, \bar{c}]$. The corresponding optimal retail price is $r^*_{\mathrm{DC}} = r_{\mathrm{DC}}(q^*_{\mathrm{DC}})$.

The market response in a simultaneous-move equilibrium with durability commitment is

$$p^*_{\mathrm{DC}} \triangleq p(q^*_{\mathrm{DC}}, r^*_{\mathrm{DC}}) = \max\left\{\frac{1}{2}, p^*_{\mathrm{SMP}}\right\},$$

by virtue of Proposition 4, independent of (δ, c).

Comparison with the Stackelberg regime. We now compare the solutions for the regimes with full commitment (Stackelberg) and partial/no commitment, respectively, in the simultaneous-move stage game, in terms of durability, retail price, and equilibrium profit.

1. *Durability:*

   $q^*_{\mathrm{DC}} \leq q^*_{\mathrm{SMP}} = q^*_{\mathrm{SP}}$.

   With full commitment the firm finds it optimal to provide full-durability as long as there is an active peer-to-peer market. With partial commitment and sufficiently low production costs, the product durability becomes imperfect (i.e., less than 1), making planned obsolescence a part of the profit-maximizing design.

2. *Retail Price*:

   $\max\left\{r^*_{\mathrm{SMP}}, r^*_{\mathrm{DC}}\right\} < r^*_{\mathrm{SP}}$ and $r^*_{\mathrm{SMP}} \lesseqgtr r^*_{\mathrm{DC}}$.

   Whereas in a Stackelberg regime the firm benefits from a positive sharing premium on each unit sold, the purchase price in a simultaneous-move play (as described in Proposition 6 or Lemma 6) is not necessarily higher than the retail price induced by a durability commitment regime (as described in Proposition 7).

3. *Profit:*

   $\Pi^*_{\mathrm{SMP}} \leq \Pi^*_{\mathrm{DC}} < \Pi^*_{\mathrm{SP}}$.

   The profit in the Stackelberg regime always exceeds the profit obtained with partial or no commitment. In terms of sensitivity to parameters, the responsiveness to changes in costs or customer patience satisfies the following inequalities:

$$\frac{\partial \Pi^*_{\mathrm{SP}}}{\partial c} < \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial c} \leq \frac{\partial \Pi^*_{\mathrm{DC}}}{\partial c} < 0 < \min\left\{\frac{\partial \Pi^*_{\mathrm{SP}}}{\partial \delta}, \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial \delta}, \frac{\partial \Pi^*_{\mathrm{DC}}}{\partial \delta}\right\},$$

   for all $\delta \in (0, 1]$ and $c \in [0, \bar{c}]$.

In addition to optimizing the product design in the presence of a sharing market, the firm can use product design to deliberately inhibit sharing, a possibility we examine next.

3.3. Sharing shutdown

By undercutting the sharing market, the firm has the option to effectively disable the peer-to-peer economy. Specifically, if for a given level of durability $q \in [0, 1]$ a retail price r is chosen below the lower bound $\underline{r}(q)$, then the liquidity condition (L') is violated and therefore the transaction volume on the sharing market drops to zero. Conditional on the resulting "sharing shutdown" (SS), the firm's best price is

$$r_{\mathrm{SS}}(q) \triangleq \min\left\{\underline{r}(q), \hat{r}(q)\right\}, \tag{39}$$

where the liquidity bound $\underline{r}(q)$ and the no-sharing price $\hat{r}(q)$ appear in Eqs. (L') and (9), respectively.

Shutdown pricing. We first assume that the firm is pre-committed to a certain durability level, for example, having disbanded its design team. When choosing an optimal retail price, conditional on q, the company would prefer a functioning peer-to-peer exchange to a sharing shutdown if and only if

$$\hat{\Pi}(q, r_{\mathrm{SS}}(q)) \leq \Pi_j(q, r_j(q)),$$

where $j \in$ {SP, DC}, depending on the relevant commitment regime. Fig. 6 shows the optimal price and the profit, for full product-design commitment (SP), durability commitment (DC), and in the absence of sharing. In order to choke off the sharing market, the firm may need to sacrifice a portion of the optimal no-sharing profit, charging the highest possible no-sharing retail price $\underline{r}(q)$ per unit. The shaded area highlights the durability levels for which the sharing market is active. In this area, the firm may not gain from the presence of a peer-to-peer market; however, it would be too costly to drop its rate to the shutdown price $\underline{r}(q)$.

Fig. 7 provides an overview of the viability of a sharing market, as controlled by the firm's choice of its product design. For any given level of production cost c, it can lower the level of durability q and/or adjust its price to $r_{\mathrm{SS}}(q)$. For very small levels of durability the no-sharing price $\hat{r}(q)$ is optimal whereas for intermediate durabilities the company prefers to disable sharing at the choke-off retail rate $\underline{r}(q)$. For large durability levels, sharing is preferred—all things considered. Given the ability to at least partially commit to its product design by fixing the level of its durability, the firm is able to extract a sharing premium when its user base is faced with an active peer-to-peer economy.

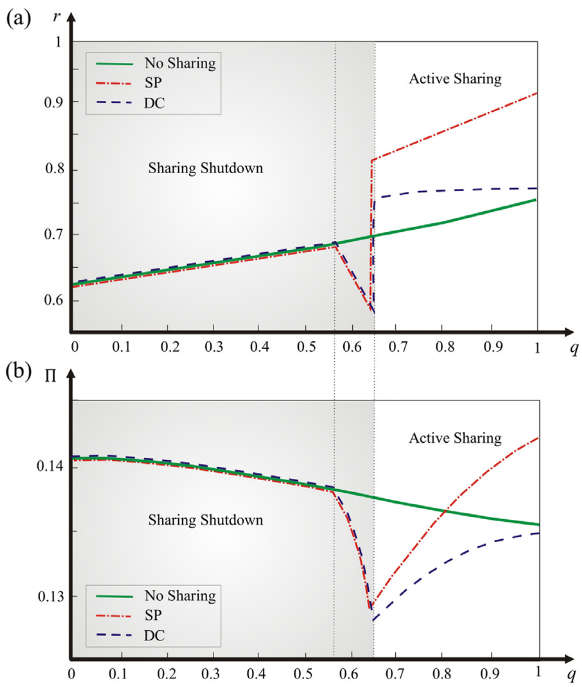

Fig. 6. (a) Optimal price; (b) optimal profit, for $(\delta, c) = (0.6, 0.25)$.

Shutdown design. We now examine the firm's option to disable the sharing market when optimizing over its full product design, including retail price and durability. Given the commitment regime $j \in \{\text{SP}, \text{DC}\}$, the firm finds it optimal to use a "sharing-shutdown design" $(r_{\text{SS}}^*, q_{\text{SS}}^*)$, with $r_{\text{SS}}^* = r_{\text{SS}}(q_{\text{SS}}^*)$, as long as

$$\max_{q\in[0,1]} \Pi_j(q, r_j(q)) \le \max_{q\in[0,1]} \hat{\Pi}(q, r_{\text{SS}}(q)) = \hat{\Pi}(r_{\text{SS}}^*, q_{\text{SS}}^*) \triangleq \Pi_{\text{SS}}^*,$$

where $r_{\text{SS}}(q)$ is the sharing-shutdown price in Eq. (39). Note in particular that the monopolist can always obtain the sharing-shutdown profit Π_{SS}^* by strategically reducing its product durability. Fig. 8 shows the optimal durability for SP and DC, as well as SS. By Proposition 9 in the Stackelberg regime, the most profitable way for the firm to disable sharing is to produce at zero durability. Note that in a simultaneous-move play, the firm might be willing to choose an intermediate durability level that is small enough to disable the sharing market and yet maximizes the no-sharing profit.

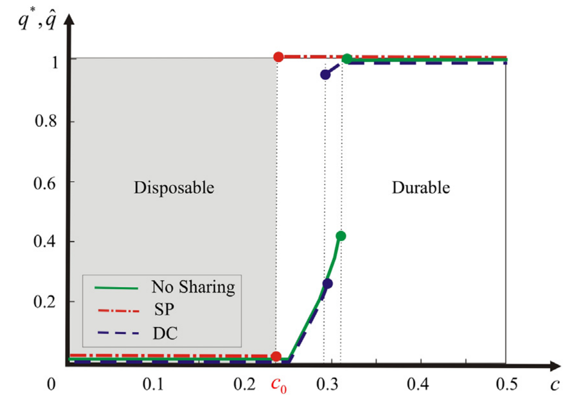

Fig. 8. Durability as a function of production cost ($\delta = 0.6$).

Proposition 9 (*Extreme Durability with Full Commitment*)**.** *In the Stackelberg regime (SP), there exists a production-cost threshold $c_0 \in [0, \check{c}]$, such that the firm's optimal durability is 0 (disposable) for $c \le c_0$ and 1 (perfect) for $c \ge c_0$.*

Some firms have started to complement their product portfolio with sharing. Producers of durable goods such as car manufacturers have launched one-time-use products and services in response to the emergence of the sharing economy. For example, BMW's DriveNow service allows its customers fee-based access to a fleet of cars (for its BMW and MINI brands).[17] The latter can be viewed as a portfolio of disposable products. Consistent with the model, the high-cost models (e.g., BMW 6 or 7 series) are *not* part of the DriveNow fleet.

Remark 8. Making products *shareable* may be costly.[18] Let κ be the additional unit cost for producing a shareable item. In the Stackelberg commitment regime, if $\kappa \ge \check{c} - c_0$, then the optimal durability is necessarily strictly greater than 0, and the rental program is no longer an option. It might therefore become more profitable to invest in promoting the sharing market. For example, in an attempt to promote peer-to-peer markets, GM has recently invested \$500 million in Lyft.

Remark 9. The optimal profit in Fig. 9 has a convex kink at $c = c_0$. The monopolist benefits from cost function uncertainty around this point. For example consider an R&D investment on

17 Similarly, Audi's pilot program "Audi On Demand" offers one-day car rentals, while Ford's GoDrive introduces a pay-per-minute car-sharing program.
18 The control and pricing of shareability is discussed by Weber (2017).

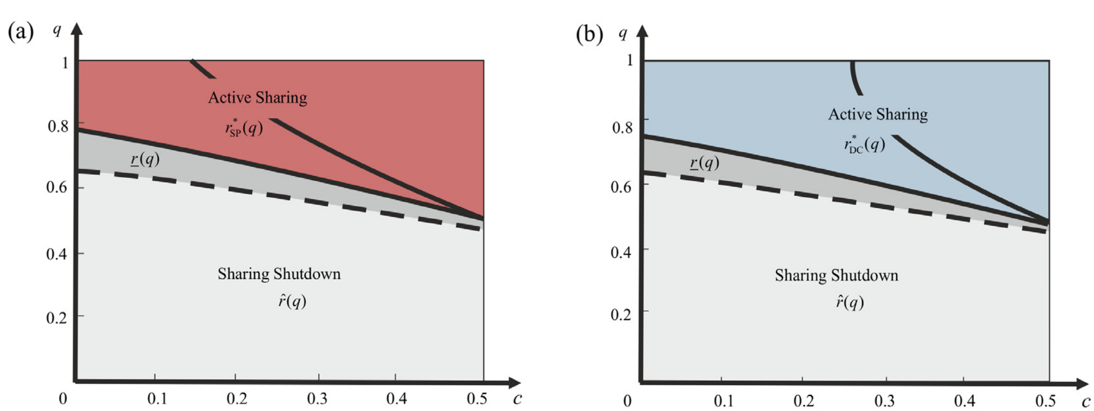

Fig. 7. Active sharing vs. sharing shutdown in the durability-cost space ($\delta = 0.6$), for both commitment regimes: (a) SP; (b) DC.

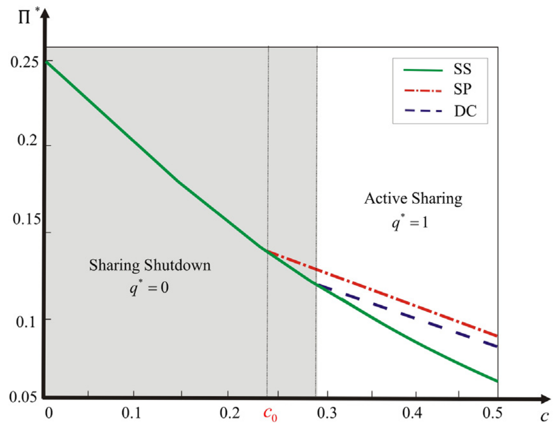

Fig. 9. Firm's profit with and without sharing markets ($\delta = 0.6$).

cost reduction with random realizations in $\{c_0 - \Delta, c_0 + \Delta\}$, with probabilities φ and $1-\varphi$, respectively, where $\varphi \in (0,1)$ and $\Delta \in (0, c_0)$. The low realization encourages the firm to deactivate the sharing market, whereas the high realization requires the firm to promote sharing. The mean-preserving lottery therefore increases the monopolist's expected payoff (by Jensen's inequality). Hence, any R&D investment of $R \le (1-\varphi)\Pi^*(c_0+\Delta) + \varphi\hat{\Pi}^*(c_0-\Delta)$ is a profitable investment for a risk-neutral firm.

4. Consumer surplus and social welfare

We now examine how social welfare depends on the presence of a sharing economy (be it active or not) and a firm that optimizes its product design in terms of retail price and durability.

Consumer surplus without sharing. At each time $t \ge 1$, the consumers' aggregate benefit is the sum of the surplus belonging to young and mature consumers currently present. At the firm's optimal no-sharing product design $(\hat{q}, \hat{r})$, total consumer surplus is

$$\widehat{\mathrm{CS}}(\hat{q},\hat{r}) = \widehat{\mathrm{CS}}_0(\hat{q},\hat{r}) + \widehat{\mathrm{CS}}_1(\hat{q},\hat{r}).$$

In the early consumption phase ($\mathcal{C}_0$), only those who purchase the product obtain a positive surplus, leading to the consumer surplus

$$\begin{aligned}\widehat{\mathrm{CS}}_0(\hat{q},\hat{r}) &= \int_0^1 \left(\int_{\hat{r}/(1+\delta\hat{q}\theta)}^1 (\nu - \hat{r}) d\nu \right) \theta \, d\theta \\ &= \frac{1}{4} + \frac{\hat{r}}{\delta\hat{q}} \left(\frac{(3/2)+\delta\hat{q}}{1+\delta\hat{q}} r - \frac{\delta\hat{q}}{2} \right) \\ &\quad - \frac{3}{2}\left(\frac{\hat{r}}{\delta\hat{q}}\right)^2 \ln(1+\delta\hat{q}). \end{aligned} \tag{40}$$

In the late consumption phase ($\mathcal{C}_1$), consumer surplus is

$$\begin{aligned}\widehat{\mathrm{CS}}_1(\hat{q},\hat{r}) &= \int_0^1 \left[\hat{q} \left(\int_{\hat{r}/(1+\delta\hat{q}\theta)}^1 \nu d\nu \right) \right. \\ &\quad \left. + (1-\hat{q}) \left(\int_{\hat{r}}^1 (\nu - \hat{r}) d\nu \right) \right] \theta^2 d\theta \\ &\quad + \int_0^1 \left(\int_{\hat{r}}^1 (\nu - \hat{r}) d\nu \right) (1-\theta)\theta \, d\theta \\ &= \frac{(3-2\hat{q})(1-\hat{r})^2}{12} \\ &\quad + \frac{\hat{q}}{6} - \left(\frac{\hat{r}}{\delta\hat{q}}\right)^2 \left(\frac{\hat{q}(2+\delta\hat{q})}{2(1+\delta\hat{q})} - \ln(1+\delta\hat{q}) \right). \end{aligned} \tag{41}$$

The first term is the surplus of those who are in a high-need state over the full lifecycle, depending on whether they buy the item once or twice (after product failure), respectively. The second term corresponds to the surplus of first-time purchasers in their late consumption phase.

Consumer surplus with active sharing market. Similarly, when the peer-to-peer economy is liquid, for a market price p and a product design (q, r) in equilibrium, total consumer surplus at each time $t \ge 1$ is, as before, the sum of the surplus gained by both overlapping generations,

$$\mathrm{CS}(p,q,r) = \mathrm{CS}_0(p,q,r) + \mathrm{CS}_1(p,q,r).$$

Consumer surplus of the young generation (in $\mathcal{C}_0$) consists of the purchasers' ownership benefits,

$$\mathrm{CS}_0(p,q,r) = \int_0^1 \left(\int_{r-\delta qp}^1 (\nu - r) d\nu \right) \theta d\theta = \frac{(1-r)^2 - (\delta qp)^2}{2}. \tag{42}$$

Consumer surplus for the mature generation (in $\mathcal{C}_1$) is

$$\begin{aligned}\mathrm{CS}_1(p,q,r) &= \int_0^1 \left[q \left(\int_p^1 \nu d\nu \right) + (1-q) \left(\int_p^1 (\nu - p) d\nu \right) \right] \theta^2 d\theta \\ &\quad + \int_0^1 \left(\int_p^1 (\nu - p) d\nu \right) (1-\theta)\theta \, d\theta \\ &\quad + qp \int_0^1 \left[\left(\int_{r-\delta qp}^p d\nu \right) + (1-\theta) \left(\int_p^1 d\nu \right) \right] \theta \, d\theta \\ &= \frac{(1-p)^2}{12} + \frac{qp}{2}(p - r + \delta qp) + \frac{q}{6}(1-p)(1+2p). \end{aligned} \tag{43}$$

The first term collects the surplus of those always in high need (including owners who do not participate in sharing and former owners who borrow after product failure); the second term corresponds to the non-owners in a high-need state who borrow the item; finally, the third term contains the surplus of those who lend out items acquired in the early consumption phase.

Fig. 10(a) shows the consumer surplus as a function of the production cost. In the absence of sharing, the consumers' aggregate surplus is decreasing in the production cost for disposable products (where $\hat{q} = 0$). This is because the higher production costs tend to increase the purchase price, so that less consumers purchase the item at a smaller surplus. As c increases further, the optimal durability becomes strictly positive, and the consumer surplus goes up as long as the durability is less than perfect ($\hat{q} < 1$). This happens because the price increase is more than offset by a higher product quality. For perfectly durable goods, the consumer surplus decreases once again in c, for the consumers are charged more but cannot obtain compensation because the quality is already maximal. With active sharing on the other hand, consumers are never worse off in a durability-commitment regime, where $(q, r) = (q^*_{\mathrm{DC}}, r^*_{\mathrm{DC}})$ and $p = p^*_{\mathrm{DC}}$. However, in a Stackelberg regime, where $(q, r) = (q^*_{\mathrm{SP}}, r^*_{\mathrm{SP}})$ and $p = p^*_{\mathrm{SP}}$, when the firm charges a high sharing premium, consumers do not necessarily benefit from the peer-to-peer economy. In fact, when the production cost is high enough (such that the optimal durability is 1), consumers are better off without sharing. Conversely, they benefit from sharing when the active secondary exchange provides sufficient incentives for the firm to provide highly durable items.

Social welfare. The social welfare in the economy is the sum of the consumer surplus and the firm's profit. Without sharing it is

$$\hat{W} = \hat{\Pi} + \widehat{\mathrm{CS}},$$

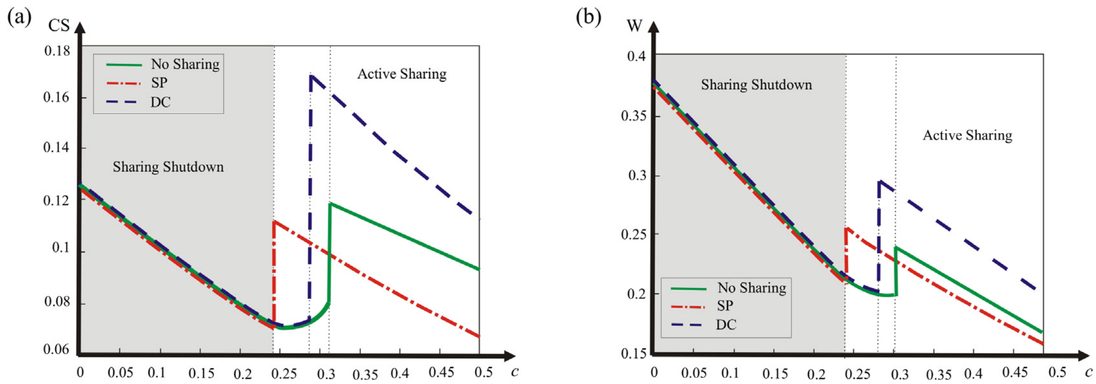

Fig. 10. (a) Consumer surplus; (b) welfare, as a function of production cost ($\delta = 0.6$).

where – in the presence of an active sharing economy – social welfare amounts to

$$W_j = \Pi_j^* + \mathrm{CS}_j,$$

where $j \in \{\mathrm{DC}, \mathrm{SP}\}$ denotes the commitment regime. Fig. 10(b) shows the social welfare in the benchmark case of no sharing, as well as in the two regimes with sharing, i.e., durability-commitment and Stackelberg play. Social welfare follows a similar pattern as consumer surplus: in DC, when the sharing market and the firm adjust their pricing decisions fast, social welfare increases in production cost. In SP, the economy benefits from active sharing for intermediate production costs only. Sharing increases social welfare when otherwise imperfect durability would be optimal; it also provides incentives for the firm to produce perfectly durable goods.

5. Conclusion

With the advent of collaborative consumption, firms face the possibility of a secondary sharing market for their products. The design of the product offering in terms of durability and retail price can serve as a strategic tool to exploit or mitigate the impact of a peer-to-peer economy on a manufacturer's bottom line. The dynamic overlapping-generations model in this paper is the first to analyze the strategic choice of product durability in the presence of sharing markets. As the results clearly indicate, a monopolist's optimal product design depends on the manufacturing cost as well as on the consumers' level of patience. Another key determinant is the firm's ability to commit to its price, its product design, or both.

Strategic shutdown of sharing. By controlling the product's durability, the firm retains the option of disabling the sharing market. Indeed, a fully disposable product cannot be offered on a secondary market, by its lack of durability violating an essential requirement for shareability. As shown in Fig. 4, for any given retail price there is a durability level below which liquidity on the sharing market disappears (because condition (L') is violated). Conversely, provided the product is *not* perfectly durable, it is also possible for the firm to shut down a sharing market using retail price as instrument: a sufficiently large price of ownership dries up the demand for ownership and thus also the sharing supply, whereas a sufficiently small retail price makes ownership so attractive that the sharing market cannot compete with new products. In general, the possibility of "sharing shutdown" is profitable for the firm when product costs are small, or when consumers are impatient, i.e., they do not care about future consumption. Small production costs imply a relatively small price, which make quantity an important driver for profit, thus creating an incentive for the firm to produce disposable products, thus draining the sharing market. The strategic shutdown of a peer-to-peer aftermarket becomes much less attractive when production cost is high, or when consumers care about the future, so that their discount factor is close to 1. In that case, it becomes extremely costly for the firm to force a sharing shutdown, while in fact there is an opportunity to gain from sharing and, in that vein, a strong incentive to provide very durable products.

Strategic coexistence with sharing. If the firm is unable or unwilling to kill an aftermarket for its goods, it can tailor its product design so as to extract a maximum of rent in the presence of a peer-to-peer economy. Which precise design is optimal depends on the company's production cost, the consumers' discount factor, as well as on the firm's commitment ability. Furthering the complexity of the firm's optimization problem is that consumers in their early consumption phase strategically anticipate their own and the firm's actions. Specifically, they perceive the product durability as a compounding multiplier of their discount factor, so that product quality is a significant driver of lifecycle benefits when consumers are patient. Interestingly, an active sharing economy has a positive effect on product durability: it encourages the manufacturer to produce more durable items. The downside for the monopolist, namely the loss of sales to consumers – whose products either failed due to planned obsolescence (limited durability) or who happened to not need the products in their early consumption phase – in their late consumption phase,[19] is compensated by a "sharing premium" added to the retail price. In other words, the presence of sharing markets allows for a markup that extracts a portion of the surplus that consumers gain by being able to trade and thus to collectively better match their needs with their access to the products. Because of its ability to extract rent via price, the firm has an incentive to provide durability, provided this can be done at sufficiently little additional cost. As noted earlier, the firm's optimal actions and the market's reactions are determined in large part by the firm's ability to commit to its actions, which in turn is at least in part enabled by the market's (*in*)ability to adjust sufficiently fast. As a rule, the larger the firm's commitment power, the more rent it can extract and the stronger it feels about enabling sharing. Fig. 9 illustrates that provided the production cost exceeds a minimal threshold, the firm's equilibrium profit increases in the power to commit to its actions. At one extreme is the "Stackelberg regime" which allows the firm to credibly preannounce both the level of durability and the retail price at the beginning of each period. At the other extreme is the "simultaneous-move play" where the market's reaction is so fast that the firm is effectively engaging in a simultaneous-move stage game. The intermediate, perhaps most realistic, "durability commitment" scenario is where the

[19] Put more simply: a higher durability decreases the number of repeat buyers.

firm is credibly locked into a certain level of product durability (because it may not be easy to change at short notice) but cannot commit on its price.[20] By imposing costs on modifying its own choices in this dynamic game, the firm may be able to – in reality – transition to a regime with a higher level of commitment and greater profitability.[21]

Limitations. The model developed in this paper is an abstraction of a more complex and interesting reality, focused on examining the firm's incentives to provide durability, as it relates to salient features of demand (discount factor) and supply (production cost), the adjustment speed of the market, and the commitment ability of the firm. To limit the complexity of our analysis, it was assumed that durability can be provided at no cost to the firm. From the differences in the firm's profits obtained by higher durability levels, it is easy to derive the company's implied willingess to pay for such quality increases. Furthermore, assuming a quadratic cost of durability would not change the qualitative nature of the incentives, since the marginal cost of durability becomes arbitrarily small for sufficiently low levels of durability. As far as the (marginal) production costs are concerned, in the model their values are meaningful *relative* to the consumers' maximum use value (which was normalized to 1). The cost thresholds implied by the model are to be seen as relative to the largest price the firm can charge so as to choke off the demand for ownership. An interesting feature we have neglected is the fact that firms can make product-line extensions to offer the same product at a lower level of (virtual) durability. As noted in the main text, BMW's DriveNow allows drivers to rent a car (effectively reducing its product to a one-period consumable) instead of buying it. While the coexistence of different car sizes and purchasing options is not a part of our model, it is interesting to observe that high-cost models such as the BMW 6 or 7 series are *not* part of DriveNow, in line with the robust model conclusion that high-cost products should be provided at full durability. More intricate, possibly nonlinear, pricing schemes are subject to future research. Lastly, we note that durability in our model can be viewed as a proxy for product quality. The main difference to the standard notion of quality is that its increase would also increase the consumers' value of using the item, which in our model remains unaffected. Durability increases the expected lifecycle utility of a product, but not its immediate need-contingent use value. Finally, in our model durability determines product survival between periods. As the sharing market clears in each period, there are no unused items and there is no intensity of usage, so that different notions of failure (e.g., through use vs. through age) are in fact equivalent. The interesting topic of endogenizing product usage in the consideration of sharing decisions and markets is left for future research.[22]

Societal impact. The presence of an active sharing market encourages firms to produce more durable products. This finding is robust across all scenarios, and can be viewed as a vector of encouragement for regulators to create favorable boundary conditions for an active sharing economy. That said, we also showed that under certain conditions, such as low production costs or myopic customers, firms would prefer to disable sharing markets and produce less durable products in order to accomplish this. Unsurprisingly the planned obsolescence of products serves exactly the purpose of disabling sharing markets and thus triggering the repurchase of items. By encouraging (or mandating) companies with low-cost products or those in high-pressure settings (e.g., when the consumers' cost of capital is high) to enable sharing markets by opting for better durability standards, the ensuing presence of a peer-to-peer economy will in turn create a self-fulfilling prophecy of providing the incentives that might have been lacking in the first place. If sharing markets exist and cannot be easily disabled by the company, then a quest for high-durability products follows, almost by necessity. We emphasize that this quest is moderated by the additional cost that may need to be incurred to achieve better durability, which – as discussed earlier – was left unmodeled in order to create a level playing field for conclusions.[23]

In terms of social welfare, it is remarkable that consumer surplus may not necessarily increase in the presence of sharing markets. This runs counter to the intuition that adding options to the consumers' choice set (in this case participation in a sharing market) necessarily increases the aggregate benefit. The reason is that much of the benefit may be captured by the firm in the form of its sharing premium. The precise conclusions depend on the commitment regime: while in a simultaneous-move setting sharing markets always augment consumer surplus and social welfare, in the Stackelberg regime, the firm may be able to syphon off consumer surplus by charging a substantial sharing premium on each unit sold. This also decreases the social welfare compared to the no-sharing case. The sharing market improves the social welfare only for a range of intermediate costs, where an active peer-to-peer market increases product durability. The results may inform policy makers about which durable good would benefit from incentives to provide sharing-friendly products. This is often the case if the unit production cost is small. Otherwise, firms do not need financial motivation, for it is naturally in their best interest to promote the sharing economy and enjoy additional profits.[24]

Acknowledgments

The authors would like to thank David Myatt, Armin Schmutzler, several anonymous referees, and participants of the 2017 Latin American Meeting of the Econometric Society in Buenos Aires, the 2017 Asian Meeting of the Econometric Society in Hong Kong, as well as the 2018 Inaugural Baltic Economics Conference and 7th Annual Lithuanian Conference on Economic Research in Vilnius for helpful comments and suggestions.

Appendix A. Analytical details

A.1. Auxiliary results

Lemma A.1. *Let $\varkappa \triangleq f/g$, where $f, g : \mathcal{X} \to \mathbb{R}_{++}$ are differentiable functions defined on the closed interval $\mathcal{X} \subset \mathbb{R}$ such that $f(x_0) = g(x_0)$ for some $x_0 \in \mathcal{X}$. If furthermore $f' \geq g'$ on $\mathcal{X}$, then $\varkappa \geq 1$ on $\mathcal{X} \cap [x_0, \infty)$.*

Proof. Since $f' \geq g'$ on $\mathcal{X}$, it is $f(x) \geq g(x) \geq f(x_0) = g(x_0) > 0$ for all $x \in \mathcal{X}$ with $x \geq x_0$, which implies that necessarily $\varkappa \geq 1$ on $\mathcal{X} \cap [x_0, \infty)$. □

[20] The intermediate durability-commitment regime allows for unforeseen contingencies where the firm for a given durability can adjust its price to cope with a given economic situation (as represented in the model by a cost and a level of patience).

[21] In future research, it would be interesting to extend our model to include the competitive interactions between several product manufacturers in the presence of a sharing market.

[22] Weber (2018) examines an agent's dynamic sharing decisions when collaborative consumption may lead to a faster degradation of a durable good.

[23] In the no-sharing case, the optimal durability problem is nonconcave and cannot be solved in closed form, even without additional durability cost.

[24] Some firms are responding proactively to collaborative consumption. For example, General Motors (GM) is working with RelayRides to make it easier for drivers to rent out their under-utilized GM vehicles. All else equal, a peer-to-peer economy never decreases the firm's incentives to provide durability.

Lemma A.2. *Let $\varkappa \triangleq f/g$, where $f, g : \mathcal{X} \to \mathbb{R}_+$ are differentiable functions defined on the closed interval $\mathcal{X} \subset \mathbb{R}$, with $g > 0$, such that $f(x_0) \geq g(x_0)$ for some $x_0 \in \mathcal{X}$. If furthermore $0 \geq f' \geq g'$ on $\mathcal{X}$, then $\varkappa' \geq 0$ on $\mathcal{X} \cap [x_0, \infty)$.*

Proof. Since $f' \geq g'$ on $\mathcal{X}$, it is $f(x) \geq g(x) \geq f(x_0) \geq g(x_0) > 0$ for all $x \in \mathcal{X}$ with $x \geq x_0$. Thus, $0 \geq f'(x)g(x) \geq g'(x)g(x) \geq g'(x)f(x)$ for all $x \in \mathcal{X} \cap [x_0, \infty)$. But this implies that $\varkappa'(x) = (f'(x)g(x) - g'(x)f(x))/g^2(x) \geq 0$ for all $x \in \mathcal{X} \cap [x_0, \infty)$. □

Lemma A.3. *For all $q \in [0, 1]$: $\rho'(q) \leq 0 \leq \rho''(q)$.*

Proof. Let $\hat{\rho}(q) \triangleq [1 - \ln(1 + \delta q)/(\delta q)]/(\delta q)$. Then, $\rho'(q) = \hat{\rho}'(q) - 1/3$. To prove that $\rho'(q) < 0$, it is sufficient to show that $\hat{\rho}(q)$ is decreasing in q. Differentiating $\hat{\rho}(q)$ yields

$$\hat{\rho}'(q) = -\frac{1}{\delta^2 q^3}\left(\delta q + \frac{\delta q}{1+\delta q} - 2\ln(1+\delta q)\right) = -\frac{f(q)}{\delta^2 q^3},$$

where we have set $f(q) \triangleq \delta q + (\delta q)/(1 + \delta q) - 2\ln(1 + \delta q)$. Note that

$$f'(q) = \delta + \frac{\delta}{(1+\delta q)^2} - \frac{2\delta}{(1+\delta q)} = \delta\frac{(1+\delta q)^2 + 1 - 2\delta(1+\delta q)}{(1+\delta q)^2}$$
$$= \delta\frac{(\delta q)^2}{(1+\delta q)^2} \geq 0, \quad q \in [0, 1].$$

This inequality, together with the initial value $f(0) = 0$, implies that $f(q) \geq 0$ on $[0, 1]$, so that $\rho'(q) \leq 0$ for all $q \in [0, 1]$, as claimed. The convexity of ρ is established by first introducing $h(q) \triangleq \delta^2 q^3$, so

$$\rho''(q) = \hat{\rho}''(q) = -\frac{f'(q)h(q) - h'(q)f(q)}{h(q)^2}, \quad q \in (0, 1].$$

Furthermore, $f'(q)h(q) - h'(q)f(q) \leq 0$ if and only if

$$\frac{(\delta q)^3/3}{(1+\delta q)^2} \leq \delta q + \frac{\delta q}{1+\delta q} - 2\ln(1+\delta q).$$

By setting $g(q) \triangleq 3(\delta^3 q^3)/(1 + \delta q)^2$ the preceding inequality is equivalent to $f(q)/g(q) \geq 1$ for all $q \in (0, 1]$. Note that $f(0) = g(0) = 0$, and

$$f'(q) = \delta\frac{(\delta q)^2}{(1+\delta q)^2} \geq \frac{\delta}{3}\frac{(3+\delta q)(\delta q)^2}{(1+\delta q)^3} = g'(q), \quad q \in (0, 1].$$

By Lemma A.1. (including a continuous completion at $q = 0^+$), it follows that $f/g \geq 1$ a.e. on the interval $[0, 1]$, which implies the monotonicity of ρ' and thus the convexity of ρ on $[0, 1]$, as claimed. □

A.2. Proof of the main results

Proof of Proposition 1. For $r \in [0, 1]$, the no-sharing demand by consumers in the early consumption phase is[25]

$$\hat{\Omega}_0(r) = \frac{1-r}{2} + \int_{r/(1+\delta q)}^{r}\left(\int_{(r-\nu)/(\delta q \nu)}^{1} \theta\, d\theta\right) d\nu$$
$$= \frac{1-r}{2} + \frac{r}{2\delta q}\int_0^1 \frac{1-\xi^2}{((1/(\delta q)) + \xi)^2}\, d\xi,$$

so that

$$\hat{\Omega}_0(r) = \frac{1}{2} - \frac{r}{\delta q}\left(1 - \frac{\ln(1+\delta q)}{\delta q}\right), \quad r \in [0, 1]. \tag{44}$$

[25] The inner integral evaluates to $\int_{(r-\nu)/(\delta q \nu)}^{1} \theta\, d\theta = \frac{1}{2} - \frac{((r/\nu)-1)^2}{2(\delta q)^2}$.

For $r \in [1, 1 + \delta q]$, one obtains – by similar calculations – that

$$\hat{\Omega}_0(r) = \int_{r/(1+\hat{\delta})}^{1}\left(\int_{(r-\nu)/(\hat{\delta}\nu)}^{1} \theta\, d\theta\right) d\nu$$
$$= \frac{1}{2}\left[\left(1 - \frac{1}{\hat{\delta}^2}\right)\left(1 - \frac{r}{1+\delta}\right) + \frac{r^2}{\hat{\delta}^2}\left(1 - \frac{1+\delta}{r}\right)\right]$$
$$- \frac{r}{\hat{\delta}^2}\ln\left(\frac{r}{1+\hat{\delta}}\right). \tag{45}$$

Lastly, for $r \geq 1 + \delta q$ it is $\hat{\Omega}_0(r) = 0$. Combining Eqs. (44) and (45) yields Eq. (6). On the other hand, the ownership demand by consumers in the late consumption phase is

$$\hat{\Omega}_1(q, r) = (1 - r)\left(\int_0^1 (1-\theta)\theta\, d\theta + (1-q)\int_0^1 \theta^2\, d\theta\right)$$
$$= (1 - r)\left(\frac{1}{2} - \frac{q}{3}\right), \quad r \in [0, 1],$$

and $\hat{\Omega}_1(q, r) = 0$ for $r \geq 1$, corresponding to Eq. (7). The aggregate demand for ownership is $\hat{\Omega} = \hat{\Omega}_0 + \hat{\Omega}_1$. □

Proof of Corollary 1. It is straightforward to show that the partial derivative of Ω_1 with respect to q is nonpositive,

$$\frac{\partial\hat{\Omega}_1}{\partial q} = -\max\left\{0, \frac{1-r}{3}\right\} \leq 0.$$

We now prove that the sales in the early consumption phase is increasing in q. Note that by Proposition 1, for all $r \in [0, 1]$, the demand for ownership by the young generation (in $\mathcal{C}_0$) is

$$\hat{\Omega}_0(q, r) = \frac{1}{2} - r\hat{\rho}(q),$$

where $\hat{\rho}(q) = (1 - \ln(1 + \delta q)/(\delta q))\,/(\delta q)$. Partially differentiating $\hat{\Omega}_0$ with respect to q yields

$$\frac{\partial\hat{\Omega}_0}{\partial q} = \frac{\partial\hat{\rho}}{\partial q} \leq 0,$$

thus implying the result by virtue of Lemma A.3. □

Proof of Lemma 1. Differentiating $\hat{r}$ in Eq. (9) with respect to δ and taking into account Lemma A.3, one obtains

$$\frac{\partial\hat{r}}{\partial\delta} = -\left(1 - \frac{q}{3}\right)\frac{\rho'(q)}{2\rho^2(q)} \geq 0, \quad \delta, q \in (0, 1).$$

Similarly, differentiating $\hat{r}$ with respect to q yields

$$\hat{r}'(q) = -\frac{1}{2\rho^2}\left(\left(1 - \frac{q}{3}\right)\rho'(q) + \frac{\rho}{3}\right) = \frac{\partial\hat{r}}{\partial\delta} - \frac{1}{6\rho} < \frac{\partial\hat{r}}{\partial\delta}.$$

Using the abbreviation $f(q) \triangleq 1 - q/3$, it is $f(0) = \rho(0) = 1$, and

$$\hat{r}'(q) = \frac{1}{2}\frac{d}{dq}\left(\frac{k(q)}{\rho(q)}\right).$$

Note that $f(q)$ and $\rho(q)$ are positive for all $q \in [0, 1]$. Furthermore,

$$f' = \frac{1}{3} \geq \hat{\rho}' - \frac{1}{3} = \rho',$$

where $\hat{\rho}(q) = [1 - \ln(1 + \delta q)/(\delta q)]/(\delta q)$ and $\hat{\rho}'(q) \leq 0$ by Lemma A.3. Thus, applying Lemma A.2 to the fraction $\varkappa \triangleq f/\rho$ yields that $\varkappa$ is nondecreasing, whence, by construction, $\hat{r}' \geq 0$, which completes our proof. □

Proof of Proposition 2. It is $\rho'(0) = -(1+\delta)/3$. Thus, differentiating the profit function in Eq. (10) yields $d\hat{\Pi}(q, \hat{r}(q))/dq\big|_{q=0} \geq 0$, if and only if

$$c \geq \frac{1-\delta}{1+\delta} \triangleq \check{c}. \tag{46}$$

(i) Let $c \in [0, \check{c}]$, so that the no-sharing profit has a nonpositive slope at $q = 0$. Note that $\hat{\Pi}'(q)$ is unimodal with the local maximum always preceding the local minimum (if it exists in the domain). As $\hat{\Pi}'(q) \leq 0$ for all $c \in [0, \check{c}]$, in order to find the global maximum it is enough to consider the profit at the boundary, i.e., where $q \in \{0, 1\}$. Using l'Hôpital's rule it is $\lim_{q\to 0^+} \rho(q) = 1$, so that the profit at zero durability becomes

$$\lim_{q\to 0^+} \hat{\Pi}(\hat{r}(q), q) = \frac{(1-c)^2}{4}.$$

On the other hand, perfect durability generates a profit equal to

$$\hat{\Pi}(\hat{r}(1), 1) = \frac{(2 - 3c\rho(1))^2}{36\,\rho(1)}.$$

Comparing the profit at the boundary points yields that $\hat{\Pi}(1, \hat{r}(1)) \leq \hat{\Pi}(0, \hat{r}(0))$ if and only if

$$c < \frac{3\sqrt{\rho(1)} - 2}{3\sqrt{\rho(1)} - 3\rho(1)} \leq \check{c},$$

which is satisfied for all $c \in [0, \check{c}]$. This completes the proof of part (i).

(ii) For any $c \geq \check{c}$, the corner solution $\hat{q} = 0$ is suboptimal. The other corner solution, $\hat{q} = 1$, is also suboptimal, as long as

$$\hat{\Pi}(1, \hat{r}(1)) < \hat{\Pi}(\hat{q}, \hat{r}(\hat{q})),$$

where $\hat{q}$ is the interior maximizer implicitly defined in Eq. (12) of Corollary 3. Substituting Eq. (12) in Eq. (10) and rearranging the terms provides us with the global optimality of the interior maximizer as long as

$$0 \leq c \leq \frac{2 - \sqrt{8\rho(1)\rho(\hat{q})/\rho'(\hat{q})}}{3\rho(1)} \triangleq \hat{c} \in (\check{c}, \bar{c}), \tag{47}$$

as claimed.

(iii) As already established in our proof of (ii), for all $c \in (\hat{c}, \bar{c}]$ one obtains $\hat{\Pi}(\hat{q}, \hat{r}(\hat{q})) < \hat{\Pi}(1, \hat{r}(1))$, so $\hat{q} = 1$.

Parts (i)–(iii) together establish the result. □

Analytical Details for Footnote 2: see the proof of Proposition 2(i). □

Proof of Corollary 2. Part (i) follows from solving the inequality in Eq. (46) in the proof of Proposition 2 for the discount factor: $\delta \geq (1 - c)/(1 + c) \triangleq \check{\delta}$. Parts (ii) and (iii) revolve around the discount-factor threshold $\hat{\delta} \in (\check{\delta}, 1]$ which can be obtained, for a given c, by (numerically) solving the second inequality in Eq. (47). □

Proof of Corollary 3. An interior maximizer $\hat{q} \in (0, 1)$ of the firm's profit $\hat{\Pi}(q, \hat{r}(q))$ in Eq. (10) satisfies the Fermat condition

$$\begin{aligned}&\frac{d\hat{\Pi}(q, \hat{r}(q))}{dq}\\ &= \frac{2(1 - c\rho(q) - q/3)(-c\rho'(q) - 1/3)\rho(q) - (1 - c\rho(q) - q/3)^2\rho'(q)}{2\rho^2(q)}\\ &= 0,\end{aligned}$$

which – provided a positive optimal profit – is equivalent to Eq. (12). □

Proof of Proposition 3. We first consider the dependence of the optimal durability $\hat{q}$ on the production cost c. To establish the monotone dependence, it is sufficient to show that the firm's objective function in Eq. (10) has increasing differences in (q, c). The corresponding cross-partial derivative is

$$\frac{\partial^2 \hat{\Pi}(q, \hat{r}(q))}{\partial q \partial c} = c\rho'(q) + \frac{1}{3} \geq c\rho'(0) + \frac{1}{3} = \frac{1 - (1+\delta)c}{3}.$$

The last expression on the right-hand side is positive if $c > \check{c}$, which by Corollary 2 is a necessary and sufficient condition for the optimal durability to be nonzero. Since $1/(1 + \delta) > \check{c}$ (see Eq. (46)), this guarantees that $\hat{q}$ is (weakly) increasing in c as claimed. — Next, we consider the monotonicity of $\hat{q}$ in δ, restricting attention to an interior solution ($0 < \hat{q} < 1$) which satisfies

$$\left(1 + c\rho(\hat{q}) - \frac{q}{3}\right)(-\rho'(\hat{q})) = \frac{2\rho(\hat{q})}{3}, \tag{48}$$

as indicated in Corollary 3. Differentiating implicitly with respect to δ yields

$$\frac{\rho'(\hat{q})}{3} = \left[\left(\frac{2}{3} - c\right)\rho'(\hat{q}) + \left(1 + c\rho(\hat{q}) - \frac{q}{3}\right)\rho''(\hat{q})\right]\frac{\partial \hat{q}}{\partial \delta},$$

so that $\partial\hat{q}/\partial\delta \geq 0$ if and only if

$$\left(1 + c\rho(\hat{q}) - \frac{\hat{q}}{3}\right)\rho''(\hat{q}) \leq \left(\frac{2}{3} - c\right)(-\rho'(\hat{q})).$$

Using again the optimality condition (48), the preceding inequality is equivalent to

$$\left.\frac{\rho\rho''}{(\rho')^2}\right|_{q=\hat{q}} \leq 1 - \frac{3c}{2},$$

which holds for all $(\delta, c) \in (0, 1) \times (0, \bar{c})$. Note also that the transition from zero durability to intermediate durability takes place at a lower cost threshold $\check{c}$ as δ goes up. The same holds for the cost threshold $\hat{c}$, which is increasing in δ. □

Proof of Lemma 2. Given the optimal profit $\hat{\Pi}^* = \hat{\Pi}(\hat{q}, \hat{r}(\hat{q}))$ in Eq. (10), for intermediate durability levels the claims obtain immediately by using the envelope theorem:

$$\frac{\partial \hat{\Pi}^*}{\partial c} = -\left(1 - c\rho(\hat{q}) - \frac{\hat{q}}{3}\right) < 0 < -\left(c + \hat{\Pi}^*\right)\frac{\hat{q}\hat{\rho}'(\hat{q})}{\delta\rho(\hat{q})} = \frac{\partial \hat{\Pi}^*}{\partial \delta},$$

for all $(\delta, c) \in (0, 1) \times (0, \bar{c})$. In the boundary cases where the optimal durability is either 0 or 1, the conclusion of the envelope theorem continues to hold, given that $\partial\hat{q}/\partial(\delta, c) = 0$. □

Proof of Proposition 4. For any given $r \in (\underline{r}(q), \bar{r}(q)]$, the equilibrium sharing price solves the market-clearing condition by equating the supply and demand obtained in Eqs. (22)–(23). That is,

$$\frac{q}{2}\left(\frac{1}{3} + \left(\frac{2}{3} + \delta q\right)p - r\right) = \frac{1-p}{2}\left(1 - \frac{2q}{3}\right),$$

which results in the best market response

$$p(q, r) = \frac{1 - q + qr}{1 + \delta q^2}, \tag{49}$$

as claimed. The sharing price from Eq. (49) in the sharing demand of Eq. (23) yields the transaction volume:

$$Q(q, r) = D(q, p(q, r)) = \frac{q}{2}\left(1 - \frac{2q}{3}\right)\left(\frac{1 + \delta q - r}{1 + \delta q^2}\right).$$

Substituting $\bar{r}(q) = 1 + \delta q$ in the preceding equation completes the proof. □

Proof of Lemma 3. (i) Partially differentiating the clearing price in Proposition 4 with respect to q yields

$$\frac{\partial p}{\partial q} = \frac{1(1-r)(1+\delta q^2) - 2\delta q(rq + 1 - q)}{(1 + \delta q^2)^2} \leq 0.$$

Similarly, partial differentiation of the sharing price with respect to r yields

$$\frac{\partial p}{\partial r} = \frac{q}{1+\delta q^2} \geq 0.$$

(ii) For the transaction volume in Eq. (26), partial differentiation with respect to r yields

$$\frac{\partial Q}{\partial r} = -\frac{q}{2(1+\delta q^2)}\left(1-\frac{2q}{3}\right) \leq 0.$$

With respect to q, partial differentiation yields

$$\frac{\partial Q}{\partial q} = \left(1-\frac{4q}{3}\right)\left(\frac{1+\delta q - r}{1+\delta q^2}\right) + \delta q\left(1-\frac{2q}{3}\right)\left(\frac{1-\delta q^2 - 2q(1-r)}{(1+\delta q^2)^2}\right).$$

One can verify that for all $\delta \in [0,1]$, there exists $\bar{q} \in [0,1]$, below which $\partial Q/\partial q > 0$, and above which $\partial Q/\partial q < 0$. This completes the proof. $\square$

Proof of Lemma 4. The result follows directly from Eq. (21) and the liquidity condition (L'). $\square$

Proof of Lemma 5. Differentiating $r(q) - \hat{r}(q)$ with respect to q yields

$$\frac{d}{dq}[r(q) - \hat{r}(q)] = \frac{1}{2}\frac{d}{dq}\left[1+\delta q - \frac{1-\frac{q}{3}}{\rho(q)}\right] = \frac{1}{2}\left(\delta + \frac{1}{3\rho(q)} + \left(1-\frac{q}{3}\right)\frac{\rho'(q)}{\rho^2(q)}\right).$$

Let $\phi(q) \triangleq 1/(3\rho(q)) + (1-q/3)(\rho'(q)/\rho^2(q))$. It is easy to show that $\phi(q)$ is positive as $q \to 0^+$. That is,

$$\lim_{q\to 0^+}\left(\delta + \frac{1}{3\rho} + \left(1-\frac{q}{3}\right)\frac{\rho'(q)}{\rho^2(q)}\right) = \delta + \frac{1}{3} - \left(\frac{\delta}{3}+\frac{1}{3}\right) > 0.$$

We complete the proof by showing that $\phi'(q) > 0$. Differentiating with respect to q, together with Lemma A.3, gives

$$\phi'(q) = -\frac{\rho'(q)}{3\rho^2} + \rho''(q)\left(1-\frac{q}{3}\right) - \frac{\rho'(q)}{3} > 0,$$

as claimed. $\square$

Proof of Proposition 5. Provided the firm's profit is positive, the derivative $d\Pi_{SP}(q, r(q))/dq$ is positive (i.e., warrants increasing the durability level) if and only if $(1+\delta q - c)(1+\delta q^2)\delta - (1+\delta q - c)^2\delta q > 0$. But the last inequality is automatically satisfied for all $q \in (0,1)$, which therefore implies that with sharing the optimal level of durability is $q^* = 1$. $\square$

Proof of Lemma 6. Given that $p < r$, the firm's profit is $\Pi(q, r; p) = (1 - r + \delta pq)(r - c)/2$. The first-order necessary optimality condition with respect to the purchase price, combined with the nonnegativity of the profit function, yields the monopolist's best response,

$$r(q;p) = \max\left\{c,\ 1+\delta pq,\ \frac{1+c+\delta pq}{2}\right\};$$

equivalently, for all $p \in [0, (1+c)/(2-\delta q))$:

$$r(q;p) = \begin{cases} c, & \text{if } p \in [0, c/(1+\delta q)), \\ p(1+\delta q), & \text{if } p \in [c/(1+\delta q), (1+c)/(2+\delta q)], \\ (1+c+\delta pq)/2, & \text{if } p \in ((1+c)/(2+\delta q), \\ & \quad (1+c)/(2-\delta q)). \end{cases} \tag{50}$$

Note that $p \geq (1+c)/(2-\delta q)$ results in $p > r(q;p)$, which violates the liquidity condition (L'). The firm is able to deactivate the sharing market and its best response is to charge the optimal price $\hat{r}(q)$, as obtained in Eq. (9). – We now examine the optimal durability for a given $p \in [0,1]$. for all $c < (1+c)/(2-\delta q)$ such that the $p < r$. The first-order necessary optimality condition with respect to q yields

$$\frac{\partial \Pi(q, r; p)}{\partial q} = \delta p(r - c) > 0,$$

which results in the optimality of the corner solution $q(p) = 1$, as claimed. For all $c \geq (1+c)/(2-\delta q)$, the liquidity condition (L') is not satisfied and the sharing market is not active. Hence, the results in Proposition 2 are applicable. Plugging the optimal $q(p)$ into Eq. (50) completes the proof. $\square$

Proof of Proposition 6. The simultaneous-move stage-game Nash equilibrium in each time period is obtained at the intersection of the best-response functions indicated by Eqs. (25) and (35)–(36), respectively. That is,

$$p^*_{SMP} = \frac{r^*_{SMP}}{1+\delta}, \tag{51}$$

and

$$r^*_{SMP} = (1+c+p^*_{SMP})/2. \tag{52}$$

Solving the system of Eqs. (51)–(52) yields the desired results. Note that for the equilibrium sharing price p^*_{SMP}, the liquidity condition (L') is satisfied regardless of the value of c. This completes the proof. $\square$

Proof of Lemma 7. The proof is similar to the proof of Lemma 6, for a fixed $q \in [0,1]$. $\square$

Proof of Proposition 7. The Nash equilibrium is obtained as the intersection of the two best-response functions in Eqs. (25) and (37). That is,

$$p^*(q) = \frac{2-q+qc}{2+\delta q^2},$$

and

$$r^*(q) = \frac{1}{2}\left(1+c+\delta q\left(\frac{2-q+qc}{2+\delta q^2}\right)\right).$$

Note that the liquidity condition (L') is satisfied if

$$c > \frac{1-q-\delta q}{1-q+\delta q^2} \triangleq \underline{c}(q),$$

as claimed. $\square$

Proof of Proposition 8. Note first that $\Pi_{DC}(q) = (\Phi(q))^2/2$ with $\Phi(q) \triangleq (1-c+\delta q)/(2+\delta q^2)$ for $q \in [0,1]$. A maximizer of $\Phi(q)$ also maximizes $\Pi_{DC}(q)$. The first-order necessary optimality condition to the optimal durability problem solves $\max_{q\in[0,1]} \Pi_{DC}(q)$ is $(2+\delta q^2) - 2q(1-c+\delta q) = 0$, with unique nonnegative solution

$$q^*_{DC} = \frac{\sqrt{(1-c)^2+2\delta} - (1-c)}{\delta} \in [0,1], \quad c \in [0, \delta/2].$$

Perfect durability is optimal ($q^*_{DC} = 1$) if and only if $\delta + (1-c) \geq \sqrt{(1-c)^2+2\delta}$, which is equivalent to $c \geq \delta/2$. By Proposition 7, the firm's equilibrium retail price is $r^*_{DC} = r_{DC}(q^*_{DC})$, which completes our proof. $\square$

Proof of Proposition 9. By Eq. (8) and Corollary 2, for all $c \in [0, \check{c}]$, the optimal profit is $\hat{\Pi}^*(c) = (1-c)^2/4$. On the other hand, by Eq. (34) the optimal profit in the presence of an active sharing market in the Stackelberg regime is

$$\Pi^*(c) = \frac{(1+\delta-c)^2}{8(1+\delta)}.$$

Note that at $\hat{\Pi}^*(0) \leq \Pi^*(0)$, and both functions are strictly decreasing in c. Hence there exists at most one intersection c_0, below which for all $0 \leq c \leq c_0$, the monopolist prefers no sharing, i.e., $\hat{\Pi}^*(c) \leq \Pi^*(c)$ for all $c \in [0, c_0]$, where

$$c_0 \triangleq 1 - \frac{\delta}{\sqrt{2(1+\delta)} - 1}.$$

One can verify that $c_0 \leq \check{c}$ as follows:

$$1 - \frac{\delta}{\sqrt{2(1+\delta)}-1} \leq \frac{1-\delta}{1+\delta} \Leftrightarrow \frac{1}{\sqrt{2(1+\delta)}-1} \geq \frac{2}{1+\delta}$$
$$\Leftrightarrow 9 + \delta^2 + 6\delta \geq 8 + 8\delta \Leftrightarrow (1-\delta)^2 \geq 0,$$

which completes the proof. □

Appendix B. Notation

Symbol	Description	Range
c	Unit production cost	$[0, \bar{c}]$
$\hat{c}/\check{c}$	Cost threshold for perfect durability/disposability (without sharing)	$[0, \bar{c}]$
c _	Cost threshold for existence of Nash Equilibrium ($j = \text{DC}$)	$[0, \bar{c}]$
c_0	Cost threshold for perfect durability versus full disposability ($j = \text{SP}$)	$[0, \bar{c}]$
$\mathcal{C}_0/\mathcal{C}_1$	Early/late consumption phases	N/A
$\text{CS}/\widehat{\text{CS}}$	Consumer surplus (with/without sharing)	$\mathbb{R}$
D	Demand on the sharing market	$[0, 2]$
i	Item availability	$\mathcal{I} = \{0, 1\}$
j	Commitment regime	$\{\text{DC}, \text{SMP}, \text{SP}\}$
n	Total per-period sales (items in circulation before potential failure)	$[0, 2]$
p	Sharing price (clearing price on the peer-to-peer market)	$[0, 1]$
q	Product durability	$[0, 1]$
Q	Transaction volume on the sharing market	$[0, 1]$
r	Retail price	$\mathbb{R}_+$
$\underline{r}/\bar{r}$	Lower/upper bound on the retail price in the liquidity condition (L')	$\mathbb{R}_+$
s	Agent's need state	$\mathcal{S} = \{L, H\}$
S	Supply on the sharing market	$[0, 2]$
t	Time	$\mathbb{N}$
T_s	Agent's total expected payoff (with sharing)	$\mathbb{R}$
u_0/u_1	Agent's (indirect) utility without/with the item	$\mathbb{R}$
U_s	Non-owner's payoff in $\mathcal{C}_1$ in need state s (with sharing)	$\mathbb{R}$
$\hat{U}_s$	Non-owner's payoff in $\mathcal{C}_1$ in need state s (without sharing)	$\mathbb{R}$
$\bar{U}$	Non-owner's *ex ante* expected payoff in $\mathcal{C}_1$ (with sharing)	$\mathbb{R}$
V_s	Owner's payoff in $\mathcal{C}_1$ (with sharing)	$\mathbb{R}$
$\bar{V}$	Owner's *ex ante* expected payoff in $\mathcal{C}_1$ (with sharing)	$\mathbb{R}$
$\hat{V}_s$	Owner's payoff in $\mathcal{C}_1$ (without sharing)	$\mathbb{R}$
$W/\hat{W}$	Per-period social welfare (with/without sharing)	$\mathbb{R}$
y	Agent's income	$\mathbb{R}_+$
δ	Discount factor	$(0, 1]$
$\hat{\delta}/\check{\delta}$	Discount-factor threshold for optimality of perfect durability/disposability	$[0, 1]$
θ	Agent's likelihood of need	$\Theta = [0, 1]$
θ_0	Purchase threshold in the early consumption phase (without sharing)	$[0, 1]$
ν	Agent's consumption value contingent on high need	$\mathcal{V} = [0, 1]$
π	Sharing premium as a function of the durability	$\mathbb{R}_+$
$\Pi/\hat{\Pi}$	Firm's per-period profit (with/without sharing)	$\mathbb{R}$
$\Omega/\hat{\Omega}$	Firm's per-period demand (with/without sharing)	$[0, 2]$

References

Arrow, K.J., 1953. Le rôle des valeurs boursières pour la répartition la meilleure des risques. Économetrie, Centre Natl. Rech. Sci. 11, 41–47, [Reprinted: The Role of Securities in the Optimal Allocation of Risk Bearing, Review of Economic Studies, 31(2) (1964) 91–96].

Bardhi, F., Eckhardt, G.M., 2012. Access-based consumption: the case of car sharing. J. Consum. Res. 39 (4), 881–898.

Belk, R., 2007. Why not share rather than own? Ann. Am. Acad. Polit. Soc. Sci. 611, 126–140.

Belk, R., 2010. Sharing. J. Consum. Res. 36 (5), 715–734.

Benjaafar, S., Kong, G.C., Li, X., Courcoubetis, C., 2015. Peer-To-Peer Product Sharing: Implications for Ownership, Usage and Social Welfare in the Sharing Economy, Working Paper. University of Minnesota, Minneapolis, MN.

Benkler, Y., 2004. Sharing nicely: On shareable goods and the emergence of sharing as a modality of economic production. Yale Law J. 114 (2), 273–358.

Bulow, J., 1986. An economic theory of planned obsolescence. Q. J. Econ. 101 (4), 729–749.

Chamberlin, E.H., 1953. The product as an economic variable. Q. J. Econ. 67 (1), 1–29.

Coase, R.H., 1972. Durability and monopoly. J. Law Econ. 15 (1), 143–149.

Deneckere, R.J., McAfee, R.P., 2005. Damaged goods. J. Econ. Manag. Strategy 5 (2), 149–174.

Fethke, G., Jagannathan, R., 2002. Monopoly with endogenous durability. J. Econom. Dynam. Control 26 (6), 1009–1027.

Fraiberger, S.P., Sundararajan, A., 2015. Peer-To-Peer Rental Markets in the Sharing Economy, Working Paper. New York University, New York, NY.

Fudenberg, D., Tirole, J., 1998. Upgrades, tradeins, and buybacks. Rand J. Econ. 29 (2), 235–258.

Goering, G.E., 2007. Durability choice with differentiated products. Res. Econ. 61 (1), 105–112.

Gul, F., Sonnenschein, H., Wilson, R.B., 1986. Foundations of dynamic monopoly and the coase conjecture. J. Econom. Theory 39 (1), 155–190.

Hawkins, A.J., 2016. GM's New Car-Sharing Startup, Maven, Is Launching in Three more US Cities. The Verge, Vox Media, Washington, DC, May 19. Available at http://www.theverge.com/2016/5/19/11712764/maven-gm-care-share-boston-chicago-dc-launch-announced.

Horton, J.J., Zeckhauser, R.J., 2016. Owning, Using and Renting: Some Simple Economics of the 'Sharing Economy', NBER Working Paper, No. 22029.

Jiang, B., Tian, L., 2018. Collaborative consumption: strategic and economic implications of product sharing. Manage. Sci. 64 (3), 1171–1188.

Lamberton, C.P., Rose, R.L., 2012. When is ours better than mine? A framework for understanding and altering participation in commercial sharing systems. J. Mark. 76 (4), 109–125.

MacMillan, D., 2016. Toyota and uber reach investment, lease partnership. Wall Street J. Available at http://www.wsj.com/articles/toyota-and-uber-reach-investment-lease-partnership-1464122403.

Martin, D.D., 1962. Monopoly power and the durability of durable goods. South. Econ. J. 28 (3), 271–277.

Orbach, B., 2004. The durapolist puzzle: Monopoly power in durable-goods markets. Yale J. Regul. 21 (1), 67–118.

Ozanne, L.K., Ballantine, P.W., 2010. Sharing as a form of anti-consumption? An examination of toy library users. J. Consum. Behav. 9 (6), 485–498.

Razeghian, M., Weber, T.A., 2015. To Share Or Not to Share: Adjustment Dynamics in Sharing Markets, Working Paper. Swiss Federal Institute of Technology (EPFL, Lausanne, Switzerland.

Selten, R., 1965. Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit. Z. Gesamte Staatswissenschaft (J. Inst. Theoret. Econ.) 121 (2/4), 301–324/667–689.

Stokey, N.L., 1981. Rational expectations and durable goods pricing. Bell J. Econ. 12 (1), 112–128.

Swan, P.L., 1972. Optimum durability, second-hand markets, and planned obsolescence. J. Polit. Econ. 80 (3), 575–585.

Swan, P.L., 2006. How well has the market structure irrelevance principle for durability, product quality and innovation withstood the test of time? ANU inaugural trevor swan distinguished lecture, May 23. Available at https://crawford.anu.edu.au/pdf/seminars/Inaugural_lecture.pdf.

Waldman, M., 2003. Durable goods theory for real world markets. J. Econ. Perspect. 17 (1), 131–154.

Weber, T.A., 2014. Intermediation in a sharing economy: Insurance, moral hazard, and rent extraction. J. Manage. Inf. Syst. 31 (3), 35–71.

Weber, T.A., 2016. Product pricing in a peer-to-peer economy. J. Manage. Inf. Syst. 33 (2), 573–596.

Weber, T.A., 2017. Controlling and pricing shareability. In: Proceedings of the 50th Annual Hawaii International Conference on System Sciences (HICSS), pp. 5572–5581.

Weber, T.A., 2018. The dynamics of asset sharing and private use. In: Proceedings of the 51st Annual Hawaii International Conference on System Sciences (HICSS), pp. 5205–5211.